MGCR 341: Finance 1

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Assignment 1: Solutions

Topic: Time value of money: Retirement savings problem

1) Today is t = 0. You plan to take a year off after you graduate and then get a job. Your first salary payment will be $200,000, received at t = 2. Each year thereafter, your salary will grow by 3%, until your last salary payment at t = 30. You plan to save a certain fixed percentage X of your salary every year to meet the following retirement needs: at t = 31 you want to withdraw an amount of money that will allow you to buy the same amount of goods that $50,000 can buy at t = 0; each year thereafter, up to and including t = 50, you want to withdraw an amount of money that will allow you to consume 5% more goods than in the previous year; after your final withdrawal, you want there to be $100,000 left in the bank at t = 50. The risk free rate is 8% and the inflation rate is 1%.

a) What percent X of your salary must you save each year to meet your retirement needs? Solve this question using the nominal approach, then again using the real approach.

Nominal Approach:

In order to buy at t = 31 what $50,000 can buy today, you will need withdraw $50,000(1+0.01)^{31}$ at t = 31. Thereafter, you want to consume 5% more real goods, which in nominal terms means you will have to withdraw (1.05)(1.01)-1 = 6.05% more each year. Your withdrawals are in the form of a forward starting growing perpetuity.

Your first savings at t = 2 is equal to X(200,000), and each year thereafter you save 3% more than the previous year. Your savings are also in the form of a forward starting growing annuity.

Setting the PV of your savings equal to the PV of your withdrawals plus the PV of what`s left in the bank at t = 50 gives:

$$\frac{X(200,000)}{0.08-0.03}\left(1-\left(\frac{1+0.03}{1+0.08}\right)^{(30-2+1)}\right)\frac{1}{1.08} = \frac{50,000(1+0.01)^{31}}{0.08-0.0605}\left(1-\left(\frac{1+0.0605}{1+0.08}\right)^{(50-31+1)}\right)\frac{1}{1.08^{30}}+\frac{100,000}{1.08^{50}}$$

➔ **X = 3.91%**

Real Approach:

In the real approach, the differences are that: the first real cash flow withdrawal is 50,000; this grows at a 5% real growth rate; the first salary has to be converted to a real amount; need to convert the salary growth rate to a real growth rate; have to convert the final balance at t = 50 to a real amount; have to use the real interest rate

The real growth rate of the salary is (1.03/1.01)-1 = 1,98%

The real interest rate is (1.08/1.01)-1 = 6.93%

Setting the PV of your savings equal to the PV of your withdrawals plus the PV of what`s left in the bank at t = 50 gives:

$$\frac{X(200,000)/1.01^2}{0.0693-0.0198}\left(1-\left(\frac{1+0.0.0198}{1+0.0693}\right)^{(30-2+1)}\right)\frac{1}{1.0693}$$
$$=\frac{50,000}{0.0693-0.05}\left(1-\left(\frac{1+0.05}{1+0.0693}\right)^{(50-31+1)}\right)\frac{1}{1.0693^{30}}+\frac{100,000/1.01^{50}}{1.0693^{50}}$$

➔ **X = 3.91%**

Note: you could have also solved the problem by setting the FV (at any common point) of the savings equal to the FV of the withdrawals and the FV of the remaining balance. If you did it correctly, you would get the same answer as above.

b) How much money are you spending at t = 35 in order to meet your t = 35 consumption needs?

$$50,000(1+0.01)^{31}(1.0605)^4 = 86,094.48$$

Topic: Time value of money: Lamborghini gift purchase decision

2) After several years of successfully implementing strategies you learned in MGCR 341 you are considering purchasing a new Lamborghini for your old professor as a small token of your appreciation. You can obtain the car by making monthly payments of $20,000 a month for 4 years. You have $100,000 at your disposal, and if you pay $100,000 today the car is yours for only $17,000 a month for 4 years. In each case, the first monthly payment is one month from today.

a) If the APR with monthly compounding is 12%, which option do you prefer?

The monthly interest rate is 12%/12 = 1%.

For the first option, the PV is given by

$$= \frac{20,000}{0.01}\left(1 - \left(\frac{1}{1.01}\right)^{4x12}\right) = 759,479.19$$

For the second option, the PV is given by

$$= 100,000 + \frac{17,000}{0.01}\left(1 - \left(\frac{1}{1.01}\right)^{4x12}\right) = 745,557.31$$

Thus, the second option is preferred since the PV of your payments is lower.

b) Suppose the second option has the lower PV. Might you prefer the first option if you had an amazing investment opportunity for the $100,000 you have at your disposal? How does your answer depend on whether or not you have access to borrowing/lending at the risk free rate?

If you can borrow at the risk free rate (which is always the assumption unless noted otherwise), then you would still prefer the 2nd option, because you can always borrow another $100,000 if you need it for your amazing investment opportunity.

If you can't borrow, however, then you might actually prefer the 1st option because you might want to use your $100,000 for your amazing investment opportunity.

Topic: Investment Decision Rules

3) You are considering building a power plant. Today is t = 0. Building the power plant will require a $2bn investment at t = 1, and another $300M investment at t = 2. Starting at t = 4, there will be a maintenance cost each year. The maintenance cost at t = 4 is expected to be $30M, and is expected to grow by 3% each year thereafter. The project is expected to generate $200M in revenues at t = 5, $300M in revenues at t = 6, and revenues are expected to grow 4% each year thereafter, forever.

Assume the appropriate discount rate for a project with this type of risk is 10%.

Based on the NPV rule, should you proceed with the project?

$$NPV = -\frac{2bn}{1.10} - \frac{300M}{1.10^2} - \frac{30M}{0.10 - 0.03}\frac{1}{(1.10)^3} + \frac{200M}{1.10^5} + \frac{300M}{0.10 - 0.04}\frac{1}{(1.10)^5} = \$840.68M$$

Since NPV > 0, accept the project.

Note: The maintenance costs are a growing perpetuity, where the first cash flow is 30M at t = 4. The expression $\frac{30M}{0.10-0.02}$ gives you the value of those cash flows as of t = 3. So, to get it back to t = 0, you have to divide that expression by $(1.10)^3$. The same logic applies to the growing revenues. You can view the $300M cash flow at t = 6 as the first cash flow of a growing perpetuity. Thus you would have to discount the value of those cash flows back five periods to get the present value.