MATH 1402A **Term test #2** **17 October 2013**

Maximum score: **35** marks

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Non-programmable calculators are permitted. This test is closed book.

Supply your answers on this sheet, but TA's have extra paper if you need it.

PLEASE PRINT _____ _____

First name Last name Student number

Please show your work where appropriate!

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1. [5] You are asked to **determine the (*K*, *L*) settings** that will minimize the total cost *TC*, with the constraint that the production levels *Q* must remain at a constant value of $Q_0 = 32$. The model for total cost is $TC = 9L + 72K$ and the production function uses the standard Cobb-Douglas model, with $A = 1, \alpha = \frac{1}{4}$, and $\beta = \frac{1}{2}$. **USE ANY METHOD**

2. [4] Let $y^3 + 3xy^2 + 3x^2y + x^3 - 8y + 4x = 0$. Determine *dy*/*dx* at the point (1, 1).

3. [4] Let $z = f(x, y) = x^3 - x^2y + xy^2 - y^3$ and $y = g(x) = 3x - 8$. Find *dz*/*dx* , the total derivative, **using any method**. Express the answer as a function of *x* only. Do not DEVELOP the expression.

4. Let $f(w, x, y) = \sqrt{w^3 - x^2 - y}$ be a 3-variable function that needs to be optimized (i.e. you need to determine a max./min.). Suppose also that the independent variables are linked together by the equation $wxy^3 + w^3xy = wx^3y$.

a. [1] WRITE the Langrangian Λ for the problem (without solving).

b. [3] Write down the equation $\Lambda_w = \dfrac{\partial \Lambda}{\partial w} = 0$

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5. [6] Let $z = f(x, y) = \sqrt{2x^2 + xy + y^2}$. Use the differential dz to estimate the change in z (i.e. Δz) when moving from $(x, y) = (2, -2)$ to $(1.9, -1.9)$. Compare the result with the actual value of Δz.

6. [5] Let $z = f(x, y) = 4x^3 + 3y^2 - 12xy + 144x - 120y + 16$. **Determine the (x, y) coordinates of the critical point(s)**. *Do NOT determine if these are a max, min, or saddle*.

7. [5] Let $z = f(x, y) = 2x^3 - y^3 - 6x + 12y + 18$. Determine the **nature** (i.e. min., max. or saddle) of the critical point $(-1, 2)$. Make sure to do all the work necessary to justify your answer.

8. [2] Let $Q = f(K, L) = 4K^{1/4} L^{1/2}$ be the rule for a production function, where ***K*** corresponds to ***input capital***, ***L*** corresponds to ***input labour*** and ***Q*** corresponds to ***output production.***
Assuming $Q = Q_0 = 8$:

Determine *MRS* using ANY METHOD