CHAPTER – 02 : Solution to Selected Back of the Chapter Problems

2.1 Using 12% simple interest per year, how much interest will be owed on a loan of $500 at the end of two years.

Note that here:

i = 12% = 0.12 per year
P = $500
N = 2 years

Simple interest formula is:

$I = P * i * N = 500 (0.12) (2) = 120$

Thus, at the end of two years, the amount of interest owed will be $120.

2.2 That principal amount will yield $150 in interest at the end of three months when the interest rate is 1% simple interest per month?

Note that following information are given:

the total interest amount, $I = 150$

$N = 3$ months

$i = 0.01$ per month

Simple interest formula is:

$I = P * i * N$

$\Rightarrow P = I / (i * N) = 150 / [(0.01)(3)] = 5000$

A principal amount of $5000 will yield $150 in interest at the end of 3 months when the interest rate is 1% per month.

2.6 How much is accumulated in each of these savings plans over two years?
(a) Deposit €1000 today at the 10% compounded annually.
(b) Deposit €900 today at 12% compounded monthly.

(a) Note that following information are given:

P = 1000
i = 10% per year = 0.10 per year
N = 2 years

We need to use the future value formula:

$F = P(1+i)^N = 1000(1+0.10)^2 = 1210$

The balance at the end of 2 years will be €1210.

(b) Again the following information are given:

P = 900
i = 12% compounded monthly = (0.12/12) per month = 0.01 per month
N = 2 years = 24 months

Hence:

$F = P(1+i)^N = 900\ (1 + 0.01)^{24} = 1142.76$

The balance at the end of 24 months (2 years) will be €1143.

2.8 Greg wants to have $50 000 in five years. He has $20 000 today to invest. The bank is offering five-year investment certificates that pay interest compounded quarterly. What is the minimum nominal interest rate he would have to receive to reach his goal?

The following information are given:

F = 50 000

P = 20 000

i = unknown (to be found; but it compounds quarterly that is every 3 months or 4 times in a year)

N = 5 years = 60 months = (60 months / every 3 months Or, 5 years * 4 quarters in every year =) 20 quarters

Hence:

$F = P(1 + i)^N$

$\Rightarrow 50\ 000 = 20\ 000(1 + i)^{20}$

$\Rightarrow 5 = 2\ (1 + i)^{20}$ [dividing both sides by 10 000]

$\Rightarrow (1+i)^{20} = 5/2$

$\Rightarrow i = (5/2)^{1/20} - 1 = 0.04688 = 4.688\%$ per quarter = (4.688% per quarter * 4 quarters in a year =) 18.75% per year

The investment certificate would have to pay at least 18.75% nominal interest, compounded quarterly.

2.13 Simple interest of $190.67 is owed on a loan of $550 after four years and four months. What is the annual interest rate?

Information contained in the question:

$I = 190.67$
$P = 550$

N = 4 years and 4 months = 4 years and 4/12 year = 4 year and 1/3 year = 13/3 years

Therefore:

$I = P * i * N$
$\Rightarrow i = I/(P*N) = 190.67/[550(13/3)] = 0.08$

The simple interest rate is 8% per year.

2.15 Greg will invest $20 000 today in five-year investment certificates that pay 8 percent <u>nominal</u> interest, compounded <u>quarterly</u>. How much money will this be in five years?

Information contained in the question:

P = 20 000

i = 8% <u>nominal</u> interest, compounded quarterly = annual interest of 8% that compounds every 3 months or 4 times in a year = (8%/4 =) 2% compounded 4 times in a year

N = quarterly over five years = (4*5=) 20 quarters

Hence:

$$F = P(1 + i)^N$$
$$= 20\,000(1 + 0.02)^{20} = 29\,718.95$$

Greg would have accumulated about $29 719.

2.17 You have a bank deposit now worth $5 000. How long will it take for your deposit to be worth more than $8 000 if:
(a) The account pays 5% <u>actual</u> interest every half-year and is compounded every <u>half year</u>?
(b) The account pays 5% <u>nominal</u> interest, compounded <u>semi-annually</u>?

