DEPARTMENT OF MATHEMATICS

SOLUTIONS MIDTERM TEST #1

MTH 140 – Calculus I

Last Name (Print):__________________. First Name:__________________. Student Number: ____________

Section (circle one)

Dr. Alvarez :	1 2 3 4 5
Dr. Ha :	6 7 8 9 10
Dr. Kim :	11 12 13 14 15
Dr. Fisseha:	16 17 18 19 20
Dr. Liu :	21 22 23 24 25

Signature:__________________.

Date: Sept. 27, 2013, 4:00 pm

Duration: 1.5 hours

Instructions:

1. This is a closed-book test. **Notes, calculators and other aids are not permitted.**
2. Verify that your test has pages 1-8.
3. (a) Unless otherwise instructed, **make sure you include all significant steps in your solution, presented in the correct order. Unjustified answers will be given little or no credit. Cross out or erase all rough work not relevant to your solution.** Put a box around your final **answer.**
   (b) For multiple choice questions make sure to write your answers in the box at the end of each question **carefully**. There are no part marks in the multiple-choice section and **only** the answer in the box will be marked. The correct response gets full marks, an incorrect response or no response gets no marks.
   (c) Write your solutions in the space provided. If you need more space, use the back of the page. Indicate this fact on the original page, making sure that your solution cannot be confused with any rough work which may be there. Marks (out of 50) are shown in brackets.
4. Do not separate the sheets.
5. Have your student card available on your desk.

For Instructor's use only.

Page	Mark
MC	/ 8
3	/5
4	/6
5	/7
6	/8
7	/9
8	/7
Total	/50

1. [2 marks] (Multiple choice question). The domain of the function $f(x) = \dfrac{1}{\sqrt[3]{2t-2}}$ is the set:

   A) $[1, \infty)$ B) $(1, \infty)$ C) $(-\infty, 1) \bigcup (1, \infty)$ D) $\mathbb{R}$ E) None of these

   Write the (capital) letter of the answer in this box. **Only** the answer in the box will be marked. C)

2. [2 marks] (Multiple choice question). If $\tan\beta = -3/4$ and $\pi/2 < \beta < \pi$, then $\csc\beta$ is equal to:

   A) -4/3 B) 3/5 C) -3/5 D) 5/3 E) None of these

   Write the (capital) letter of the answer in this box. **Only** the answer in the box will be marked. D)

3. [2 marks] (Multiple choice question). If $f(x) = \sqrt{x}$, $g(x) = x/4$ and $h(x) = 4x - 8$, then $f \circ g \circ h$ is equal to:

   A) $\sqrt{2x} - 8$ B) $\sqrt{x-2}/2$ C) $\sqrt{x-2}$ D) $\sqrt{x-8}$ E) None of these

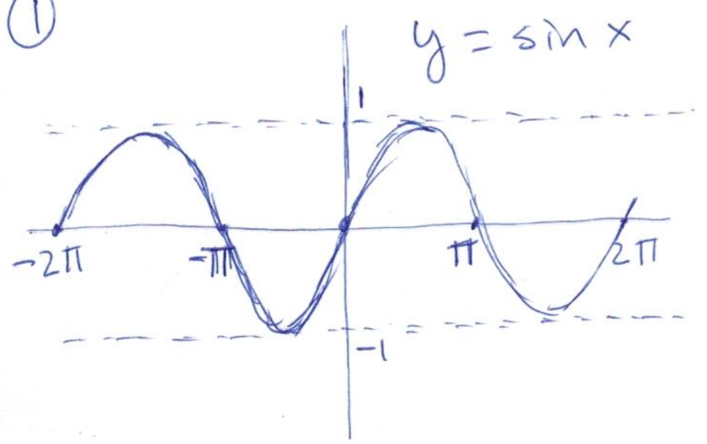
   Write the (capital) letter of the answer in this box. **Only** the answer in the box will be marked. C)

