

ECOR 1010

Lecture 4

Measurements, Units
and Errors

SI Magnitude Prefixes

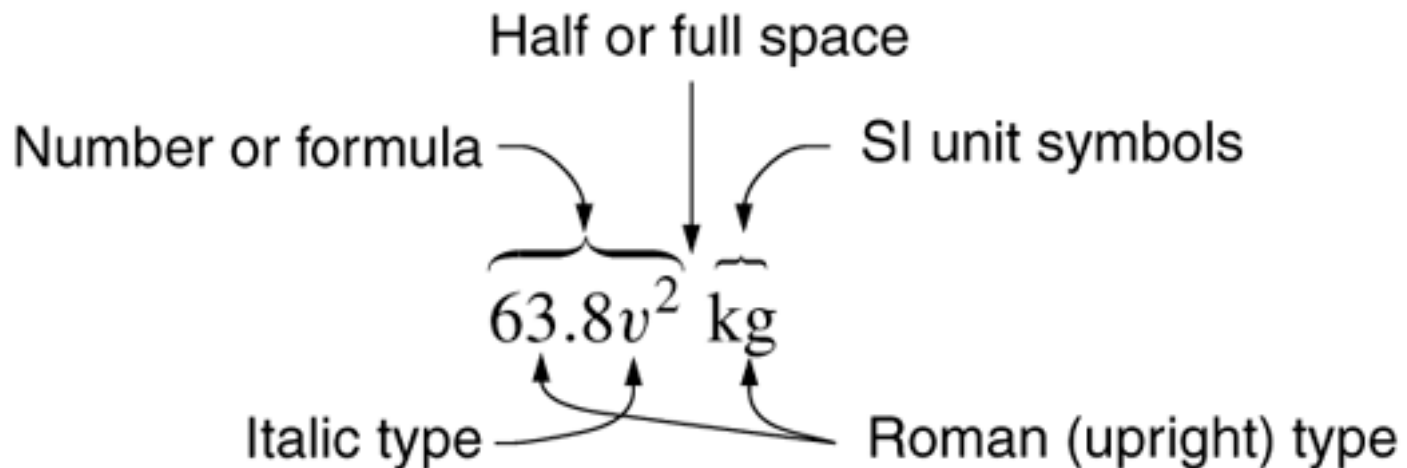
- The unit may be preceded by a magnitude prefix;
 - The prefix is regarded as an inseparable part of the symbol.

Table 10.3 Magnitude prefixes in the SI unit system by symbol, name, and numerical value.

Y	yotta	10^{24}	M	mega	10^6	f	femto	10^{-15}
Z	zetta	10^{21}	k	kilo	10^3	a	atto	10^{-18}
E	exa	10^{18}	m	milli	10^{-3}	z	zepto	10^{-21}
P	peta	10^{15}	μ	micro	10^{-6}	y	yocto	10^{-24}
T	tera	10^{12}	n	nano	10^{-9}			
G	giga	10^9	p	pico	10^{-12}			

Writing Quantities With Units

- Standardized rules for correctly representing quantities with units:



A physical quantity is written as a number or a formula representing a number, followed by unit symbols.

Rules for Writing With Units - 1

- Always a space between numeric value and unit:
 - 23.67 kg not 23.67kg
- Unit symbols are treated like algebraic factors that can be multiplied or divided with numbers or other unit symbols:
 - Multiplication between units is indicated by a centred dot or a half-space,
 - Division by a slash or a negative superscript:
 - 27 kg·m/s² or 27 kg m/s² or 27 kg·m·s⁻²

Rules for Writing With Units - 2

- Unit symbols are not modified with subscripts:
 - $V_{\max} = 200 \text{ V}$ not $V = 200 V_{\max}$.
- Relation between numerical quantity and unit symbols must be clear:
 - $3.7 \text{ km} \times 2.8 \text{ km}$ for an area; not $3.7 \times 2.8 \text{ km}$
 - $87.2 \text{ g} \pm 0.4 \text{ g}$ or $(87.2 \pm 0.4) \text{ g}$; but not $87.2 \pm 0.4 \text{ g}$