(a) P = 5 000
i = 0.05 per six months
F = 8 000

From: $F = P(1 + i)^N$
$\Rightarrow F/P = (1 + i)^N$
$\Rightarrow \ln(F/P) = \ln(1 + i)^N$ [taking natural logarithm on both sides]
$\Rightarrow N \ln(1 + i) = \ln(F/P)$
$\Rightarrow N = \ln(F/P) / \ln(1 + i)$
$\Rightarrow N = \ln(8000/5000) / \ln(1 + 0.05) = 9.633$

The answer that we get is 9.633 (six-month) periods. But what does this mean? It means that after 9 compounding periods, the account will not yet have reached $8000. (You can verify yourself that the account will contain $7757). Since compounding is done only every six months, we must, in fact, wait 10 compounding periods, or 5 years, for the deposit to be worth *more* than $8000. At that time, the account will hold $8144.

(b) P = 5 000
r = 0.05 for the full year
F = 8 000
$i = r/m = 0.05/2 = 0.025$ per six months

From: $F = P(1 + i)^N$

$$N = \ln(F/P) / \ln(1 + i) = \ln(8000/5000) / \ln(1 + 0.025) = 19.03$$

We must wait 20 compounding periods, or 10 years, for the deposit to be worth more than $8 000.

2.19 (a) If you put $1 000 in a bank account today that pays 10% interest per year, how much money could be withdrawn 20 years from now?

(a) P = 1000
$i = 0.1$
N = 20

$$F = P(1 + i)^N = 1000(1+0.1)^{20} = 6727.50$$

About $6 728 could be withdrawn 20 years from now.

2.19 (b) If you put $1 000 in a bank account today that pays 10% *simple* interest per year, how much money could be withdrawn 20 years from now?

(b) $F = P\, i\, N = 1000\ (0.1)\ (20) = 2000$

Without compounding, the account would only accumulate $2 000 over 20 years.

2.21 Compute the effective annual interest rate on each of these investments.

(a) 25% nominal interest, compounded semi-annually.

$r = 0.25$ and $m = 2$

Hence, $i_e = (1 + r/m)^m - 1 = (1 + 0.25/2)^2 - 1 = 0.26563$

The effective rate is approximately 26.6%.

(b) 25% nominal interest, compounded quarterly.

$r = 0.25$ and $m = 4$

Hence, $i_e = (1 + r/m)^m - 1 = (1 + 0.25/4)^2 - 1 = 0.27443$

The effective rate is approximately 27.4%.

(c) 25% nominal interest, compounded continuously.

$i_e = e^r - 1 = e^{0.25} - 1 = 0.28403$

The effective rate is approximately 28.4%.

2.23 A Studebaker automobile that cost $665 in 1934 was sold as an antique car at $14 800 in 1998. What was the rate of return on this "investment"?

$F = P(1 + i)^N$
$\Rightarrow 14\ 800 = 665(1 + i)^{64}$
$\Rightarrow 14\ 800 / 665 = (1 + i)^{64}$
$\Rightarrow (1 + i)^{64 * (1/64)} = (14\ 800 / 665)^{1/64}$
$\Rightarrow 1 + i = 1.04967$
$\Rightarrow i = 0.04967$

The rate of return on this investment was 5%.

2.26 The Bank of Brisbane is offering a new savings account that pays a nominal 7.99% interest, compounded continuously. Will your money earn more in this account than in a daily interest account that pays 8%?

Effective interest for continuous interest account:

$i_e = e^r - 1 = e^{0.0799} - 1 = 0.08318 = 8.318\%$

Effective interest for daily interest account:

$i_e = (1 + r/m)^m - 1 = (1 + 0.08/365)^{365} - 1 = 0.08328 = 8.328\%$

No, your money will earn slightly <u>less</u> with continuous compounding.

2.27 The Crete Credit Union advertises savings account interest as 5.5% compounded weekly and chequing account interest at 7% compounded monthly. What are the effective interest rates for the two types of accounts?

Knowing that there are 52 weeks or, 12 months in a year:

$i_e(\text{weekly}) = (1 + r/m)^m - 1 = (1 + 0.055/52)^{52} - 1 = 0.0565 = 5.65\%$

$i_e(\text{monthly}) = (1 + r/m)^m - 1 = (1 + 0.07/12)^{12} - 1 = 0.0723 = 7.23\%$

2.29 April has bank deposit now worth $796.25. A year ago, it was $750. What was the nominal monthly interest rate on her account?