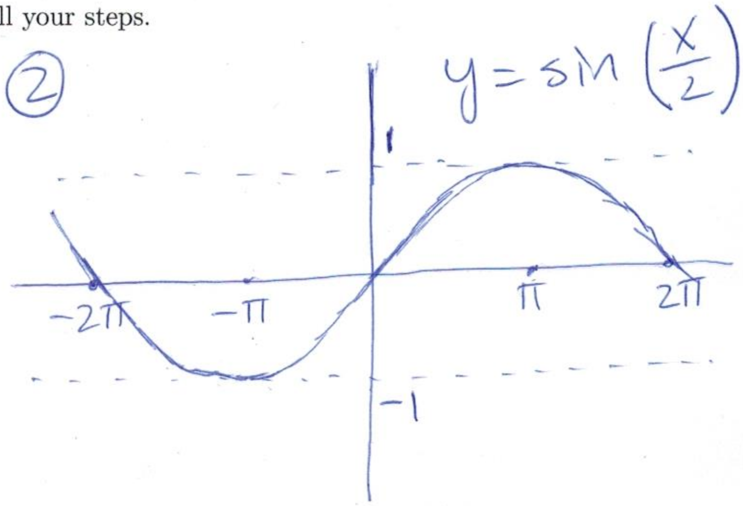
4. [2 marks] (Multiple choice question). If $f(x) = e^x + 2x^3 - \dfrac{1}{e^2}$, then $f^{-1}(-16)$ is equal to:

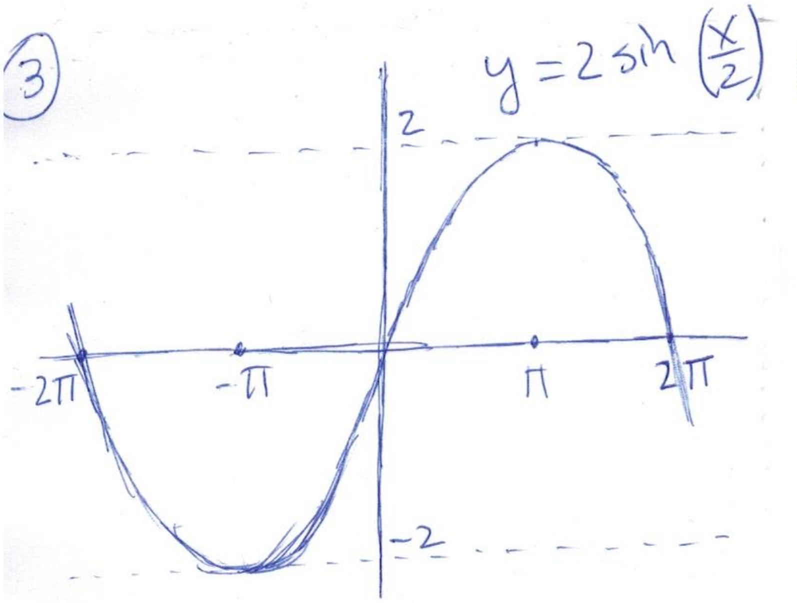
   A) 0 B) -2 C) 4 D) 2 E) None of these

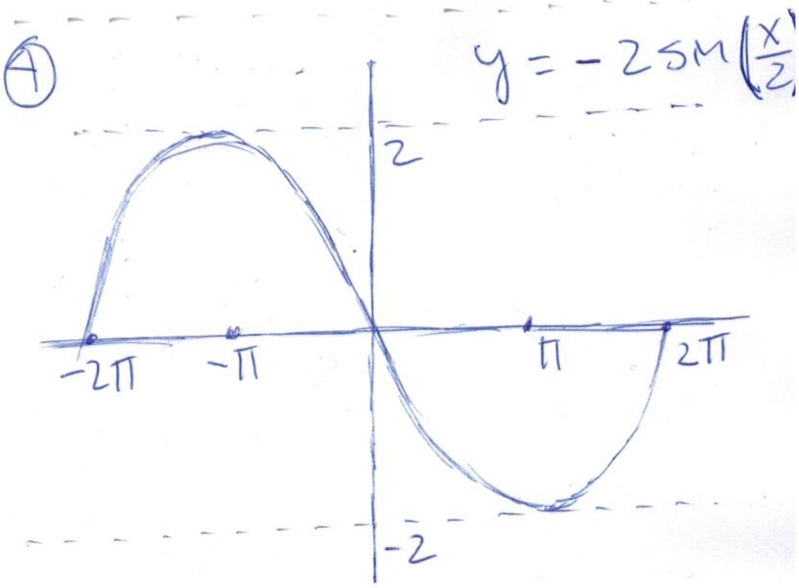
   Write the (capital) letter of the answer in this box. **Only** the answer in the box will be marked. B)

