MAST-218 FINAL EXAM PREPARATION PROBLEMS

Problem 1.

(a) Find equation of an ellipse with the foci at the points (-1,2) , (3,2), and the large semiaxis $a=3$.

(b) Find an equation of a hyperbola with the asymptots $y=\pm 2x$ and the foci $(\pm 1,0)$.

(c) Find the equation of a parabola with the directrix $y=0$ and the focus $(1,2)$.

(d) Find the equation of a hyperbola with the foci $(\pm 2,0)$ and the vertices $(\pm 1,0)$.

Problem 2.

(a) Find equation of a plane containing the point $P(1,-1,2)$ and the line $\frac{x+1}{2}=\frac{y-2}{3}=\frac{z+4}{8}$.

(b) Find equation of a line which is contained in the plane $3x+2y-z=4$ and is orthogonal to the line $x=t-4,\ y=2t++1,\ z=-2t+3$.

(c) Find the equation of a plane which is orthogonal to the plane $3x-2y+z=4$ and contains the line $x-4=2x+1=0.2z+3$.

Find equation of a line forming an angle $\frac{\pi}{4}$ with the z -axis and an angle $\frac{\pi}{3}$ with the x -axis, and contains the point $(1,2,3)$.

Problem 3. Find the length of the following curves:
(a) $r=e^{\theta}$, $0\le\theta\le\pi$;

(b) $x=\ln\cos t$, $y=t$, $0\le t\le 1$;

(c) $x=\ln t,\ y=2t,\ z=t^2$, $1\le t\le 2$.

(d) $x=\cos t,\ y=2\sin t,\ z=\cos 2t$, $0\le t\le\pi$.

(e) $x=t-\sin t,\ y=t+\sin t,\ z=\frac{1}{2\sqrt{2}}(t+\sin t\cos t)$.

Problem 4.

(a) Find the area inside the curve $r=3+\cos 2\theta$, $0\le\theta<2\pi$;

(b) Find the area of domain bounded by the curve $r=e^{\theta/4}$ and the rays $\theta=0$ and $\theta=\pi$ in the upper semi-plane.

(c) Find the area inside the closed curve $x=t-t^{2},\ y=\sin(2\pi t)$, $0\le t\le 1$.

(d) Find the area inside the closed curve $x=\sin 2t,\ y=\sin t$, $0\le t\le\pi$.

Problem 5. Find the curvature of a given curve at a given point:

(a) Ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ at the point $\left(\frac{16}{5},\frac{9}{5}\right)$;

(b) Hyperbola $\frac{y^2}{25}-\frac{x^2}{9}=1$ at the point $\left(\frac{25}{4},\frac{9}{4}\right)$;

(c) $r=\cos\theta+1$ at the point $\theta=0,\ r=2$;

(d) $r=e^{\theta/4}$ at any point.

Problem 6.

(a) Find all the points on the surface $x^2+\frac{y^2}{4}+\frac{z^2}{9}=1$ such that the tangent plane at those points is parallel to the plane $x+y+z=1$.

(b) Find all the points on the surface $x^2+\frac{y^2}{4}-\frac{z^2}{16}=-1$ such that the tangent plane at those points is parallel to the plane $x+y-z=-1$.

(c) Find the tangent plane and the normal line to the graph of the function $z=f(x,y)=\cos x\,\cos 2y$ at the point $x=y=\frac{\pi}{6}$.

(d) Consider the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, $0<a<b<c$. Find all the points (x,y,z) on this ellipsoid such that the normal vector $\boldsymbol{n}$ at these points is parallel to the position vector $\boldsymbol{r}=(x,y,z)$.

Problem 7.

(a) Let $f(x,y,z)=x^4y^2+y^4z^2$. Find the unit vector $\boldsymbol{u}$ such that it shows the direction of the fastest decay of the function f at the point $(1,2,3)$.

(b) Let $f(x,y,z)=\sin(x+y)\sin z$; let $x=u+v,\ y=u-v,\ x=2u-v$. Find $\frac{\partial f}{\partial u},\ \frac{\partial f}{\partial v}$ at the point $u=v=1$.

(c) Let the function $y=f(x)$ satisfy the following relation: $\sin^3 x+\sin^3 y=1$. Find $\frac{dy}{dx}$ in terms of $x,\ y$.

(d) Let $f(x,y,z)=x^2+4y^2-9z^2$. Consider the line $x=t+1$, $y=t-2$

, $z=t+3$. Find t such that $\frac{df}{dt}=0$; prove that for this t , $\nabla f(x(t), y(t), z(t))$ is orthogonal to to the vector $(1,1,1)$.

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Problem 8.

(a) Find the best linear approximation for the function $f(x, y)=e^{2x} \sin y$ near the point $(1,1)$. Evaluate numerically the error for $(x, y)=(1.01, 0.99)$.

(b) Find the best linear approximation for the function $f(x, y)=\ln(x^2+y^2)$ near the point (1, 2). Evaluate numerically the error for $(x, y)=(0.99, 2.01)$.

Problem 9.

(a) Find all critical points of the function $f(x, y)=\sin 2x-2\sin y$ and classify them.

(b) Find all critical points of the function $f(x, y)=(2x^2-x^4)(1-y^2)$ and classify them.

(c) Find all critical points of the function $f(x, y)=\sin(x+y)+\sin(x-y)$ and classify them.

(d) Find all critical points of the function $f(x, y)=(e^x+e^{-x})\sin y$ and classify them.

Problem 10.

(a) Find the absolute maximum of the function $f(x, y)=x^3-3xy^2$ in the disk $x^2+y^2\le 1$.

(b) Find the absolute maximum of the function $f(x,y)=(x-1)^2+(y-2)^2$ in the square $0\le x,\ y\le 1$.

(c) Find absolute minimum of the function $f(x,y)=(x+2\mathrm{y})^2$ in the square $1\le x,\ y\le 2$.

(d) Find the point P on the graph of the function $f(x,y)=1-x-2y$ such that (i) (x,y) lies in the disk $x^2+y^2\le 1$, and (ii) the distance between the points P and $Q(1,2,3)$ is minimal.