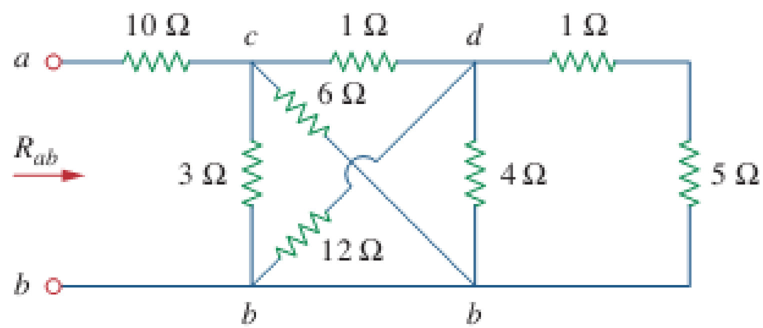
Assignment-1: 100 Marks

(10 Questions, 10 Marks each)

Problem-1:

Calculate the equivalent resistance R_{ab} in the circuit.



Solution:

The 3-Ω and 6-Ω resistors are in parallel because they are connected to the same two nodes c and b. Their combined resistance is

$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3 + 6} = 2\ \Omega \qquad \textbf{(2.10.1)}$$

Similarly, the 12-Ω and 4-Ω resistors are in parallel since they are connected to the same two nodes d and b. Hence

$$12\ \Omega \parallel 4\ \Omega = \frac{12 \times 4}{12 + 4} = 3\ \Omega \qquad \textbf{(2.10.2)}$$

Also the 1-Ω and 5-Ω resistors are in series; hence, their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega \qquad \textbf{(2.10.3)}$$

With these three combinations, we can replace the circuit in Fig. 2.37 with that in Fig. 2.38(a). In Fig. 2.38(a), 3-Ω in parallel with 6-Ω gives 2-Ω, as calculated in Eq. (2.10.1). This 2-Ω equivalent resistance is now in series with the 1-Ω resistance to give a combined resistance of $1\ \Omega + 2\ \Omega = 3\ \Omega$. Thus, we replace the circuit in Fig. 2.38(a) with that in Fig. 2.38(b). In Fig. 2.38(b), we combine the 2-Ω and 3-Ω resistors in parallel to get

$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

This 1.2-Ω resistor is in series with the 10-Ω resistor, so that

$$R_{ab} = 10 + 1.2 = 11.2\ \Omega$$

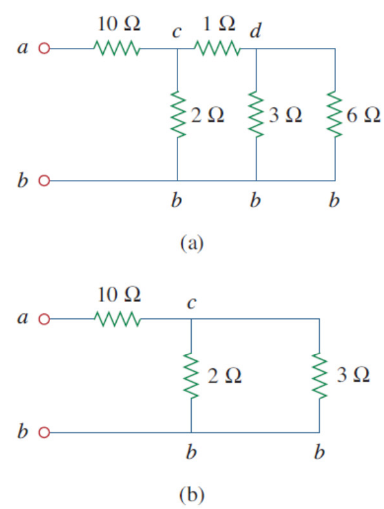
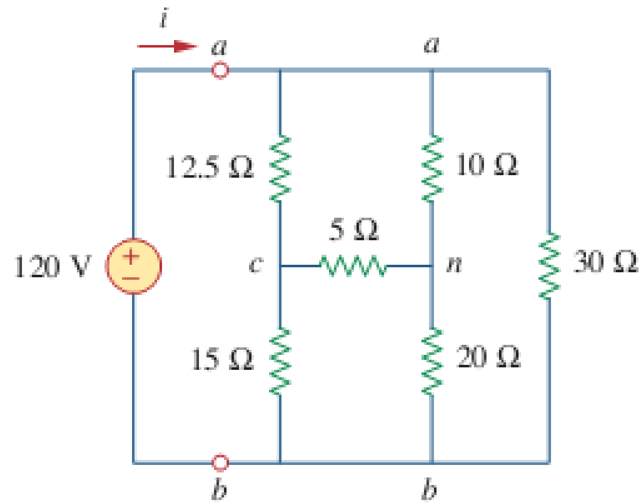


Figure 2.38
Equivalent circuits for Example 2.10.

