MIDTERM 1

Microeconomic Theory III
ECO 3153A

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October 2, 2013

Notes

There are 7 questions in this test and 50 points in total. This means that you should budget about 1.5 minutes per mark.

Read over all of the questions. Do the questions that you know well first, then go to the ones that you are less sure of. I do not care in which order you answer the questions.

Basic calculators are allowed. No electronic device that can be programmed with text, or that has transmitting possibilities can be used.

No cellular phones are allowed.

1. A consumer is willing to trade 3 units of x for 1 unit of y when she has 6 units of x and 5 units of y. She is also willing to trade in 6 units of x for 2 units of y when she has 12 units of x and 3 units of y. She is indifferent between bundle (6, 5) and bundle (12, 3). Write out her utility function. (4 marks)

2. Suppose a strictly quasi-concave utility function U(X), where X = x_1, x_2, … x_n. Prove that the marginal rate of substitution between any two goods is invariant to any positive monotonic transformation of this function. (3 marks)

3. An individual enjoys the consumption of coffee (c) and biscuits (b) according to the function: $U(c,b) = 20c - c^2 + 18b - 3b^2$

a) If the individual has an unlimited budget, what is his optimal consumption of c and b? (4 marks)
b) Are these preferences monotonic? Explain. (3 marks)? if sus
c) Suppose the individual is constrained so that the sum of c plus b cannot exceed 5. How many b and c will she consume? (3 marks)

4. Let the utility function for an individual be $U(x_i, x_2) = x_1 x_2$ find the following:

a) Hicksian demands for goods 1 and 2; (6 marks)
b) the Expenditure function; (2 marks)
c) Prove that Shephard's lemma holds (3 marks)

5. Using the expenditure function $E(p,U) = 2p_1^{0.5}p_2^{0.5}U^{0.5}$ find the following:

a) The indirect utility function; (1 mark)
b) Marshallian demand for good 1. (3 marks)

6. The function $V(p,I) = \frac{I}{p_1 + 0.25{p_2}^{1/2}}$ conforms to the requirements of an indirect utility function. True/False/Uncertain? Explain your answer. (3 marks)

7. Suppose that $U(x_i, x_2) = x_1^{1/2}x_2^{1/2}$. Let $p_1 = 1$, $p_2 = 2$, and I (income) = \$100. The demand function for good i (i=1,2) is: $x_i = \frac{I}{2p_i}$.

a) Calculate **the substitution effect** and **the income effect** associated with a change in the price of good 1 from \$1 to \$2. (6 marks)

b) Suppose that the individual is offered the following choice: "The price of good 1 is returned to \$1 **OR** you receive \$50 and the price of good 1is \$2". Will the individual prefer to have an additional \$50 and p_1 = \$2, or no additional cash and p_1=\$1, or is the individual indifferent between these two options? Explain very carefully your answer. (5 marks)

c) Calculate the Compensating Variation associated with this increase in price. (1 marks)

d) Calculate the Equivalent Variation associated with this increase in price. (3 marks)

① Answers Midterm #1 Eco 3153.

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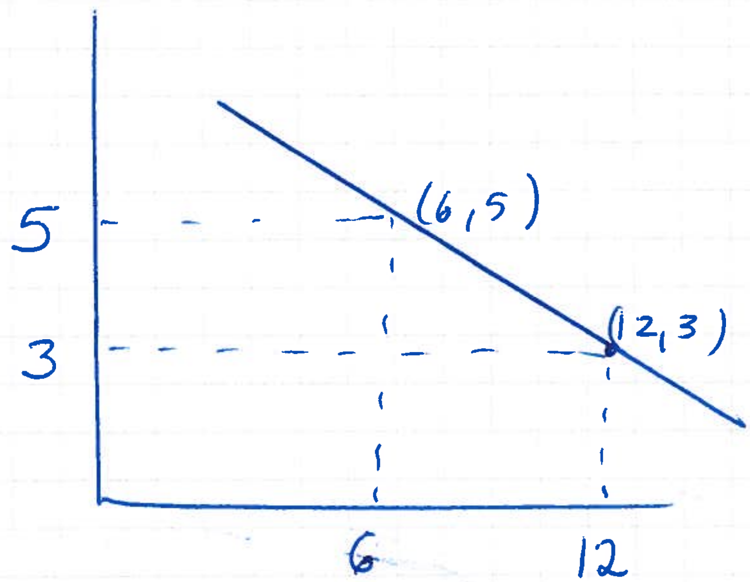
Q1. Book problem 3.7.

a diagram is useful.

These goods are perfect substitutes b/c the indifference curve is a straight line:



One possible u fn:

$$u = x_1 + 3x_2.$$

any u fn with $MRS_{2,1} = 1/3$ will work.