Rules for Writing With Units - 3

- Unit symbols not mixed with unit names in sentences:
 - Use kg/m² or kilogram per square metre, but not kilogram/m².
- Long numbers, written in groups of 3, the groups should be separated by a non-breaking half space rather than a comma or period:
 - 93 000 000 miles not 93,000,000 miles
 - 21 298.046 83 m not 21298.04683 m

Common Units

	SI	FPS
Force	newton (N) = 1 kg·m/s ²	pound (lb)
Length	metre (m)	foot (ft)
Mass	kilogram (kg)	slug = 1 lb·sec ² /ft
Time	s	sec
Pressure	pascal (Pa) = 1 N/m ²	psi (lbs/in ²) or lbs/ft ²
Work & Energy	joule (J) = N·m	ft·lb
Power	watt (W) = N·m/s	horsepower HP = 550 ft·lb/sec
Temperature	K T(K) = T(°C)+273.15	°F T(°F) = 9/5 T(°C)+32

Fixed, Scientific and Engineering Notation

- Fixed notation
 - Ordinary decimal notation
 - e.g. 30 140.0,
- Scientific notation
 - 3.014×10^4 ,
- Engineering notation
 - Exponent of 10 is a multiple of 3 to correspond to SI prefixes
 - e.g. 30.14×10^3 .

Unit Algebra - 1

- Algebra can be performed on units,
 - with restriction that only quantities with same units can be equated, added, or subtracted.
- Consider conversion from one unit to another
 - 20 000 ft·lb/min = ? HP

$$20\,000 \frac{\cancel{\text{ft}} \cdot \cancel{\text{lb}}}{\cancel{\text{min}}} \times \frac{\cancel{1 \text{ min}}}{60 \text{ s}} \times \frac{1 \text{ HP}}{550 \cancel{\text{ft}} \cdot \cancel{\text{lb}}/\cancel{\text{s}}} = 0.606 \text{ HP}$$

Unit Algebra - 2

- Consider the equation $s = vt + (1/2)at^2$, where
 - v is the initial velocity in m/s,
 - t is time in s,
 - a is acceleration in m/s^2 .
- Then s has units $(\text{m/s})s + (\text{m/s}^2)s^2 = \text{m} + \text{m}$
 - Since we can add quantities with same units,
 - s has units m.
 - s is a measure of length.

Dimensions

- Dimensions differ slightly from units
 - e.g., Speed
 - Dimensions = distance / time.
 - Units may be km/hr, or mm/s, etc.
- Fundamental dimensions
 - Distance, time, mass etc.

Derived Dimensions

- Products or ratios of basic dimensions

Acceleration: $a = \frac{[l]}{[t]^2}$

Force: $f = ma = \frac{[m][l]}{[t]^2}$

Dimensional Homogeneity: The dimensions on either side of an equation must be the same

↳ Dimensions of $f =$ dimensions of m times a

Derived Dimensions with Units (Pressure)

$$\text{Pressure} = \text{Force per area} \rightarrow \frac{[f]}{[l^2]}$$

Units :

$$\frac{\text{N}}{\text{m}^2} = \text{Pascal (Pa) in the SI system}$$

$$\frac{\text{lb}}{\text{in}^2} = \text{psi in the FPS system}$$

Derived Dimensions with Units (Work)

$$\text{Work} = \text{force 'dot' displacement} = f \bullet d$$
$$\rightarrow [f] [l]$$

Units:

$$\text{N} \cdot \text{m} = \text{Joules (J)}$$

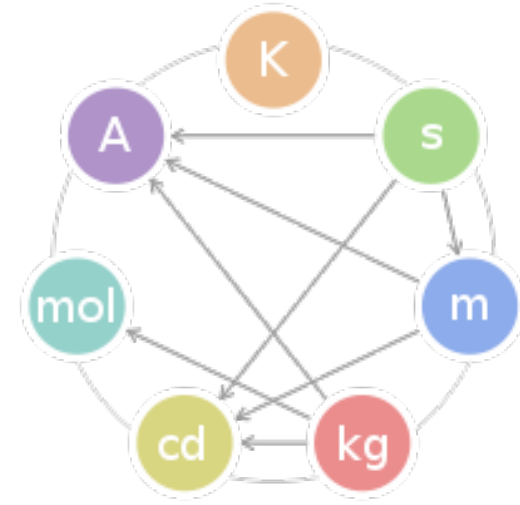
Derived Dimensions with Units (Power)

$$\text{Power} = \frac{\text{work}}{\text{time}} \rightarrow \frac{[f] [l]}{[t]}$$