Problem-2:

Calculate the equivalent resistance R_{ab} in the circuit and then find the current i



Solution:

1. **Define.** The problem is clearly defined. Please note, this part normally will deservedly take much more time.
2. **Present.** Clearly, when we remove the voltage source, we end up with a purely resistive circuit. Since it is composed of deltas and wyes, we have a more complex process of combining the elements together. We can use wye-delta transformations as one approach to find a solution. It is useful to locate the wyes (there are two of them, one at n and the other at c) and the deltas (there are three: *can, abn, cnb*).
3. **Alternative.** There are different approaches that can be used to solve this problem. Since the focus of Sec. 2.7 is the wye-delta transformation, this should be the technique to use. Another approach would be to solve for the equivalent resistance by injecting one amp into the circuit and finding the voltage between a and b; we will learn about this approach in Chap. 4.

   The approach we can apply here as a check would be to use a wye-delta transformation as the first solution to the problem. Later we can check the solution by starting with a delta-wye transformation.
4. **Attempt.** In this circuit, there are two Y networks and three Δ networks. Transforming just one of these will simplify the circuit. If we convert the Y network comprising the 5-Ω, 10-Ω, and 20-Ω resistors, we may select

$$R_1 = 10\ \Omega, \qquad R_2 = 20\ \Omega, \qquad R_3 = 5\ \Omega$$

Thus from Eqs. (2.53) to (2.55) we have

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10}$$
$$= \frac{350}{10} = 35\ \Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$

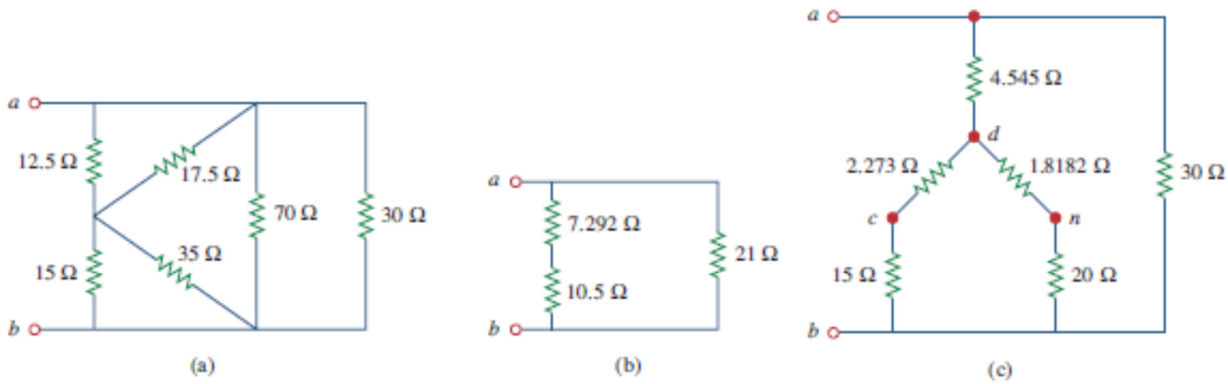


Figure 2.53
Equivalent circuits to Fig. 2.52, with the voltage source removed.

With the Y converted to Δ, the equivalent circuit (with the voltage source removed for now) is shown in Fig. 2.53(a). Combining the three pairs of resistors in parallel, we obtain

$$70 \parallel 30 = \frac{70 \times 30}{70 + 30} = 21\ \Omega$$

$$12.5 \parallel 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292\ \Omega$$

$$15 \parallel 35 = \frac{15 \times 35}{15 + 35} = 10.5\ \Omega$$

so that the equivalent circuit is shown in Fig. 2.53(b). Hence, we find

$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = \mathbf{9.632\ \Omega}$$

Then

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = \mathbf{12.458\ A}$$

We observe that we have successfully solved the problem. Now we must evaluate the solution.

5. **Evaluate.** Now we must determine if the answer is correct and then evaluate the final solution.

   It is relatively easy to check the answer; we do this by solving the problem starting with a delta-wye transformation. Let us transform the delta, *can*, into a wye.

   Let $R_c = 10\ \Omega$, $R_a = 5\ \Omega$, and $R_n = 12.5\ \Omega$. This will lead to (let d represent the middle of the wye):

$$R_{ad} = \frac{R_c R_n}{R_a + R_c + R_n} = \frac{10 \times 12.5}{5 + 10 + 12.5} = 4.545\ \Omega$$

$$R_{cd} = \frac{R_a R_n}{27.5} = \frac{5 \times 12.5}{27.5} = 2.273\ \Omega$$

$$R_{nd} = \frac{R_a R_c}{27.5} = \frac{5 \times 10}{27.5} = 1.8182\ \Omega$$

This now leads to the circuit shown in Figure 2.53(c). Looking at the resistance between *d* and *b*, we have two series combination in parallel, giving us

$$R_{db} = \frac{(2.273 + 15)(1.8182 + 20)}{2.273 + 15 + 1.8182 + 20} = \frac{376.9}{39.09} = 9.642\ \Omega$$

