MAT 2375
Midterm Examination

February 14, 2013 **Professor G. Ivanoff**
Time: 80 minutes

Student Number:__________________

Family Name: ____________________ **First Name:** _____________

- This is an open book examination. No electronic devices other than Faculty approved calculators are permitted.
- Record your answer (a,b,c,d, or e) to each question in the table below.

Question	Answer
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

NOTE: At the end of the examination, hand in only this page. You may keep the questionnaire.

.************************.

Professor's use only: Grade=______/10

1. Let $X_1, ..., X_n$ be a random sample from a continuous distribution with probability density function

$$f(x) = \theta x^{\theta-1}, 0 \leq x \leq 1,$$

where $\theta > 0$ is a constant. Find the maximum likelihood estimator of θ.

(a) $-\frac{1}{n}\sum_{i=1}^{n} \ln X_i$ (b) $-\frac{n}{\sum_{i=1}^{n} \ln X_i}$ (c) $\overline{X}$ (d) $\frac{\overline{X}}{1-\overline{X}}$ (e) $\frac{1-\overline{X}}{\overline{X}}$

2. Let $X_1, ..., X_n$ be a random sample from a continuous distribution with probability density function

$$f(x) = \theta x^{\theta-1}, 0 \leq x \leq 1,$$

where $\theta > 0$ is a constant. Find the method of moments estimator of θ.

(a) $-\frac{1}{n}\sum_{i=1}^{n} \ln X_i$ (b) $-\frac{n}{\sum_{i=1}^{n} \ln X_i}$ (c) $\overline{X}$ (d) $\frac{\overline{X}}{1-\overline{X}}$ (e) $\frac{1-\overline{X}}{\overline{X}}$

3. A scientist is attempting to monitor the mean daily intake of dairy products by Canadian women following a publicity campaign promoting the importance of calcium in the diet. Before the campaign, a woman's daily intake of dairy products could be represented by a normal random variable with mean 740 grams and standard deviation 30 grams. After the campaign, a sample of 25 women yielded an average of $\overline{x} = 756$ grams per day and a sample standard deviation $s = 35$ grams per day. Assuming that the daily intake is still normal and that the standard deviation has not changed, find a 95% confidence interval for the mean intake after the campaign. Based on this confidence interval, does the campaign appear to have increased the mean daily intake?

(a) 756 ± 2.15, yes (b) 756 ± 14.48, no (c) 756 ± 14.48, yes
(d) 740 ± 11.76, no (e) 756 ± 11.76, yes

4. The scientist in question 3 wants to better control his sampling error. If he want to be 95% certain that the maximum error of his estimate of the mean intake is at most 5, what size of sample should he have taken?

   (a) 139 (b) 189 (c) 98 (d) 188 (e) 138

5. An assumption was made in question 3 above that the variance of the daily intake did not change after the campaign. Still assuming normality, use the data provided in question 3 to find a 90% confidence interval for the variance σ^2 of the daily intake following the campaign. Based on this confidence interval, is it reasonable to assume that the variance did not change?

   (a) [21.34,67.47], yes (b) [807.25,2122.74], no (c) [807.25,2122.74], yes
   (d) [746.95,2370.97], yes (e) [746.95,2370.97], no

6. A firm polled 100 Canadians, and 90 of those questioned are in favour of reducing the GST. Based on this data, the polling firm claims that the true proportion of Canadians in favour of reducing the GST is at least 0.849 . What level of confidence should we have in this lower bound?

   (a) .9500 (b) .9750 (c) .9108 (d) .9000 (e) .9554

7. In question 6, in fact all of those polled were men. A second sample of 100 women was taken, and 80 of the women were in favour of reducing the GST. Find a 90% confidence interval for the difference $p_M - p_F$, where p_M and p_F are, respectively, the proportion of males and females in favour of reducing the GST. Based on this interval, would you conclude that there is a difference between the proportions?

   (a) $.10 \pm 0.098$, yes (b) $.10 \pm 0.098$, no (c) $.10 \pm 0.082$, yes
   (d) $.10 \pm 0.082$, no (e) $.10 \pm 0.064$, no

8. The manager of a car plant is investigating how the monthly electricity usage in kWh (Y) depends on the monthly production in millions of dollars (x). After 12 months of observations, the following summary data was obtained:

$$\sum_{i=1}^{12} x_i = 58,\ \ \sum_{i=1}^{12} x_i^2 = 291,\ \ \sum_{i=1}^{12} y_i = 33,\ \ \sum_{i=1}^{12} y_i^2 = 99,$$

$$\sum_{i=1}^{12} x_i y_i = 169$$

Based on this data, the least squares regression line was constructed. If a production level of $\$5$ million dollars is planned for next month, what electricity usage (in kWh) will the manager predict?

(a) 2.898 (b) 1.602 (c) 7.203 (d) 2.847 (e) 5.655

9. An experimenter is trying to investigate the water absorption properties of cotton run between two rollers containing a bath of water. He wants to see if the pressure of the rollers (in pounds per square inch) affects the percent water pickup of the cotton (the percent increase in the weight of the cotton due to water absorption). He observes 15 cases (X) with the pressure of the rollers at 10 pounds per square inch and 15 cases (Y) with the rollers at 20 pounds per square inch. His observations were:

$$\overline{x} = 59.81,\ s_X = 4.943,\ \overline{y} = 48.56,\ s_Y = 4.991$$

Assuming that the two distributions are independent and normal with equal variances, find a 90% confidence interval for the difference in the mean percent water pickup ($\mu_X - \mu_Y$).

(a) 11.25 ± 2.984 (b) 11.25 ± 3.085 (c) 11.25 ± 3.714 (d) 11.25 ± 2.381
(e) 11.25 ± 1.384

10. Let S_X^2 be the sample variance of a sample of size 10 taken from a $N(\mu_X, \sigma_X^2)$ distribution and let S_Y^2 be the sample variance of a sample of size 6 taken from $N(\mu_Y, \sigma_Y^2)$ distribution. If the two samples are independent and if it is known that $\sigma_X^2 = 2\sigma_Y^2$, find a constant c such that

$$P\left(\frac{S_X^2}{S_Y^2} \le c\right) = 0.99.$$

(a) 12.12 (b) 3.03 (c) 20.32 (d) 5.08 (e) 10.16