Q2. $u(x)$ $\quad MRS_{2,1} = -\frac{u_1}{u_2} = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2}$

Let T be a positive, monotonic transformation. $\left(\frac{dT(u(x))}{du(x)} > 0\right)$

define $V(x) = Tu(x)$. $MRS_{2,1} = -\frac{V_1}{V_2} = -\frac{T'u_1}{T'u_2}$

$= -u_1/u_2$ = same as above

②

Q3 see problem 4.3 text

a) $\max_{c,b} u(c,b) = 20c - c^2 + 18b - 3b^2$

FOCs $\frac{\partial u(c,b)}{\partial c} \Rightarrow 20 - 2c = 0$ ①

$\frac{\partial u(c,b)}{\partial b} \Rightarrow 18 - 6b = 0$ ②

from ① ② $\boxed{\begin{matrix} c = 10 \\ b = 3 \end{matrix}}$

b) No. These preferences are not monotonic as more is not always better than less.

Suppose $c = 15$ $b = 0$ $u = -25$

$c = 20$ $b = 0$ $u = -200$

because u fn has $-$quadradic $\Rightarrow u < 0$ as $c \uparrow$ or $b \uparrow$

c) $\max 20c - c^2 + 18b - 3b^2$

st $b + c \leq 5$

$\mathcal{L} = 20c - c^2 + 18b - 3b^2 + \lambda(5 - b - c)$

$\frac{\partial L}{\partial c} = 20 - 2c - \lambda = 0$ ①

③

$$\frac{\partial \mathcal{L}}{\partial b} = 18 - 6b - \lambda = 0 \quad ②$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 5 - b - c = 0 \quad ③$$

① + ②

$20 - 2c = 18 - 6b$

$-2c = -2 - 6b$

④ $- \quad c = 1 + 3b.$

put ④ into ③

$$5 - b - 1 - 3b = 0$$

$$4 - 4b = 0 \quad \boxed{\begin{array}{l} b = 1 \\ c = 4 \end{array}}$$

Q4. $u(x_1, x_2) = x_1 x_2$

a) Hicksian demand. Solve min costs

$$\left.\begin{array}{l} \min\limits_{x_1, x_2} \; p_1 x_1 + p_2 x_2 \\ \quad \text{st} \quad x_1 x_2 = u \end{array}\right\} \text{set up Lagrangean.}$$

$$\mathcal{L} = p_1 x_1 + p_2 x_2 + \lambda(u - x_1 x_2)$$

note $\min\limits_{x_1, x_2 \lambda.} \mathcal{L} = \max\limits_{x_1, x_2, \lambda} -\mathcal{L} = -p_1 x_1 - p_2 x_2 - \lambda(u - x_1 x_2$

$$\left.\begin{array}{l} \frac{\partial \mathcal{L}}{\partial x_1} \Rightarrow -p_1 + \lambda x_2 = 0 \quad ① \\ \frac{\partial \mathcal{L}}{\partial x_2} \Rightarrow -p_2 + \lambda x_1 = 0 \quad ② \end{array}\right\} \left.\begin{array}{l} \lambda = p_1 / x_2 \\ \lambda = p_2 / x_1 \end{array}\right\} \frac{p_1}{x_2} = \frac{p_2}{x_1}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = u - x_1 x_2 = 0 \quad ③$$

$$\Rightarrow \frac{x_1}{x_2} = \frac{p_2}{p_1}$$

$\Rightarrow x_1 = \frac{p_2}{p_1} x_2$ — ④ put into ③

(4) $\left(\frac{p_2}{p_1}\right) x_2 x_2 = u$ $\qquad x_2^2 = u \cdot \frac{p_1}{p_2}$

$$\boxed{\begin{aligned} x_2^h &= u^{1/2}\left(\frac{p_1}{p_2}\right)^{1/2} \\ x_1^h &= u^{1/2}\left(\frac{p_2}{p_1}\right)^{1/2} \end{aligned}}$$

$\Rightarrow \quad x_1^h = \frac{p_2}{p_1}\left(u^{1/2}\left(p_1/p_2\right)^{1/2}\right)$

b) Expenditure fn: $E = p_1 x_1^h + p_2 x_2^h$

$$\boxed{E = 2u^{1/2} p_1^{1/2} p_2^{1/2}.}$$

c) Shephard's Lemma $\Rightarrow \frac{\partial E}{\partial p_i} = x_i^h$

eg $\frac{\partial E}{\partial p_1} = 2u^{1/2}\, {1/2}\, p_1^{-1/2} p_2^{1/2} = u^{1/2}\left(\frac{p_2}{p_1}\right)^{1/2}$

Verified ✓

5 a) $V(x) = E^{-1}$

$2p_1^{1/2} p_2^{1/2} u^{1/2} = E = I$

$u^{1/2} = \frac{I}{2p_1^{1/2} p_2^{1/2}}$ $\qquad u = \frac{I^2}{4p_1 p_2} = \text{IUF}$

5 b) Marshallian demand. Use Roy's Identity

$$x_1 = -\frac{\partial V/\partial p_1}{\partial V/\partial I} \quad \text{where } V = \text{IUF}.$$

$$\frac{\partial v}{\partial p_1} = -I^2 (4p_1p_2)^{-2} 4p_1$$

$$= -\frac{I^2}{4p_1^2 p_2}$$

$$x_1 = \frac{I^2/(4p_1^2 p_2)}{2I/4p_1p_2}$$

$$\frac{\partial v}{\partial I} = \frac{2I}{4p_1p_2}.$$

$$= \frac{I^2 4p_1p_2}{2I(4p_1^2p_2)}$$

$$\boxed{x_1 = \frac{I}{2p_1}}$$ Marshallian dem. good 1.

