RYERSON UNIVERSITY
DEPARTMENT OF CIVIL ENGINEERING
CVL 313 — STRUCTURAL ANALYSIS
Deflection: Moment Area Theorems

Assignment # 2

Available: September 09, 2014 **Due**: Week of Sept 23, 2014

Question 1

(a) Apply both Moment Area Theorems to determine the slope and deflection at point C and D as well as rotation at A of the beam shown in Figure 1. EI constant. **(10 Marks)**

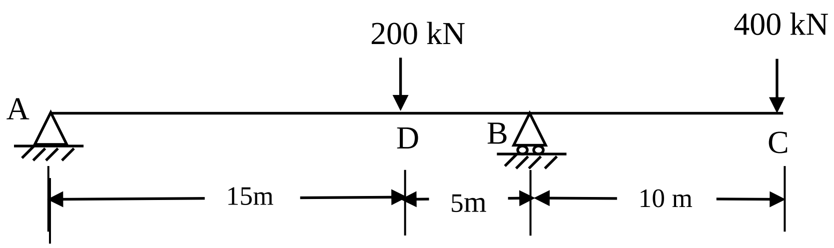


Figure 1

(b) Repeat the problem (a) using variable second moment of area of the beam shown in Figure 1. In addition, calculate rotation at A, D, B and midpoint of AD. Assume second moment of area for AD, DB and BC as 2I, 3I and 4I, respectively. E is constant. **(10 Marks)**

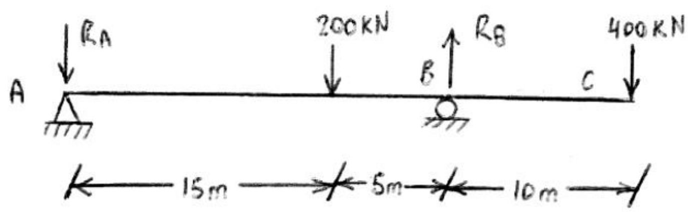
(c) Compare the results obtained in (a) and (b) and comments on it**.** **(2 Mark)**

(d) Explain and illustrate moment area theorems**.** **(2 Marks)**

(e) Discuss the advantages and disadvantages of the Moment Area Theorems. Why it is important to determine the deflection of a structure from design point of view? **(2 Marks)**

Question 1(a)

Using Moment-Area Theorems



$\theta_C = ?$ $\Delta_C = ?$

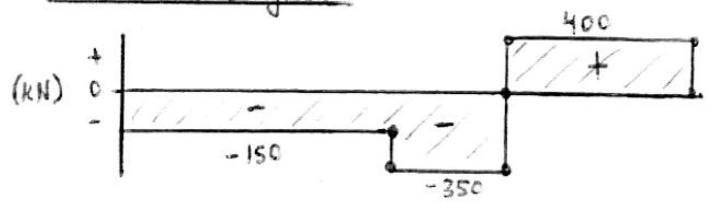
$EI = const.$

Finding Reactions:

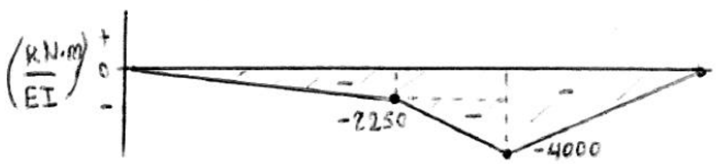
$\circlearrowright + \sum M_A = 0$; $200(15) - R_B(20) + 30(400) = 0$; $R_B = 750$ kN [↑]

$\uparrow + \sum F_y = 0$; $R_A - 200 + 750 - 400$; $R_A = -150$ kN $= 150$ kN [↓]

