MAT 1341 Assignment 4 - DGD#_____

Summer 2014

Total: 14 points; Due: July 14, **beginning of DGD**

Family Name:______________________

First Name:_______________________

Student Number: __________________

PLEASE READ THESE INSTRUCTIONS VERY CAREFULLY

1. For each question, explain how you arrive at your answer. You earn points by correctly using mathematical notation, using correct reasoning and logic, and by identifying the key aspects of the problems.

2. You are allowed to discuss the problems with your classmates, but the work you hand in should be your own. Copying other people's solutions counts as plagiarism and will be dealt with as such.

1. **[1 point]** Let A be 2×2 invertible matrix, and let 2 be an eigenvalue of A.

   One eigenvalue of $A^3 + 2A^{-1}$ is ________.

2. **[2 points]** Let $S = \{\vec{u}, \vec{v}, \vec{w}\}$ be a subset set of $\mathbb{R}^5$, such that $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w} = \vec{w} \cdot \vec{v} = \vec{0}$. Show that the set S is linearly independent.

3. **[4 points]** Let $A = \begin{bmatrix} 2 & 4 & 3 & 9 \\ 1 & 2 & 1 & 4 \\ -3 & -6 & -1 & -10 \\ -1 & -2 & -1 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

   (a) Give the rank of the matrix A.

   (b) Give the dimension for each of the following subspaces row(A), col(A) and null(A).

   (c) Give a basis for each of the following subspaces row(A), col(A) and null(A).

4. **[3 points]** Consider the following independent set $S = \{X_1,\ X_2,\ X_3\}$ of vectors from $\mathbb{R}^4$:

$$X_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, X_2 = \begin{bmatrix} 2 \\ 1 \\ 2 \\ -1 \end{bmatrix}, X_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ -2 \end{bmatrix}.$$

   Use the Gram-Schmidt algorithm to convert the set $S = \{X_1,\ X_2,\ X_3\}$ into an orthogonal set $B = \{F_1,\ F_2,\ F_3\}$.

5. **[2 points]** Let U be a subspace of $\mathbb{R}^4$ and let the set $S = \{X_1,\ X_2,\ X_3\}$ be an orthogonal basis of U, where

$$X_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ -1 \end{bmatrix}, X_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}, X_3 = \begin{bmatrix} -1 \\ 3 \\ -1 \\ 0 \end{bmatrix}.$$

(a) Given $X = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 42 \end{bmatrix}$. Find the orthogonal projection of X onto U, i.e., the best approximation in U to the vector X.

(b) Compute the approximation error $||X - proj_U X||$.

6. **[2 points]** Let $\mathbb{M}_2$ denote the set of all 2×2 matrices. We define addition with the standard addition of matrices, but with scalar multiplication given by

$$k \otimes \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & b \\ c & kd \end{bmatrix},$$

where k is a scalar.

Is $(k+s) \otimes \begin{bmatrix} a & b \\ c & d \end{bmatrix} = k \otimes \begin{bmatrix} a & b \\ c & d \end{bmatrix} + s \otimes \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ for scalars k, s? Explain.

7. **[Bonus: 1 point]** Is the set $\{\cos^2 x, 3, \sin^2 x\}$ linearly independent?