MACM 101 — Discrete Mathematics I

Exercises on Propositional Logic. Due: Friday, September 26th (at the beginning of the class)

Reminder: the work you submit must be your own. Any collaboration and consulting outside resources must be explicitly mentioned on your submission. Please, write with a pen, not a pencil. 30 points will be taken off the grade of works written in pencil.

1. Construct a truth table for the following compound statement: $(p \to r) \oplus (\neg q \to \neg r)$.

2. Determine whether the following compound statement is a tautology or contradiction

$$(p \to (q \to r)) \to ((p \wedge q) \to r).$$

3. Show that $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent.

4. Show that $(s \to (p \wedge \neg r)) \wedge ((p \to (r \vee q)) \wedge s$ and $p \wedge \neg r \wedge q$ are not logically equivalent. Do not use truth tables.

5. Simplify the compound statement

$$(p \vee (\neg p \wedge \neg(q \vee r))) \to (p \vee \neg(r \vee q)).$$

6. Prove that the Rule of Syllogism is a valid argument.

7. Each of two rooms (room I and room II) contains either a lady or a tiger. If a room contains a lady, the sign on its door is true. If it contains a tiger, the sign is false. The signs are

I	II
THERE IS A LADY IN THE OTHER ROOM	IT MAKE NO DIFFERENS WHICH ROOM TO PICK

Which rooms contain ladies?

8. What relevant conclusion or conclusions can be drawn from this set of premises? Explain the rules of inference used to obtain each conclusion from the premises. "I am either dreaming or hallucinating."
"If I am dreaming, I am snoring."
"If I am hallucinating, I see elephants running down the road."
"I am not snoring."

9. Write the following argument in symbolic form. Then establish the validity of the argument or give a counterexample to show that it is invalid.

   > If it does not rain or it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on. If the sailing race is held, then the trophy will be awarded. The trophy was not awarded. Therefore, it rained.

10. Using rules of inference and logic equivalences give the reasons for the steps verifying the following argument.
    Premises: $(t \vee q) \to (p \wedge \neg r)$, $(u \vee \neg s) \to t$, $(p \wedge \neg r) \to q$, $\neg s \vee t \vee q$, $\neg t \wedge u$, $p \vee \neg q$.
    Conclusion: p.

Steps	Reasons
1) $\neg t \wedge u$	
2) $\neg t$	
3) $(u \vee \neg s) \to t$	
4) $\neg t \to \neg(u \vee \neg s)$	
5) $\neg t \to (\neg u \wedge s)$	
6) $\neg u \wedge s$	
7) s	
8) $(t \vee q) \to (p \wedge \neg r)$	
9) $\neg s \vee t \vee q$	
10) $s \to (t \vee q)$	
11) $s \to (p \wedge \neg r)$	
12) $p \wedge \neg r$	
13) $(p \wedge \neg r) \to q$	
14) q	
15) $p \vee \neg q$	
16) p	