QUEEN'S UNIVERSITY

DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 112

Final Examination

April 26, 2013

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HAND IN
answers recorded
on question paper

Instructions: This is a **3-hour** exam. There are **6** questions worth a total of 70 marks as indicated in the box below. Answer all questions in the space provided. If you need more room, answer on the back of the previous page. Show all your work and explain how you arrived at your answers, unless explicitly told to do otherwise. Only **CASIO FX-991** or **Gold/Blue Sticker** calculators are permitted.

Please Note: Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.

Question	Possible	Received
Problem 1	20	
Problem 2	6	
Problem 3	8	
Problem 4	12	
Problem 5	12	
Problem 6	12	
Total	70	

Problem 1. Multiple Choice (10 questions, 2 marks each)

Each question has four possible answers, labeled (A), (B), (C), and (D). Choose the most appropriate answer. Write your answer in the space provided. Write clearly using **UPPERCASE** letters. Illegible answers will be given a zero. You DO NOT need to justify your answer.

(1) Determine which vector is an eigenvector of $A = \begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix}$

(A) $\boldsymbol{X}^T = [1, 2]$

(B) $\boldsymbol{X}^T = [2, 1]$

(C) $\boldsymbol{X}^T = [-1, 1]$

(D) $\boldsymbol{X}^T = [-1, 0]$

ANSWER TO (1): ______

(2) If $||\boldsymbol{\alpha}|| = 2$, $||\boldsymbol{\beta}|| = 5$, and $\boldsymbol{\alpha} \cdot \boldsymbol{\beta} = -3$, what is $||\boldsymbol{\alpha} + \boldsymbol{\beta}||$?

(A) 7

(B) 23

(C) $\sqrt{23}$

(D) 1

ANSWER TO (2): ______

(3) Let $AX = B$ be a linear system of m equations and n variables. Which one of the following is true?

(A) The system has a unique solution if the rank of the system is m

(B) The system has a unique solution if the rank of the system is n.

(C) The system has a unique solution if $\text{rank}(A) = m$.

(D) The system has a unique solution if $m = n$.

ANSWER TO (3): ______

(4) Let $\lambda_1, \lambda_2, \lambda_3$ be eigenvalues of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ x & -1 & 3 \\ y & z & 4 \end{bmatrix}$. If $\lambda_1 = 5, \lambda_2 = -3$, what is the possible value of λ_3?

(A) 1

(B) 2

(C) 3

(D) 4

ANSWER TO (4): ______

(5) If $\det(A) = 2, \det(B) = 3$, what is $\det(A^T B^{-1})$?

(A) 1/6

(B) 6

(C) 3/2

(D) 2/3

ANSWER TO (5): ______

(6) Find the standard matrix for the linear transformation

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x - 3y \\ x - y \\ y - 4x \end{bmatrix}$$

(A) $\begin{bmatrix} 2 & -3 \\ 1 & -1 \\ 1 & -4 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 1 & 1 \\ -3 & -1 & -4 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & -3 \\ 1 & -1 \\ -4 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}$

ANSWER TO (6): ______

(7) Determine which one of the following sets of vectors is linearly independent.

(A) $\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}$

(B) $\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}$

(C) $\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}$

(D) $\begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\3\end{bmatrix}$

ANSWER TO (7): ______

(8) Which one of the following is not a orthogonal set?

(A) $\begin{bmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{bmatrix}, \begin{bmatrix}1/\sqrt{2}\\-1/\sqrt{2}\end{bmatrix}$

(B) $\begin{bmatrix}1\\-1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\-2\\0\end{bmatrix}, \begin{bmatrix}-1\\1\\0\\2\end{bmatrix}$

(C) $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}$

(D) $\begin{bmatrix}1/\sqrt{2}\\0\\1/\sqrt{2}\end{bmatrix}, \begin{bmatrix}1/\sqrt{3}\\-1/\sqrt{3}\\-1/\sqrt{3}\end{bmatrix}, \begin{bmatrix}0\\1/\sqrt{2}\\1/\sqrt{2}\end{bmatrix}$

ANSWER TO (8): ______

(9) Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$. Then cof$(A)$ is

(A) $\begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 1 \\ -1 & 2 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 2 \\ -1 & 1 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & -2 \\ 1 & -1 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & -1 & 1 \\ 0 & 1 & -1 \\ 1 & -2 & 1 \end{bmatrix}$

ANSWER TO (9): ______

(10) The matrix is invertible if

(A) the corresponding homogeneous system of equations has a unique solutions.

(B) the column vectors are linearly dependent.

(C) the reduced row echelon form is $\begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix}$.

(D) the determinant of the matrix is zero.

ANSWER TO (10): ______

Problem 2. Prove that if A and B are $n \times n$ orthogonal matrices, then AB is also an orthogonal matrix.

Problem 3. Let $S = \{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\alpha}_3\} = \{ \begin{bmatrix} 2 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -12 \\ 13 \end{bmatrix} \}$. Write the vector $\boldsymbol{\alpha} = \begin{bmatrix} 2 \\ 8 \\ 3 \end{bmatrix}$ as a linear combination of the vectors in S

Problem 4. Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ such that

$$T\begin{bmatrix}0\\2\\1\end{bmatrix}=\begin{bmatrix}1\\-7\end{bmatrix},\quad T\begin{bmatrix}-1\\-2\\2\end{bmatrix}=\begin{bmatrix}0\\12\end{bmatrix},\quad T\begin{bmatrix}2\\1\\-1\end{bmatrix}=\begin{bmatrix}3\\0\end{bmatrix}$$

(a) Find the standard matrix A.

(Problem 4 continued)

(b) Use the matrix you found in part (a), find the image of $\begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix}$.

(c) Find the preimage of $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

Problem 5. Consider the difference equation $x_n = -x_{n-1} + 2x_{n-2}$ with initial conditions $x_0 = 1, x_1 = 2$, find a general formula for x_n in terms of n.

Problem 6. Consider the following quadratic equation

$$2x^2 + y^2 + z^2 + 2xy + 2xz = 5$$

(a) Express the quadratic equation in matrix form.

(b) Eliminate the product terms by finding and using an orthogonal matrix T.