Concordia University

Department of Computer Science & Software Engineering

COMP 232/2 **Mathematics for Computer Science** Fall 2014

Assignment 1 Due date: September 23, 2014

1. For each of the following statements use a truth table to determine whether it is a tautology, a contradiction, or a contingency.

   (a) $((p \vee r) \wedge (q \vee r)) \leftrightarrow ((p \wedge q) \vee r)$

   (b) $(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q)$

   (c) $(p \wedge (\neg q \rightarrow \neg p)) \rightarrow q$

   (d) $((p \rightarrow r) \vee (q \rightarrow r)) \rightarrow ((p \vee q) \rightarrow r)$

2. For each of the following logical equivalences state whether it is valid or invalid. If invalid then give a counterexample (*e.g.*, based on a truth table). If valid then give an algebraic proof using logical equivalences from Tables 6, 7, and 8 from Section 1.3 of textbook.

   (a) $p \rightarrow (q \rightarrow r) \equiv q \rightarrow (\neg p \vee r)$

   (b) $(p \rightarrow r) \wedge (q \rightarrow r) \equiv ((p \wedge q) \rightarrow r)$

   (c) $(p \rightarrow q) \wedge (p \rightarrow r) \equiv (p \rightarrow (q \wedge r))$

   (d) $((p \vee q) \wedge (\neg p \vee r)) \equiv (q \vee r)$

3. Five persons, anonymously known as P_1, P_2, P_3, P_4, P_5, are suspected of being involved in a crime. Suppose we have the following information:

   (a) If P_5 is involved then so is P_3.

   (b) If P_2 is involved then so are P_5 and P_1.

   (c) Either P_1 or P_2, or both, are involved.

   (d) Either P_3 or P_4, but not both, are involved.

   (e) P_4 and P_1 are either both involved or neither is.

   Determine which persons were involved in the crime. Explain your reasoning.

4. Write the following statements in predicate form, using logical operators $\wedge$, $\vee$, $\neg$, and quantifiers $\forall$, $\exists$. Below $\mathbb{Z}^+$ denotes all positive integers $\{1, 2, 3, \ldots\}$.

   (a) For any $x, y \in \mathbb{Z}^+$ the equation $x^2 + y^2 - z = 0$ has a solution $z \in \mathbb{Z}^+$.

   (b) The equation $x^3 + y^3 = z^3$ has no solutions $x, y, z \in \mathbb{Z}^+$.

   (c) There is no greatest positive integer.

   (d) The difference between two positive integers can be arbitrarily large.

5. Let the domain for x be the set of all students in this class and the domain for y be the set of all countries in the world. Let $P(x, y)$ denote student x has visited country y and $Q(x, y)$ denote student x has a friend in country y. Express each of the following using logical operations and quantifiers, and the propositional functions $P(x, y)$ and $Q(x, y)$.

   (a) Carlos has visited Bulgaria.
   (b) Every student in this class has visited the United States.
   (c) Every student in this class has visited some country in the world.
   (d) There is no country that every student in this class has visited.
   (e) There are two students in this class, who between them, have a friend in every country in the world.
   (f) Nobody in this class has visited a country in which they did not have a friend.
   (g) There is a country in which nobody in this class has a friend.
   (h) Everyone in this class has visited every country in which they have a friend.

6. Negate the following statements and transform the negation so that negation symbols immediately precede predicates.

   (a) $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$
   (b) $\exists x \exists y (Q(x, y) \leftrightarrow Q(y, x))$
   (c) $\forall y \exists x \exists z (T(x, y, z) \vee Q(x, y))$

7. Let P and Q be predicates on the set S, where S has two elements, say, $S = \{a, b\}$. Then the statement $\forall x P(x)$ can also be written in full detail as $P(a) \wedge P(b)$. Rewrite each of the statements below in a similar fashion, using P, Q, and logical operators, but without using quantifiers.

   (a) $\forall x \forall y (P(x) \vee Q(y))$
   (b) $\exists x P(x) \wedge \exists x Q(x)$
   (c) $\exists x \exists y (P(x) \wedge Q(y))$
   (d) $\forall x \exists y (P(x) \wedge Q(y))$

8. For each of the following equivalences, determine if it is valid for all predicates P and Q. If yes then give a full explanation. If not then provide a counterexample.

   (a) $\exists x (P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$
   (b) $\exists x (P(x) \wedge Q(x)) \equiv \exists x P(x) \wedge \exists x Q(x)$
   (c) $\forall x \forall y (P(x) \wedge Q(y)) \equiv \forall x P(x) \wedge \forall x Q(x)$
   (d) $\forall x \forall y (P(x) \vee Q(y)) \equiv \forall x P(x) \vee \forall y Q(y)$