Units:

$$\frac{\text{N} \cdot \text{m}}{\text{s}} = \text{J/s} = \text{Watts (W)}$$

Derived Dimensions with Units (Charge)



$$Q = \int I dt$$

Units : Coulomb = Ampere \times second

↓
6.241506 $\times 10^{18}$ electrons

Non-Dimensional Numbers

- Ratios where the numerator has the same dimension as the denominator

$$\text{Aspect Ratio (AR) of an aircraft wing} = \frac{(\text{Wing Span})^2}{\text{Wing Area}}$$

$$AR = \frac{[l]^2}{[l]^2} = [1]$$

Unit Consistency (Dimensional Analysis)

- When performing calculations, numbers should always be written with correct units
- Units should be carried throughout the problem
- Very useful check!

The Gimli Glider

“If a Boeing 767 runs out of fuel at 41,000 feet, what do you have? A 132 ton glider ...”

- Wade H. Nelson (Soaring Magazine, 1997)

Air Canada Flight 143: The Gimli Glider

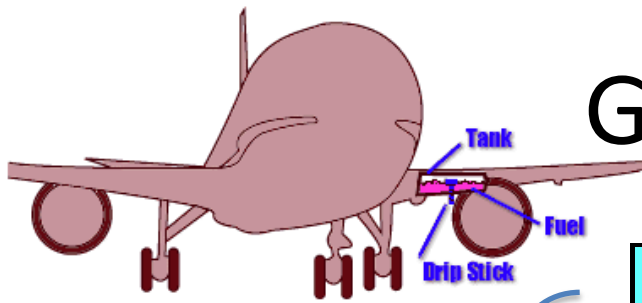


The Scenario

- There was a problem with the fuel gauges, so they calculated the volume of fuel they needed *by hand*
- Previously there was a flight engineer on board, but not on these new planes.
- Among other things, the specific gravity of fuel is needed to make the proper calculations
 - They used 1.77 lb/L as the constant
 - They should have used 0.8 kg/L for this all-metric 767 aircraft!

Recall 2.2 lb = 1 kg

Gimli Glider: What Happened



What they did

$$7,682 \text{ L} \times 1.77 = 13,597 \text{ kg of fuel remaining on board}$$

$$22,300 \text{ kg needed} - 13,597 \text{ kg on board} = 8,703 \text{ kg to be added}$$

$$8,703 \text{ kg} \div 1.77 = 4,916 \text{ L of fuel to be added}$$

If they had kept track of the units and verified that the units canceled properly, they could have calculated:

$$7,682 \text{ L} \times \frac{0.803 \text{ kg}}{\text{L}} = 6,169 \text{ kg remaining on board}$$

$$22,300 \text{ kg needed} - 6,169 \text{ kg on board} = 16,131 \text{ kg to be added}$$

$$16,131 \text{ kg} \times \frac{\text{L}}{0.803 \text{ kg}} = 20,163 \text{ L of fuel to be added}$$

What they should have done

Sideslipping a 767



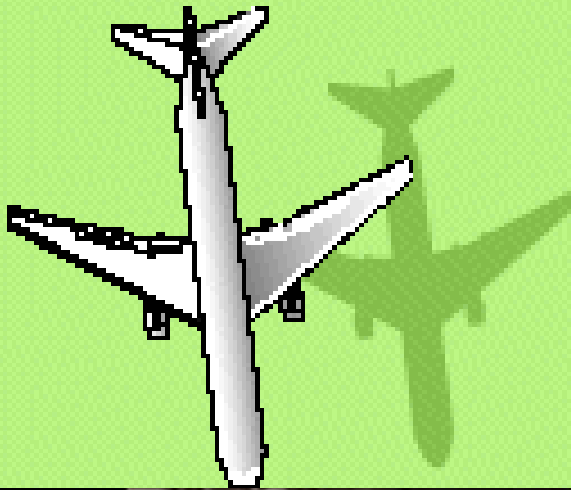
Aircraft too high. Speed slowed to 180 knots



Pilot initiates sideslip: left rudder pedal pushed while he turns the yoke to the right. Aircraft manoeuvred into a steep angle. Aircraft rapidly loses altitude.