Recognize from the information that:

$F = 796.25$
$P = 750$
$N = 1$

Thus, $F = P(1 + i_e)^N$

$\Rightarrow 796.25 = 750(1 + i_e)^1$
$\Rightarrow (796.25 / 750) = 1 + i_e$
$\Rightarrow i_e = 1.06167 - 1$
$\Rightarrow i_e = 0.06167$

To find out nominal monthly interest rate we use the fact:

$i_e = 0.06167$
$m = 12$
we need to find i_s

Thus, $i_e = (1 + i_s)^m - 1$

$\Rightarrow 0.06167 = (1 + i_s)^{12} - 1$
$\Rightarrow 1.06167 = (1 + i_s)^{12}$
$\Rightarrow (1 + i_s)^{12*(1/12)} = 1.06167^{(1/12)}$
$\Rightarrow 1 + i_s = 1.004999$
$\Rightarrow i_s = 0.004999$

The nominal monthly interest rate was 0.5%.

2.30 May has $2 000 in her bank account right now. She wanted to know how much it would be in one year, so she calculated and cape up with $2140.73. Then she realized she had made a mistake She had wanted to use the formula for monthly compounding, but instead, she had used the continuous compounding formula. Redo the calculation for May and find out how much will actually be in her account a year from now.

First, determine the effective interest rate that May used to get $2140.73 from $2000. Then, determine the nominal interest rate associated with the effective interest:

$F = P(1 + i_e)^N$
$\Rightarrow 2140.73 = 2000(1 + i_e)^1$
$\Rightarrow i_e = 0.070365$

Then : $i_e = e^r - 1$
$\Rightarrow 0.070365 = e^r - 1$
$\Rightarrow e^r = 1.070365$
$\Rightarrow \ln e^r = \ln (1.070365)$
$\Rightarrow r \ln e = 0.068$
$\Rightarrow r = 0.068$ [since ln e = 1]

The *correct* effective interest rate is then:
$i_e = (1 + r/m)^m - 1 = (1 + 0.068/12)^{12} - 1 = 0.07016$

The correct value of $2000 a year from now is:
$F = P(1 + i_e)^N = 2000(1 + 0.07016)^1 = \$2140.32$

2.36 You have just won a lottery prize of $1 000 000 collectable in 10 yearly instalments of $100 000 starting today. Why is this prize not really $1000000? What is it really worth today if money can be invested at 10% annual interest, compounded monthly?

The present worth of each instalment:

Instalment	F	P
1	100 000	100 000
2	100 000	90 521
3	100 000	81 941
4	100 000	74 174
5	100 000	67 143
6	100 000	60 779
7	100 000	55 018
8	100 000	49 803
9	100 000	45 082
10	100 000	40 809
	Total	665 270

Sample calculation for the third instalment, which is received at the end of the second year:

$$P = F/(1 + r/m)^N = 100\ 000/(1 + 0.10/12)^{24} = 81\ 941$$

The total present worth of the prize is $665 270, not $1 000 000.

2.38 You are looking at purchasing a new computer for your four-year undergraduate program. Bran 1 costs $4 000 now, and you expect it will last throughout your program without any upgrades. Brand 2 costs $2 500 now and will need an upgrade at the end of two years, which you expect to be $1 700. With 8% annual interest, compounded monthly, which is the less expensive alternative, if they provide the same level of service and will both be worthless at the end of the four year?

Brand 1: cost is $4 000

Brand 2: cost is $2 500 now and $1 700 in two years. The present worth of the $1700 is:

$$P = F/(1 + r/m)^{m} = 1700/(1 + 0.08/12)^{24} = 1449.41$$

The total present cost of Brand 2 is 2500 + 1449 = $3949.

Brand 2 is about $51 less expensive than Brand 1.

2.41 You are indifferent about whether you receive $100 today or $110 one year form now. The bank pays you 6% interest on deposits and charges you 8% for loans. Name the three types of equivalence and comment (with one sentence for each) on whether each exist for this situation and why..

Market equivalence does not apply as the cost of borrowing and lending is not the same.

Mathematical equivalence does not hold as neither 6% nor 8% is the rate of exchange between the $100 and the $110 one year from now.

Decisional equivalence holds as you are indifferent between the $100 today and the $110 one year from now.