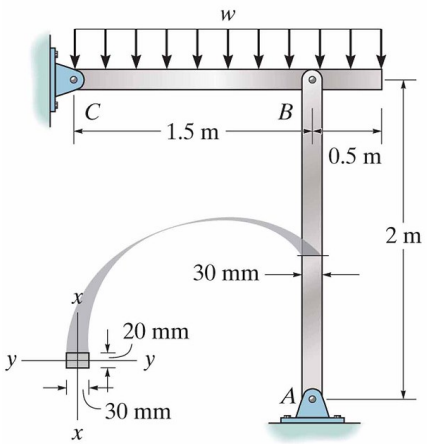
CVG3141 Mechanics of Materials II
Assignment 2 (Solution)

1. Determine if the frame can support a load of $w = 6$ kN/m if the factor of safety with respect to buckling of member AB is 3. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



Check $x-x$ axis buckling:

$$I_x = \frac{1}{12}(0.02)(0.03)^3 = 45.0(10^{-9})\ \text{m}^4$$

$$K = 1.0 \qquad L = 2\ \text{m}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(45.0)(10^{-9})}{((1.0)(2))^2}$$

$$P_{cr} = 22.2\ \text{kN}$$

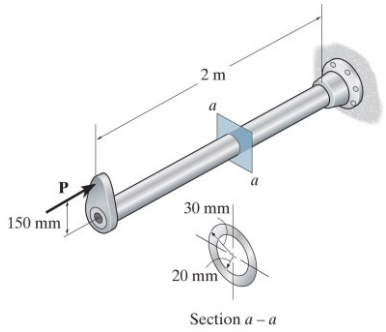
$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}(1.5) - 6(2)(1) = 0$$

$$F_{AB} = 8\ \text{kN}$$

$$P_{req'd} = 8(3) = 24\ \text{kN} > 22.2\ \text{kN}$$

No, AB will fail.

2. The hollow red brass C83400 copper alloy shaft is fixed at one end but free at the other end. Determine the maximum eccentric force P the shaft can support without causing it to buckle or yield. Also, find the corresponding maximum deflection of the shaft.
$E = 101$ GPa, $\sigma_Y = 70$ MPa.



Section Properties.

$$A = \pi\left(0.03^2 - 0.02^2\right) = 0.5\left(10^{-3}\right)\pi\,\text{m}^2$$

$$I = \frac{\pi}{4}\left(0.03^4 - 0.02^4\right) = 0.1625\left(10^{-6}\right)\pi\,\text{m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.1625\left(10^{-6}\right)\pi}{0.5\left(10^{-3}\right)\pi}} = 0.01803\,\text{m}$$

$e = 0.15$ m $\qquad\qquad$ $c = 0.03$ m

For a column that is fixed at one end and free at the other, $K = 2$. Thus,

$$KL = 2(2) = 4\,\text{m}$$

Yielding. In this case, yielding will occur before buckling. Applying the secant formula,

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{KL}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

$$70.0\left(10^6\right) = \frac{P}{0.5\left(10^{-3}\right)\pi}\left[1 + \frac{0.15(0.03)}{0.01803^2}\sec\left[\frac{4}{2(0.01803)}\sqrt{\frac{P}{101\left(10^9\right)\left[0.5\left(10^{-3}\right)\pi\right]}}\right]\right]$$

$$70.0\left(10^6\right) = \frac{P}{0.5\left(10^{-3}\right)\pi}\left(1 + 13.846\sec 8.8078\left(10^{-3}\right)\sqrt{P}\right)$$

Solving by trial and error,

$$P = 5.8697\,\text{kN} = 5.87\,\text{kN} \qquad \textbf{Ans.}$$

Maximum Deflection.

$$v_{\max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right]$$

$$= 0.15\left[\sec\left[\sqrt{\frac{5.8697\left(10^3\right)}{101\left(10^9\right)\left[0.1625\left(10^{-6}\right)\pi\right]}}\left(\frac{4}{2}\right)\right] - 1\right]$$

$$= 0.04210\text{ m} = 42.1\text{ mm}$$ **Ans.**