Sideslipping a 767

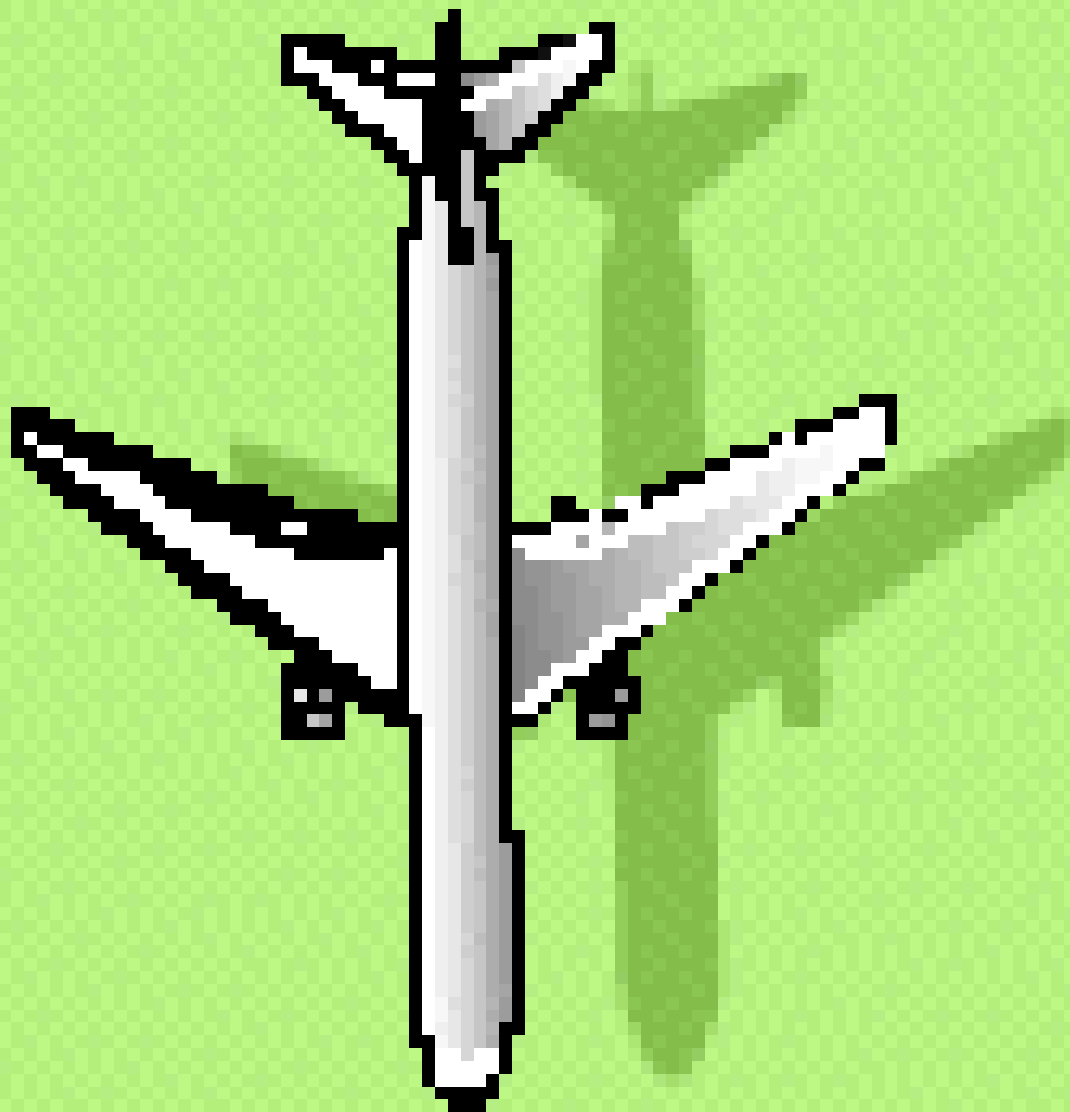


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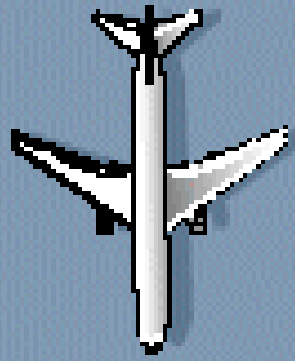