This is in series with the 4.545-Ω resistor, both of which are in parallel with the 30-Ω resistor. This then gives us the equivalent resistance of the circuit.

$$R_{ab} = \frac{(9.642 + 4.545)30}{9.642 + 4.545 + 30} = \frac{425.6}{44.19} = \mathbf{9.631\ \Omega}$$

This now leads to

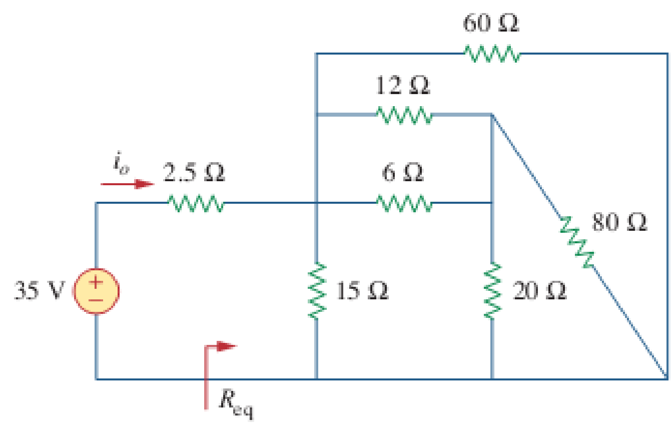
$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.631} = \mathbf{12.46\ A}$$

We note that using two variations on the wye-delta transformation leads to the same results. This represents a very good check.

6. **Satisfactory?** Since we have found the desired answer by determining the equivalent resistance of the circuit first and the answer checks, then we clearly have a satisfactory solution. This represents what can be presented to the individual assigning the problem.

Problem-3:

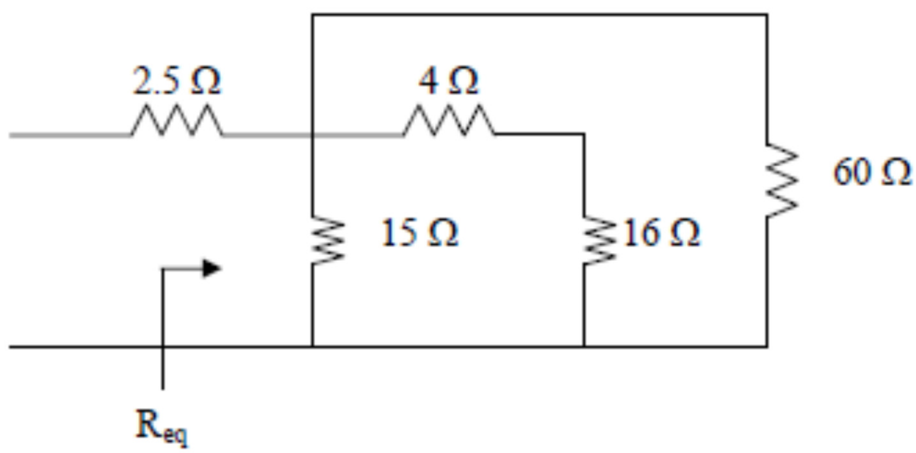
Calculate the equivalent resistance R_{eq} and then find the current i_0.



Solution:

20//80 = 80x20/100 = 16, 6//12 = 6x12/18 = 4

The circuit is reduced to that shown below.



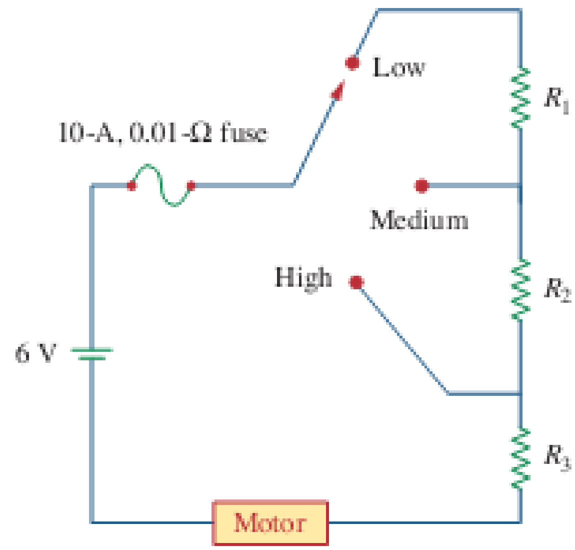
(4 + 16)//60 = 20x60/80 = 15

R_{eq} = 2.5+15||15 = 2.5+7.5 = **10 Ω** and

i_o = 35/10 = **3.5 A**.