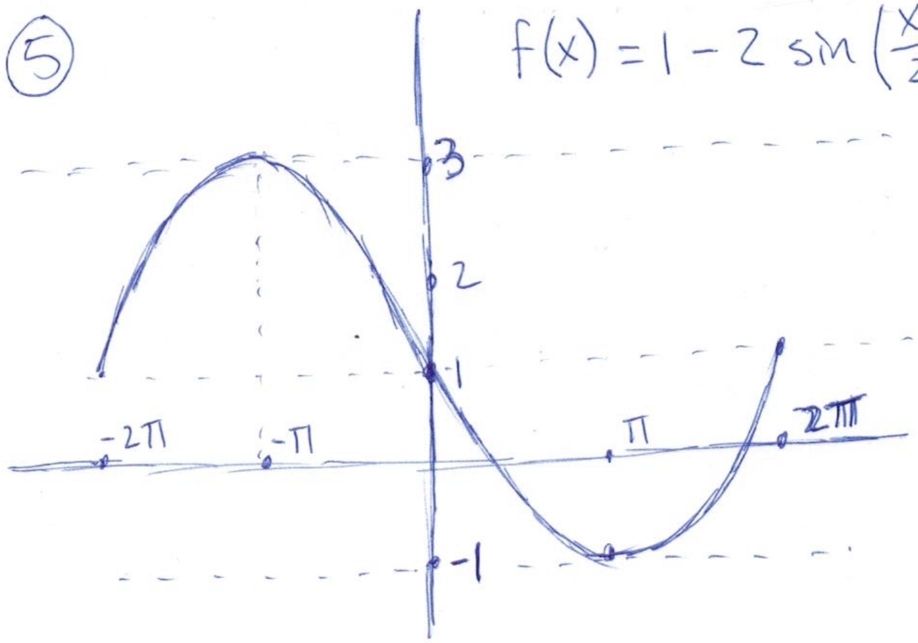
5. [5 marks] Starting with the graph of $g(x) = \sin x$, apply the appropriate transformations to sketch the graph of $f(x) = 1 - 2\sin\left(\frac{x}{2}\right)$. Show clearly all your steps.











6. [6 marks] Prove the trigonometric identity

$$\cot^2\theta - \cos^2\theta = \cot^2\theta\cos^2\theta$$

Show clearly all your steps.

Solution

$$\cot^2\theta - \cos^2\theta = \frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta = \cos^2\theta\left(\frac{1}{\sin^2\theta} - 1\right) = \cos^2\theta\left(\frac{1-\sin^2\theta}{\sin^2\theta}\right) = \cos^2\theta\left(\frac{\cos^2\theta}{\sin^2\theta}\right) = \cos^2\theta\cot^2\theta$$

7. [7 marks] Solve the equation for x

$$\ln x + \ln(x-1) = 1$$

Solution

$\ln x\,(x-1) = 1$ so

$x\,(x-1) = e^1 = e$ therefore $x^2 - x - e = 0$

The previous quadratic equation has solutions $x_{1,2} = \dfrac{1 \pm \sqrt{1+4e}}{2}$

The value $x_2 = \dfrac{1 - \sqrt{1+4e}}{2}$ is negative so $\ln x$ is not defined.

The only solution is $x = \dfrac{1 + \sqrt{1+4e}}{2}$.

8. [8 marks] Consider the function $f(x) = \ln(2 + \ln x)$

a) Find a formula for the inverse function f^{-1}

b) Find the range of f^{-1}

c) Find the range of f

Solution

a)$y = \ln(2 + \ln x)$.

Solving for x: $e^y = 2 + \ln x \Rightarrow e^y - 2 = \ln x \Rightarrow x = e^{e^y - 2}$

Interchanging x and y we get $y = e^{e^x - 2}$

Therefore $f^{-1}(x) = e^{e^x - 2}$

b) Range of f^{-1} is equal to the Domain of f

For f to be defined it must be satisfied that $2 + \ln x > 0$, so $\ln x > -2$ and $x > e^{-2}$

Range of f^{-1} : (e^{-2}, ∞)

c) Range of f is equal to the Domain of f^{-1}

As f^{-1} is defined for all values of x, the Range of f is $\mathbb{R}$

9. [9 marks] Consider the function

$$f(x) = \frac{x^2 - 3x + 2}{|x - 1|}$$

a) Find the limit $\lim_{x \to 1} f(x)$, if it exists. If the limit does not exist, explain why.