Q6. $V(p, I) = \dfrac{I}{p_1 + 0.25p_2^{1/2}}$. FALSE

Can answer this question in many ways.

① We know that the IUF is homogeneous of degree zero in p's + income.

i.e. $V(kp, kI) = k^0 V(p, I) = V(p, I)$.

but here $\dfrac{kI}{kp_1 + 0.25(kp_2)^{1/2}}$ is not.

6 Alternatively. We know several properties of the expenditure fn:

$$I = E = u\,(p_1 + 0.25 p_2^{1/2})$$

one is, that it is homogeneous of degree 1 in prices ∴ $E(p, u)$ then $E(Kp, u)$

$$= K\,E(p, u)$$

here $u\,(Kp_1 + 0.25 (Kp_2)^{1/2})$

$$\neq u\,(p_1 + 0.25 p_2^{1/2})K.$$

∴ this fn <u>cannot</u> be an I.U.F.

Q7. a) To begin, let's transform the U fn to $u(x_1, x_2) = x_1 x_2$. (easier)

a) sub. effect. – ② ways to do this

① Total effect. from marshallian demand.

$$\left.\begin{aligned} x_1^0 &= \frac{I}{2p_1} = \frac{100}{2} = 50 \\ x_1^1 &= \frac{100}{4} = 25 \end{aligned}\right\} \text{total effect } 25\downarrow$$

(7)

Q7 cont'd

sub effect: use Hicksian dem from Q4.

$$x_1^h = u^{1/2}\left(\frac{p_2}{p_1}\right)^{1/2} \qquad u = (50)(25) = 1250$$

$$x_1^h = (1250)^{1/2}\left(\frac{2}{2}\right)^{1/2} = 35.36.$$

$\therefore$ demand for x_1 goes from 50 to 35.36 because of the sub'n effect $\therefore$ $\boxed{S.e. = -14.64}$

income effect $35.36 \rightarrow 25 = \boxed{-10.36 = i.e}$

OR method (B)

begin $x_1 = 50 \quad x_2 = 25 \qquad u^0 = 1250$

end $x_1 = 25 \quad x_2 = 25$

we know that s.e. keep on $u^0 = 1250$

but prices have changed $\therefore$ i) $MRS = \frac{x_2}{x_1} = \frac{2}{2} = 1$

$\therefore x_2 = x_1$

ii) $x_1 x_2 = 1250 \Rightarrow$

$$x_1 = 35.36.$$

and hence s.e. $= -14.64$

i.e. $= -10.36.$

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b) Prefer to receive \$50 as he/she will be better off. He/she needs to purchase the bundle $x_1 = x_2 = 35.36$ in order to maintain $u = u^0$.

$$2(35.36) + 2(35.36) = \$141.44$$

He/she needs another 41.44 to be indiff between (50,25) when prices are (1,2) and (35.36, 35.36) when prices are (2,2).

$\therefore$ If given $50 > 41.44 \Rightarrow$ BETTER OFF.

c) $CV = E(p^1, u^0) - E(p^0, u^0)$

$= 141.44 - 100 = \$41.44$

d) $EV = E(p^0, u^1) - E(p^0, u^0)$

need to calculate expenditure at old prices to get new utility.

New utility $(25)(25) = 625$

again 2 ways i) $MRS = \frac{X_2}{X_1} = \frac{1}{2}$ $\quad X_2 = \frac{1}{2} X_1$

$X_1 (\frac{1}{2} X_1) = 625 \Rightarrow \frac{1}{2} X_1^2 = 625 \quad X_1 = 35.36$

9. $x_2 = 17.68$.

$E(p^0, u^1) = 35.36 + 2(17.68) = 70.72$.

$70.72 - 100 = -29.28$

income needs to fall by \$29.28 so that he/she can achieve new utility at old prices.

You could also have used the x_1^h to determine sub effect at new utility and old prices:

$$x_1^h = u^{1/2}\left(\frac{p_2}{p_1}\right)^{1/2} = (625)^{1/2}(2)^{1/2}$$

$$= 25(1.414) = 35.36.$$

$$x_2^h = 25\left(\frac{1}{2}\right)^{1/2} = 17.68.$$

Value of that bundle @ old prices

$35.36 + 17.68(2) = 70.72$.

$\therefore$ income $\downarrow$ 29.28 like before.