Problem-4:

An SFU student has been hired as a co-op student by BC Hydro to design the circuit shown to control the speed of a motor such that the motor draws currents 5A, 3A, and 1A, when the switch is at high, medium, and low positions, respectively.



The motor can be modeled as a load resistance of 20 mΩ, while the 10A fuse has very low resistance 0.01-Ω.

Determine the series dropping resistances R_1, R_2 *and* R_3.

Solution:

With the switch in high position,

$6 = (0.01 + R_3 + 0.02) \times 5 \longrightarrow R_3 = \mathbf{1.17\ \Omega}$

At the medium position,

$6 = (0.01 + R_2 + R_3 + 0.02) \times 3 \longrightarrow R_2 + R_3 = 1.97$

or $R_2 = 1.97 - 1.17 = \mathbf{0.8\ \Omega}$

At the low position,

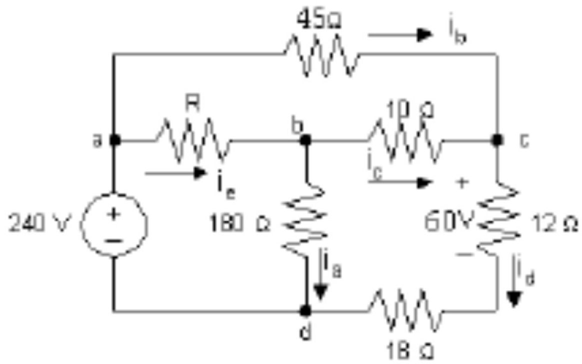
$6 = (0.01 + R_1 + R_2 + R_3 + 0.02) \times 1 \longrightarrow \quad R_1 + R_2 + R_3 = 5.97$

$R_1 = 5.97 - 1.97 = \mathbf{4\ \Omega}$

Textbook questions:
2.24, 2.28, 3.58, 4.21, 4.39, 4.93.

Problem: 2.24:

P 2.24



$i_d = 60/12 = 5\,\text{A}$; therefore, $v_{cd} = 60 + 18(5) = 150\,\text{V}$

$-240 + v_{ac} + v_{cd} = 0$; therefore, $v_{ac} = 240 - 150 = 90\,\text{V}$

$i_b = v_{ac}/45 = 90/45 = 2\,\text{A}$; therefore, $i_c = i_d - i_b = 5 - 2 = 3\,\text{A}$

$v_{bd} = 10i_c + v_{cd} = 10(3) + 150 = 180\,\text{V}$;

therefore, $i_a = v_{bd}/180 = 180/180 = 1\,\text{A}$

$i_e = i_a + i_c = 1 + 3 = 4\,\text{A}$

$-240 + v_{ab} + v_{bd} = 0$ therefore, $v_{ab} = 240 - 180 = 60\,\text{V}$

$R = v_{ab}/i_e = 60/4 = 15\,\Omega$

CHECK: $i_g = i_b + i_e = 2 + 4 = 6$ A

$p_{dev} = (240)(6) = 1440$ W

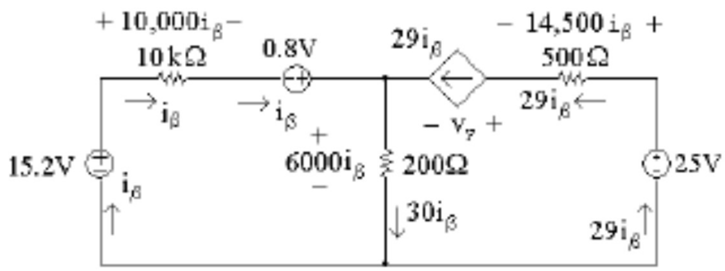
$$\sum P_{dis} = 1^2(180) + 4^2(15) + 3^2(10) + 5^2(12) + 5^2(18) + 2^2(45)$$

$$= 1440 \text{ W (CHECKS)}$$

P 2.28 First note that we know the current through all elements in the circuit except the 200 Ω resistor (the current in the three elements to the left of the 200 Ω resistor is i_β; the current in the three elements to the right of the 200 Ω resistor is $29i_\beta$). To find the current in the 200 Ω resistor, write a KCL equation at the top node:

$$i_\beta + 29i_\beta = i_{200\Omega} = 30i_\beta$$

We can then use Ohm's law to find the voltages across each resistor in terms of i_β. The results are shown in the figure below:



[a] To find i_β, write a KVL equation around the left-hand loop, summing voltages in a clockwise direction starting below the 15.2V source:

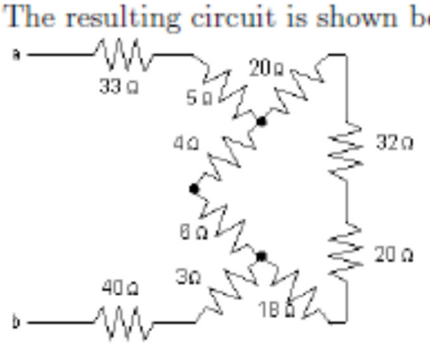
$$-15.2\,\text{V} + 10{,}000i_1 - 0.8\,\text{V} + 6000i_\beta = 0$$

P 3.58 Replace the upper and lower deltas with the equivalent wyes:

$$R_{1\text{U}} = \frac{(10)(50)}{100} = 5\,\Omega;\; R_{2\text{U}} = \frac{(50)(40)}{100} = 20\,\Omega;\; R_{3\text{U}} = \frac{(10)(40)}{100} = 4\,\Omega$$

$$R_{1\text{L}} = \frac{(10)(60)}{100} = 6\,\Omega;\; R_{2\text{L}} = \frac{(60)(30)}{100} = 18\,\Omega;\; R_{3\text{L}} = \frac{(10)(30)}{100} = 3\,\Omega$$

The resulting circuit is shown below:



Now make series and parallel combinations of the resistors:

$$(4 + 6)\|(20 + 32 + 20 + 18) = 10\|90 = 9\,\Omega$$

$$R_{\text{ab}} = 33 + 5 + 9 + 3 + 40 = 90\,\Omega$$

Solving for i_β

$10{,}000i_\beta + 6000i_\beta = 16\,\text{V}$ so $16{,}000i_\beta = 16\,\text{V}$

Thus,

$$i_\beta = \frac{16}{16{,}000} = 1\,\text{mA}$$

Now that we have the value of i_β, we can calculate the voltage for each component except the dependent source. Then we can write a KVL equation for the right-hand loop to find the voltage v_y of the dependent source. Sum the voltages in the clockwise direction, starting to the left of the dependent source:

$$-v_y - 14{,}500i_\beta + 25\,\text{V} - 6000i_\beta = 0$$

Thus,

$$v_y = 25\,\text{V} - 20{,}500i_\beta = 25\,\text{V} - 20{,}500(10^{-3}) = 25\,\text{V} - 20.5\,\text{V} = 4.5\,\text{V}$$

[b] We now know the values of voltage and current for every circuit element. Let's construct a power table:

Element	Current (mA)	Voltage (V)	Power Equation	Power (mW)
15.2 V	1	15.2	$p = -vi$	−15.2
10 kΩ	1	10	$p = Ri^2$	10
0.8 V	1	0.8	$p = -vi$	−0.8
200 Ω	30	6	$p = Ri^2$	180
Dep. source	29	4.5	$p = vi$	130.5
500 Ω	29	14.5	$p = Ri^2$	420.5
25 V	29	25	$p = -vi$	−725

The total power generated in the circuit is the sum of the negative power values in the power table:

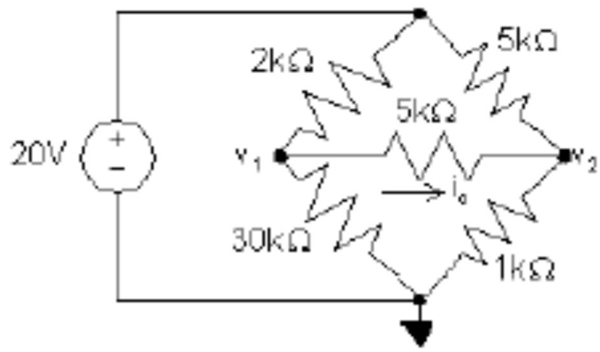
$$-15.2\,\text{mW} + -0.8\,\text{mW} + -725\,\text{mW} = -741\,\text{mW}$$

Thus, the total power generated in the circuit is 741 mW. The total power absorbed in the circuit is the sum of the positive power values in the power table:

$$10\,\text{mW} + 180\,\text{mW} + 130.5\,\text{mW} + 420.5\,\text{mW} = 741\,\text{mW}$$

Thus, the total power absorbed in the circuit is 741 mW and the power in the circuit balances.