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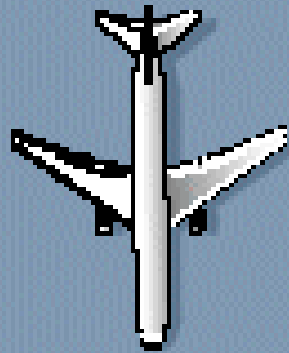




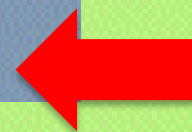
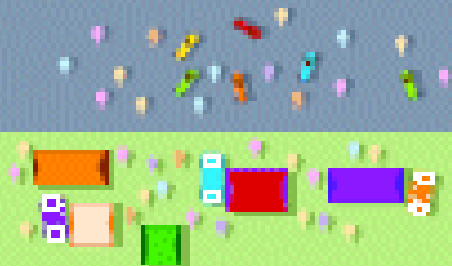
**Aircraft
straightens out
at an altitude of
40 feet.**



Aircraft touches down.



Aircraft comes to rest.



Lots of people

Gimli Glider





Measurements

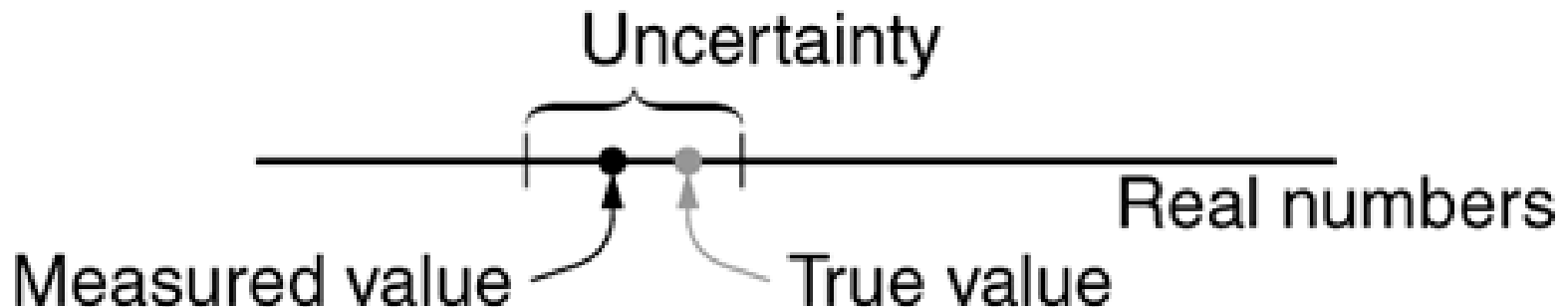
- Engineering students should know
 - Definition of traceable measurement;
 - Classification of uncertainties into systematic and random effects;
 - Correct use of words *accuracy*, *precision*, *bias*;
 - How to write inexact quantities using engineering and other notation;
 - The correct use of significant digits.

Measurement

- Measurement:
 - Property of a physical object that can be represented using a real number.
 - As opposed to counting, measurement is not exact.
 - Quantities usually measured with an instrument.
 - Calipers, scale, measuring cup etc.

Uncertainty

- Measuring instruments are not perfect;
 - Measurements typically yield
 - Estimate of true value, and
 - An interval around estimate in which true value lies.
 - This is called the uncertainty interval.



Illustrating a true (exact) value, which is a point on the line of real numbers, and its measured estimate, together with an interval of uncertainty.

True value – measured value = measurement error.

Systematic Errors

- Consistent deviation from the true value;
 - also called a *bias* or an *offset*.
- Error has same magnitude and sign when repeated measurements are made under same conditions.
- Can be detected by
 - Calibration,
 - Comparison with results obtained with an independent method

Three Types of Systematic Errors

- Natural Error
 - From environmental effects;
 - e.g. temperature changes affect electronic components and measuring instruments.
 - Correction factor can be applied to compensate if possible.
- Instrument Error
 - Also called *offset*.
 - Caused by imperfections in adjustment or construction of instrument.

Types of Systematic Errors

- Personal Error
 - Result from habits of the observer.
 - Can be reduced by proper training.