b) Sketch the graph of $f(x)$

Solution

a) $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{(x-1)(x-2)}{|x-1|} = \lim_{x \to 1^+} \frac{(x-1)(x-2)}{(x-1)} = \lim_{x \to 1^+} (x-2) = -1$

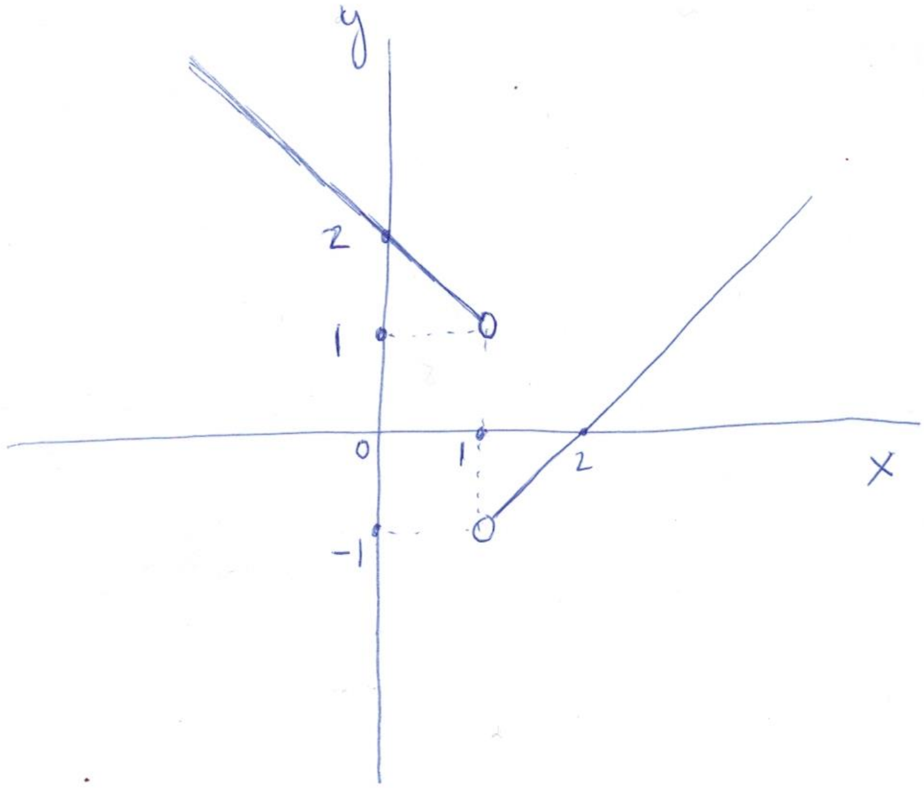
$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{(x-1)(x-2)}{|x-1|} = \lim_{x \to 1^-} \frac{(x-1)(x-2)}{-(x-1)} = \lim_{x \to 1^-} (-x+2) = 1$

As $\lim_{x \to 1^+} f(x) \neq \lim_{x \to 1^-} f(x)$ we conclude that $\lim_{x \to 1} f(x)$ DOES NOT EXIST.

b)

If $x > 1$ $f(x) = (x - 2)$

If $x < 1$ $f(x) = -(x - 2)$



10. [7 marks] Let $f(t) = \dfrac{4-\sqrt{t}}{16t - t^2}$.

a) Find the domain of f.

b) Find all vertical asymptotes of f. Show your work.

Solution

a) For f to be defined the following conditions must be satisfied:

$t \geq 0$ (for the square to be defined)

$16t - t^2 \neq 0 \Leftrightarrow t(16-t) \neq 0$ so $t \neq 0, 16$

Both conditions imply that the Domain of f is $(0, 16) \cup (16, \infty)$

b) The only possible asymptotes for f are $t = 0$ and $t = 16$

$$\lim_{t \to 0^+} f(t) = \lim_{t \to 0^+} \frac{4-\sqrt{t}}{t(16-t)} = \infty \text{ so } t = 0 \text{ is a VA}$$

$$\lim_{t \to 16} f(t) = \lim_{t \to 16} \frac{4-\sqrt{t}}{t(16-t)} \cdot \frac{4+\sqrt{t}}{4+\sqrt{t}} = \lim_{t \to 16} \frac{16-t}{t(16-t)(4+\sqrt{t})} = \lim_{t \to 16} \frac{1}{t(4+\sqrt{t})} = \frac{1}{16 \cdot 8}$$

so $t = 16$ is NOT a VA.