Assignment 1 - Solutions

MAT 1348 B : Total: 25 points

① Propositional variables:

5 points l: "The file system is locked"

f: "The system is functioning normally"

b: "Messages are sent to the message buffer"

q: "Messages are being queued"

From the text, compound propositions are:

1 each (a) $(l \vee \neg f) \rightarrow \neg b$

1p (b) $(\neg l \wedge q) \leftrightarrow f$

1p (c) $\neg q \rightarrow l$

1p (d) $\neg b \rightarrow \neg f$

1p (e) $f \rightarrow q$

② Truth table:

4 points

1p 1p 1p

p	q	r	$p \rightarrow \neg(q \wedge r)$	$(\neg p \leftrightarrow r) \vee q$	$(p \rightarrow \neg q)$	$\wedge$	$\neg(p \vee r)$
T	T	T	F	T	F	F	F
T	T	F	T	T	F	F	F
T	F	T	T	F	T	F	F
T	F	F	T	T	T	F	F
F	T	T	T	T	T	F	F
F	T	F	T	T	T	T	T
F	F	T	T	T	T	F	F
F	F	F	T	F	T	T	T

When p: F, q: T and r: F, all 3 compound propositions are T. Hence, the set is consistent.

1p conclusion

(3) 6 points

(a) $A = (p \wedge \neg q) \vee (\neg p \wedge \neg q)$

1p 1p $B = (p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

(b) $A = (p \wedge \neg q) \vee (\neg p \wedge \neg q)$

1p $\equiv \neg(\neg p \vee q) \vee \neg(p \vee q)$

1p $B = (p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

$\equiv \neg(\neg p \vee \neg q) \vee \neg(p \vee \neg q) \vee \neg(p \vee q)$

(c) Use: $x \to y \equiv \neg x \vee y$

1p $A \equiv \neg(\neg p \vee q) \vee \neg(p \vee q)$

$\equiv \neg(p \to q) \vee \neg(\neg p \to q)$

$\equiv (p \to q) \to \neg(\neg p \to q)$

1p $B = \neg(\neg p \vee \neg q) \vee \neg(p \vee \neg q) \vee \neg(p \vee q)$

$\equiv \neg(p \to \neg q) \vee \neg(q \to p) \vee \neg(\neg p \to q)$

$\equiv \big((p \to \neg q) \to \neg(\neg p \to \neg q)\big) \vee \neg(\neg p \to q)$

$\equiv (\neg p \to q) \to \big((p \to \neg q) \to \neg(\neg p \to \neg q)\big)$

Note: The solution presented is not unique. Other equivalent forms are allowed.

(4) Truth table: 6 points

1p 1p 1p

x	y	z	$(\neg x \to z)$	$\vee$	$(y \to z)$	$(\neg x \wedge y) \to z$	$((x \vee y) \wedge (x \to z) \wedge (y \to z)) \to z$
T	T	T		T		T	T
T	T	F		T	F	T	T
T	F	T		T		T	T
T	F	F		T		T	T
F	T	T		T		T	T
F	T	F	F	F	F	F	T
F	F	T		T		T	T
F	F	F	F	T		T	T

(a) (i) contingency
1p ↗ (ii) contingency
(iii) tautology

(b) (i) is F when x: F, y: T and z: F
1p ↗ (ii) is F when x: F, y: T and z: F

(c) (i) and (ii) are logically equivalent
1p ↗

(5) Define propositional variables
4 points
a: "A is a knight"
b: "B is a knight"

(a) A says: $a \to \neg b$. Hence, a and $a \to \neg b$ must have the same truth value.

1p →

a	b	$a \to \neg b$
T	T	F
T	F	T ←
F	T	T
F	F	T

It follows that a: T, b: F.
⇒ A must be a knight and B a knave.

1p ↗ conclusion

(b) A says: $a \vee b$. ⇒ a and $a \vee b$ must have the same truth value
B says: $\neg a \oplus \neg b$ ⇒ b and $\neg a \oplus \neg b$ must have the same truth value

1p →

a	b	$a \vee b$	$\neg a \oplus \neg b$
T	T	T ←	F
T	F	T ←	T
F	T	T	T ←
F	F	F ←	F ← ←

We conclude that a: F
b: F
and hence A and B must both be knaves.

1p ↗ conclusion

Note: Alternative solution is: B says $\neg a \vee \neg b$.
In this case the only value that changes in the above truth table is the last cell – it becomes T.
2p ↗ (alternative)
~~and~~ In this case, there is no solution: the statements are contradictory.