Random Errors

- Result of small variations in measurements
 - e.g. in repeated measurements, an observer may
 - exert slightly different pressures on a micrometer
 - connect voltmeter leads in slightly different locations.
- Random errors do not bias measurement,
 - They produce both positive and negative errors with zero mean value.
- If several measurements are made, their mean is normally a better estimate of true value than any single measurement.

Precision and Accuracy - 1

- Precision of a measurement
 - Repeatability.
 - Precise measurements have small random error
 - Discrepancies between repeated measurements taken under the same conditions are small.
- Accuracy
 - Accurate measurements are close to true value
 - Some authors use term to describe total error
 - Others to describe systematic error.
 - Accurate measurement has little bias.

Precision and Accuracy - 2

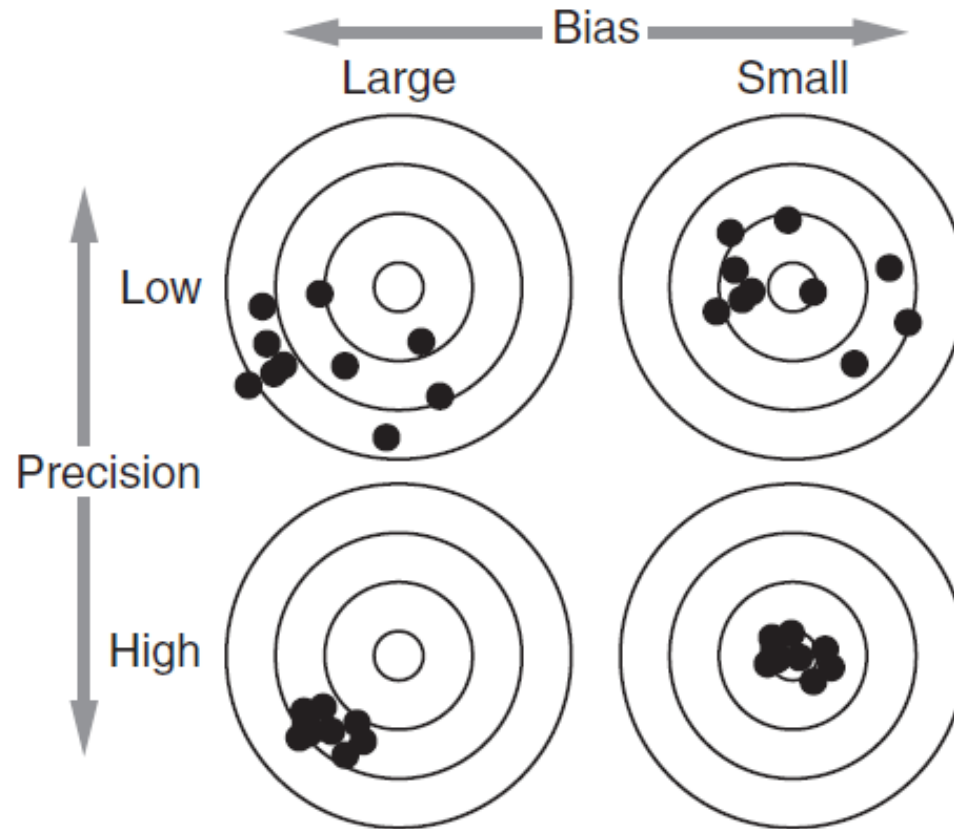


Figure 11.3 Precision, bias, and accuracy: The two right patterns have small bias (small systematic error); the two bottom patterns have high precision (small random error). The lower right pattern is accurate; ~~less commonly~~ the two right patterns would be said to be accurate.

Estimating Measurement Error - 1

- When systematic errors are quantified, the measurement can be corrected to remove bias.
 - The offset is subtracted from measurement, or
 - The instrument is recalibrated.
- Random error remains
 - Must be estimated and quoted with measurement.

Estimating Measurement Error - 2

- Source of random errors
 - Technical specifications of instrument
 - Reading errors.
 - e.g., Estimating position of needle of a gauge or meter requires interpolation between scale markings,
 - Precision of measurement is limited by ability of observer to resolve small distances.
 - Precision of the readings will be a fraction of the amount between scale markings, typically one-tenth.