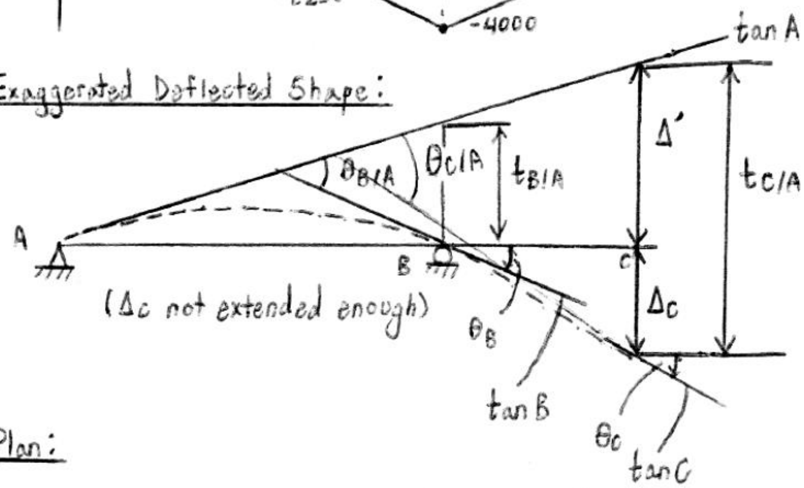
Shear Force Diagram:



M/EI Diagram:



Exaggerated Deflected Shape:



Plan:

1. Use the M/EI diagram to determine $t_{B/A}$ through moment-area.
2. Using $t_{B/A}$, through similar triangles, determine Δ'.
3. Find $t_{C/A}$; to determine Δ_C, $\Delta_C = t_{C/A} - \Delta'$.
4. Using the M/EI diagram, determine $\theta_{C/A}$; also find θ_A.
5. Perform a subtraction between θ_A and $\theta_{C/A}$ to find θ_C.

Question 1(a) — Using Moment-Area Theorems – Continued

i.e. $\theta_{b/a} = \int_b^a \frac{M}{EI}\,dx$; $t_{b/a} = \bar{x}\int_b^a \frac{M}{EI}\,dx$

Finding $t_{B/A}$:

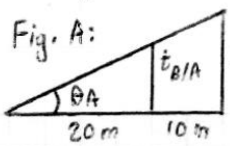
$t_{B/A} = \bar{x}\int_B^A M/EI\,dx$ (referring to M/EI diagram)

$= -(2.5)[(2250)(5)] - (5/3)[(0.5)(5)(1750)] - (10)[(15)(2250)(0.5)]$

$t_{B/A} = \frac{-204\,166.67}{EI}$ $\left(\text{or } \frac{-612500}{3} / EI\right)$

Through similar triangles,

Fig. A:



$\frac{-(612500/3)}{20} = \frac{\Delta'}{30}$; $\Delta' = \frac{-306250}{EI}$

Finding $t_{C/A}$:

$t_{C/A} = \bar{x}\int_C^A M/EI\,dx$ (referring to M/EI diagram)

$= -(2/3)(10)[(4000)(10)(0.5)] - (12.5)[(2250)(5)] - (35/3)[(1750)(0.5)(5)]$
$- (20)[(2250)(15)(0.5)]$

$t_{C/A} = -662500/EI$

Based on exaggerated deflected shape, $t_{C/A} - \Delta' = \Delta_C$.

$\Delta_C = t_{C/A} - \Delta' = -662500/EI - (-306250/EI)$

$\Delta_C = \frac{-356250 \text{ kN}\cdot\text{m}^3}{EI}$

$\therefore \Delta_C = \frac{356250 \text{ kN}\cdot\text{m}^3}{EI}$ [↓]

Finding $\theta_{C/A}$:

$\theta_{C/A} = \int_C^A M/EI\,dx$

$= -(4000)(0.5)(10) - (2250)(5)$

$- (1750)(5)(0.5) - (2250)(15)(0.5)$

$\theta_{C/A} = -52500/EI$

Finding θ_A:

(Trig.: Fig. A)

$\theta_A = t_{B/A}/20\text{m}$

$= (612500/3)/20$

$\theta_A = \frac{30625/3}{EI}$

$\theta_A = 10208.33/EI$

Finding θ_C:

$\theta_C = \theta_A + \theta_{C/A}$

$= \frac{10208.33}{EI} - \frac{52500}{EI}$

$= -42\,291.67/EI$

$\therefore \theta_C = \frac{42\,291.67 \text{ kN}\cdot\text{m}^2}{EI}$ ↺

Question 1(b): Follow the procedure as described above and consult lecture notes

Question (c) (d), and (e): consult books, lecture notes and class discussions