P 4.21



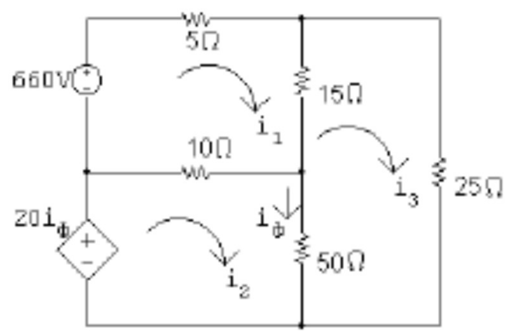
$$\frac{v_1}{30{,}000} + \frac{v_1 - v_2}{5000} + \frac{v_1 - 20}{2000} = 0 \qquad \text{so} \quad 22v_1 - 6v_2 = 300$$

$$\frac{v_2}{1000} + \frac{v_2 - v_1}{5000} + \frac{v_2 - 20}{5000} = 0 \qquad \text{so} \quad -v_1 + 7v_2 = 20$$

Solving, $v_1 = 15$ V; $\quad v_2 = 5$ V

Thus, $i_o = \frac{v_1 - v_2}{5000} = 2$ mA

P 4.39



$660 = 30i_1 - 10i_2 - 15i_3$

$20i_\phi = -10i_1 + 60i_2 - 50i_3$

$0 = -15i_1 - 50i_2 + 90i_3$

$i_\phi = i_2 - i_3$

Solving, $i_1 = 42$ A; $\quad i_2 = 27$ A; $\quad i_3 = 22$ A; $\quad i_\phi = 5$ A

$20i_\phi = 100$ V

$p_{20i_\phi} = -100 i_2 = -100(27) = -2700$ W

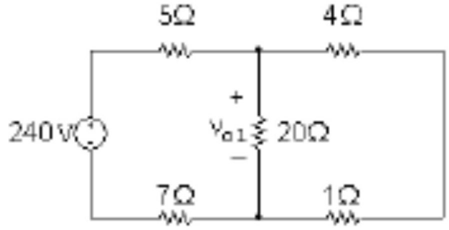
$\therefore \; p_{20i_\phi}$ (developed) $= 2700$ W

CHECK:

$p_{660\text{V}} = -660(42) = -27{,}720$ W (dev)

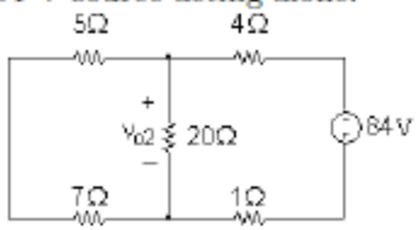
$$\begin{aligned}\therefore \; \sum P_{\text{dev}} &= 27{,}720 + 2700 = 30{,}420 \text{ W} \\ \sum P_{\text{dis}} &= (42)^2(5) + (22)^2(25) + (20)^2(15) + (5)^2(50) + \\ &\quad (15)^2(10) \\ &= 30{,}420 \text{ W}\end{aligned}$$

P 4.93 240 V source acting alone:



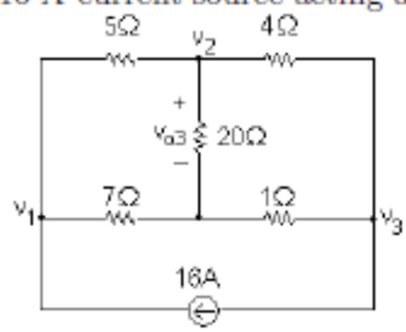
$$v_{o1} = \frac{20\|5}{5+7+20\|5}(240) = 60 \text{ V}$$

84 V source acting alone:



$$v_{o2} = \frac{20\|12}{1+4+20\|12}(-84) = -50.4 \text{ V}$$

16 A current source acting alone:



$$\frac{v_1 - v_2}{5} + \frac{v_1}{7} - 16 = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{20} + \frac{v_2 - v_3}{4} = 0$$

$$\frac{v_3 - v_2}{4} + \frac{v_3}{1} + 16 = 0$$

Solving, $v_2 = 18.4$ V $= v_{o3}$. Therefore,

$$v_o = v_{o1} + v_{o2} + v_{o3} = 60 - 50.4 + 18.4 = 28 \text{ V}$$