How to Write Inexact Quantities

- Measurement is complete only when a statement about its uncertainty is included.
- Uncertainty is normally expressed as an estimated range;
 - Can be stated explicitly or implicitly.

Explicit Uncertainty Notation

- Measurement may be expressed in fixed, scientific, or engineering notation, and
- Uncertainty may be given in either absolute or relative form.
 - Fixed notation with error in absolute form
 - $v_s = 343.5 \text{ m/s} \pm 0.9 \text{ m/s} = (343.5 \pm 0.9) \text{ m/s}$
 - Fixed notation with error in relative form
 - $v_s = 343.5(1 \pm 0.26\%) \text{ m/s}$.

Implicit Uncertainty Notation

- If a quantity is written without explicit uncertainty, uncertainty is taken to be ± 5 in digit immediately to the right of least significant digit.
- Suppose measured speed is 94 km/h
 - Without more information, the error is ± 0.5 km/h
- Indicated speed is 94 km/h
 - Error from speedometer graduations is ± 1 km/h
 - >100 km/h error is ± 2.5 km/h



Significant Digits

- Determine the precision of the number.
 - Should not use more significant digits than justified by the measurement.
- Significant digits are identified as follows:
 - All non-zeros are significant,
 - All zeros between significant digits are significant.
 - Leading zeros are not significant.
 - Trailing zeros are significant in fractional part of number.

Significant Digits

- Examples
 - 0.035 0 oz has 3 significant digits
 - 90 000 000 miles has 1 significant digit, (ambiguous)
 - 92 900 000 miles has 3 significant digits, (ambiguous)
 - $(92\,900\,000 \pm 500)$ miles has 5 significant digits

Significant Digits

- In scientific notation, integer part is normally one non-zero digit,
 - Number of significant digits is one more than number of digits after decimal point.
 - $9 \times 10^7 = 1$ significant digit
 - $9.3 \times 10^7 = 2$ significant digit
 - $9.290 \times 10^7 = 4$ significant digits
- In engineering notation, all digits except leading zeros are significant
 - 0.09×10^9 miles = 1 significant digit
 - 93×10^6 miles = 2 significant digits
 - 92.90×10^6 miles = 4 significant digits

Rounding Numbers

- Rounding is used to reduce number to the appropriate significant digits.
 - When leftmost discarded digit is less than 5, then retained digits are unchanged;
 - $3.234 \rightarrow 3.23$ or 3.2
 - When leftmost discarded digit greater than 5, or is 5 with at least one nonzero digit to its right, increase rightmost retained digit by one;
 - $3.256 \rightarrow 3.26$ or 3.3
 - $9.747 \rightarrow 9.75$ or 9.7 *not* 9.8

Rounding Numbers

– When the leftmost discarded digit is 5 and all following digits are 0, then rightmost retained digit unchanged if it is even; otherwise it is increased by one.

- $3.25 \rightarrow 3.2$
- $3.250 \rightarrow 3.2$
- $3.350 \rightarrow 3.4$

– This rule avoids introducing a bias into the mean of a set of rounded numbers.

Effect of Algebraic Operations

- Computations performed using inexact measured quantities yield inexact results.
 - Final computed value should not have more significant digits than are justified;
 - Must be rounded to imply correct interval of uncertainty.

Absolutely NEVER give all the digits you get out of your calculator or a program such as Excel: unless you can defend all of them!!

Addition and Subtraction of Inexact Quantities

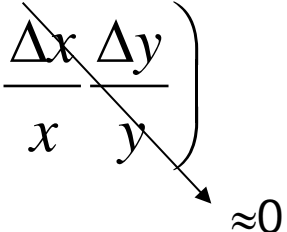
- $(x \pm \Delta x) + (y \pm \Delta y)$
 - Max: $(x + \Delta x) + (y + \Delta y) = (x + y) + (\Delta x + \Delta y)$
 - Min: $(x - \Delta x) + (y - \Delta y) = (x + y) - (\Delta x + \Delta y)$
- Therefore
$$(x \pm \Delta x) + (y \pm \Delta y) = (x + y) \pm (\Delta x + \Delta y)$$
- Similarly
$$(x \pm \Delta x) - (y \pm \Delta y) = (x - y) \pm (\Delta x + \Delta y)$$

Addition and Subtraction of Inexact Quantities

- For addition or subtraction, we add absolute uncertainties.
 - $(3.5 \pm 0.4) \text{ m} + (1.22 \pm 0.03) \text{ m} = (4.72 \pm 0.43) \text{ m}$
becomes $(4.7 \pm 0.4) \text{ m}$.
 - $(3.93 \pm 0.08) \text{ kg} - (3 \pm 0.1) \text{ kg} = (0.93 \pm 0.18) \text{ kg}$
becomes $(0.9 \pm 0.2) \text{ kg}$.

