CONCORDIA UNIVERSITY

Department of Mathematics and Statistics

Course	**Number**	**Section**
Math	209	EC
Examination	**Date**	**Pages**
Final	April 2010	3
Instructors		**EC Course Examiners**
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Instructions

- Answer all ten questions.
- Only approved calculators are allowed.
- No other material is allowed.

Evaluation

All questions are of equal value. The examination counts for 50% towards your final grade.

Questions

Question 1

(a) Find the value of each of the following:

(i) $\lim_{x\to -3} \dfrac{x^2-3x+2}{(x-1)}$ (ii) $\lim_{x\to 5} \dfrac{x^2-16}{(x-5)}$ (iii) $\lim_{x\to \infty} \dfrac{-5x^7+3x^2+2}{4-x^2}$

(b) Suppose that $\lim_{x\to 3} f(x) = -5$ and $\lim_{x\to 3} g(x) = 4$, use the properties of limits to find

(i) $\lim_{x\to 3} (-3g(x))$ (ii) $\lim_{x\to 3} \sqrt{g(x)}$ (iii) $\lim_{x\to 3} (g(x)/2f(x))$ (iv) $\lim_{h\to 0} \dfrac{(x-h)^2-x^2}{h}$

Question 2

(a) If $f(x) = 4 - 6x^{10} - 4x^3$, find $f'(x)$.

(b) If $f(x) = \left(x^2 + 5x\right) / \left(e^x - 7\right)$, find $f'(x)$.

(c) if $y = \ln\left(x^2 + 3x\right)^2$, find $\dfrac{dy}{dx}$.

(d) If $y = \sqrt[5]{x+5}$, find $\dfrac{dy}{dx}$.

(e) If $xy = e^y - 2$, find y'.

Question 3

A manufacturer currently sells sunglasses at $\$4$ a pair. The price p and the demand x for these glasses are related by price-demand equation

$$x = f(p) = 7000 - 500p$$

If the current price is increased, will revenue increase or decrease? Explain your answer.

Question 4

Find the area bounded by $f(x) = x^2 - x$ and $g(x) = 2x$ for $-2 \leq x \leq 3$.

Question 5

Consider the function $f(x) = x^4 - 2x^3$. Graph the function and find its intercepts, the values of x for which the function is increasing and decreasing, and the values of x where it is concave up and concave down.

Question 6

The price p (in dollars) and the demand x for a particular steam iron are related by the equation

$$x = 1000 - 20p$$

(a) Express the price p in terms the demand x, and find the domain of this function.

(b) Find the revenue $R(x)$ from the sale of x clock radios. What is the domain of R?

(c) Find the marginal revenue at a production level of 400 steam irons and interpret the results.

(d) Find the marginal revenue at a production level of 650 steam irons and interpret the results.

Question 7

Boyle's law for enclosed gases states that if the volume is kept constant, the pressure P and the temperature T are related by the equation $P/T = k$ where k is a constant. If the temperature is increasing at 3 Kelvins per hour, what is the rate of change of pressure when the temperature is 250 Kelvins and the pressure is 500 pounds per square inch?

Question 8

Compute the following:

(a) $\int e^{5x} dx$

(b) $\int \frac{x}{\sqrt{x-7}} dx$

(c) $\int \left(3x^2 + 5x\right) dx$

(d) $\int \frac{x^2}{4+x^3} dx$

(e) $\int \left(\left(x^2+1\right)^{12} x\right) dx$

Question 9

Evaluate the following integrals, accurate to 2 decimal places.

(a) $\int_0^5 \left(t^2 - 4\right) dt$

(b) $\int_2^3 e^{h^2} dh$

Question 10

(a) A sphere with a radius of 5 centimeters is coated with ice 0.1 centimeters thick. Use differentials to estimate the volume V of ice. Recall that $V = \frac{4}{3}\pi R^3$, where R is the radius of the sphere.

(b) Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ that is continuous at 0 but not differentiable at 0. Explain.