Multiplication and Division Using Inexact Quantities

- For multiplication, we add relative uncertainties:

$$\begin{aligned}(x \pm \Delta x) \times (y \pm \Delta y) &= xy \pm y\Delta x \pm x\Delta y + \Delta x\Delta y \\ &= xy \left(1 \pm \frac{\Delta x}{x} \pm \frac{\Delta y}{y} + \frac{\Delta x}{x} \frac{\Delta y}{y} \right) \\ &\approx xy \left(1 \pm \frac{\Delta x}{x} \pm \frac{\Delta y}{y} \right)\end{aligned}$$


- Similarly, we add relative uncertainties for division.

Multiplication and Division Using Inexact Quantities

$$\begin{aligned}(4.0 \pm 0.5) \text{ kg} \times (2.0 \pm 0.2) \text{ m/s}^2 &= 4(1 \pm 0.5/4.0) \text{ kg} \times 2(1 \pm 0.2/2.0) \text{ m/s}^2 \\ &= 4(1 \pm 0.125) \text{ kg} \times 2(1 \pm 0.1) \text{ m/s}^2 \\ &= 8(1 \pm 0.225) \text{ N} + \text{higher order terms} \\ &\approx (8 \pm 1.8) \text{ N} \\ &\approx (8 \pm 2) \text{ N}\end{aligned}$$

$$\begin{aligned}(4.0 \pm 0.5) \text{ N} / (2.0 \pm 0.2) \text{ m}^2 &= 4(1 \pm 0.5/4.0) \text{ N} / 2(1 \pm 0.2/2.0) \text{ m}^2 \\ &= 4(1 \pm 0.125) \text{ N} / 2(1 \pm 0.1) \text{ m}^2 \\ &= 2(1 \pm 0.225) \text{ Pa} + \text{higher order terms} \\ &\approx (2 \pm 0.45) \text{ Pa} \\ &\approx (2 \pm 0.5) \text{ Pa}\end{aligned}$$

Multiplication and Division Using Inexact Quantities

Product of 1.1, 1.2, 1.3, 2.4 ? (Implied uncertainty is ± 0.05)

Calculator product: $1.1 \times 1.2 \times 1.3 \times 2.4 = 4.1184$

Product rounded to two significant digits = 4.1

Product rounded with uncertainty is between 4.05 and 4.15

But, what if all errors are -0.05 ? Product = 3.5470

or, what if all errors are +0.05 ? Product = 4.7545

Worst case is much worse than 4.05 to 4.15 !

Should you give the answer as 4, with implied uncertainty of ± 0.5 ?

Multiplication and Division Using Inexact Quantities

Product of 1.1, 1.2, 1.3, 2.4 ? (Implied uncertainty is ± 0.05)

Consider propagation of errors from multiplying:

$$(1.1 \pm 0.05) \times (1.2 \pm 0.05) \times (1.3 \pm 0.05) \times (2.4 \pm 0.05)$$

$$= 1.1(1 \pm 0.05/1.1) \times 1.2(1 \pm 0.05/1.2) \times 1.3(1 \pm 0.05/1.3) \times 2.4(1 \pm 0.05/2.4)$$

$$= 4.1184 (1 \pm 0.05/1.1) \times (1 \pm 0.05/1.2) \times (1 \pm 0.05/1.3) \times (1 \pm 0.05/2.4)$$

$$= 4.1184 (1 \pm 0.05/1.1 \pm 0.05/1.2 \pm 0.05/1.3 \pm 0.05/2.4) + \dots$$

$$\approx 4.1184 (1 \pm 0.146) = 4.1 \pm 0.6$$

This is a worst case error estimate. Is this always justified?

Reading Assignment

- Read Chapter 11