b) Alpha levels must be chosen *before* examining the data. Otherwise, the alpha level could always be selected at a value that would reject the null hypothesis.

36. Safety.

a) More likely, because increasing α increases the chance of rejecting the null hypothesis.

b) The company should use stricter guidelines since sleepwalking is dangerous. Therefore, it should use a conventional value of $\alpha = 0.05$ which would be more likely to reject the null hypothesis.

37. Product testing.

Null and alternative hypotheses should involve p, not $\hat{p}$.

The question is about *failing* to meet the goal. H_A should be $p < 0.96$.

The student failed to check $nq = (200)(0.04) = 8$. Since $nq < 10$, the Success/Failure Condition is violated. Similarly, the 10% Condition is not verified.

$SD(\hat{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.96)(0.04)}{200}} \approx 0.014$. The student used $\hat{p}$ and $\hat{q}$.

Value of z is incorrect. The correct value is $z = \frac{0.94 - 0.96}{0.014} \approx -1.43$.

P-value is incorrect. $P = P(z < -1.43) = 0.076$

For the P-value given, an incorrect conclusion is drawn. A P-value of 0.12 provides no evidence that the new system has failed to meet the goal. The correct conclusion for the corrected P-value is: Since the P-value of 0.076 is fairly low, there is weak evidence that the new system has failed to meet the goal.

38. Marketing.

Null and alternative hypotheses should involve p, not $\hat{p}$.

The question asks if there is evidence that the 90% figure is *not accurate*, so a two-sided alternative hypothesis should be used. H_A should be $p \neq 0.90$.

One of the conditions checked appears to be $n > 10$, which is not a condition for hypothesis tests. The Success/Failure Condition checks $np = (750)(0.90) = 675 > 10$ and $nq = (750)(0.10) = 75 > 10$. Also, the 10% Condition is not verified.

$SD(\hat{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.90)(0.10)}{750}} \approx 0.011$. The student used rounded values of $\hat{p}$ and $\hat{q}$.

Value of z is incorrect. The correct value is $z = \frac{0.876 - 0.90}{0.011} \approx -2.18$.

The P-value calculated is in the wrong direction. To test the given hypothesis, the lower-tail probability should have been calculated. The correct, two-tailed P-value is $P = 2P(z < -2.18) = 0.029$.

The P-value is misinterpreted. Since the P-value is so low, there is moderately strong evidence that the proportion of adults who drink milk is different than the claimed 90%. In fact, our sample suggests that the proportion may be lower. There is only a 2.9% chance of observing a $\hat{p}$ as far from 0.90 simply from natural sampling variation.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.52 \pm 1.96\sqrt{\frac{(0.52)(0.48)}{(1000)}} = 0.52 \pm 0.031 = (0.489, 0.551) = (48.9\%, 55.1\%)$$

We are 95% confident that between 49.9% and 55.1% of workers have invested in individual retirement accounts.

b) Since 44% is not in the 95% confidence interval, there is strong evidence that the percentage of workers who have invested in individual retirement accounts was not 44%. In fact, our sample indicates an increase in the percentage of adults who invest in individual retirement accounts.

58. iPod reliability.

a) $H_0: p = 0.2$; $H_A: p = 0.2$

b) The observed proportion of failure rates $\hat{p} = 64/517 = 0.1238$.

$$z = \frac{(\hat{p} - p_0)}{SD(\hat{p})}; SD(\hat{p}) = \sqrt{\frac{p_0 q_0}{n}} = \sqrt{\frac{0.20 * 0.80}{517}} = 0.017592 \qquad z = \frac{(0.1238 - 0.20)}{0.017592} = -4.33$$

c) For a one-sided significance, the alternative hypothesis using $\alpha = 0.1\% = 0.001$.
The critical *z*-value for 0.001 is 3.09; -4.33 < -3.09

d) We conclude that the failure rate of the new model is lower (although we should be careful to continue to monitor to see if this is just because the newer model hasn't been out as long).

59. Testing cars.

a) In this context, a Type I error is concluding that the shop is not meeting standards when it actually is.

b) In this context, a Type II error is when the regulators certify the shop when it is not meeting the standards.

c) The shop's owner would consider a Type I error to be more serious because that error would state that the shop is not meeting standards when it actually is and would affect business.

d) Environmentalists would consider a Type II error to be more serious because that error would state that the shop is meeting standards when it actually is not and could be polluting.

60. Quality control.

H_0: The assembly process is working fine.
H_A: The assembly process is producing an increase in defective items.

a) In this context, a Type I error means that the factory is identified as producing significantly more defective products and stops the assembly line when it actually is not.

b) In this context, a Type II error means that the factory is identified as not producing significantly more defective products when it actually is producing more defective products.

c) The factory owner would probably consider a Type II error to be more serious, depending on the costs of shutting the line down. Generally, because of warranty costs and lost customer loyalty, defects that are caught in the factory are much cheaper to fix than defects found after items are sold.

d) Customers would consider a Type II error to be more serious because that error would state that the factory is not producing significantly more defective products when it actually is and that affects the consumer.

strong evidence that the true proportion of those who are at least 50 is above 2/3. The 95% confidence interval $(69.1\%, 76.9\%)$ (generated in Chapter 10, Exercise 52) does not include the 2/3 proportion of 67% for a two-sided test.

69. Drugs in Canada.

H_0: p=0.5 (The proportion of Conservative voters that supports scrapping the marijuana decriminalization legislation is 0.5.)

H_A: p>0.5 (The proportion of Conservative voters that supports scrapping the marijuana decriminalization legislation is > 0.5.)

Independence Assumption and Randomization Condition: Since this is a professionally conducted survey by Angus Reid, we can be confident these conditions are met.

10% Condition: The number of people surveyed is way below 10% of the population of interest.

Success/Failure Condition: Due to the large sample size and the percentage responses, *np* and *nq* are both >10.

$\hat{p}$ = 0.59; n=1028×0.4 =411

SD($\hat{p}$) = sqrt(0.5×0.5/411) = 0.02466

z = (0.59-0.5)/0.02466 = 3.649

P = 0.000132 very small

The null hypothesis can be rejected. We can be 99.99% confident that scrapping the marijuana decriminalization legislation has the support over 50% of Conservative voters.

70. Drugs in Canada Part 2.

H_0: p=0.5 (The proportion of Conservative voters that supports eliminating the “harm reduction” programs is 0.5.)

H_A: p>0.5 (The proportion of Conservative voters that supports eliminating the “harm reduction” programs is > 0.5.)

Independence Assumption and Randomization Condition: Since this is a professionally conducted survey by Angus Reid, we can be confident these conditions are met.

10% Condition: The number of people surveyed is way below 10% of the population of interest.

Success/Failure Condition: Due to the large sample size and the percentage responses, *np* and *nq* are both >10.

$\hat{p}$ = 0.54; n=1028×0.4 =411

SD($\hat{p}$) = sqrt(0.5×0.5/411) = 0.02466

z = (0.54-0.5)/0.02466 = 1.622

P = 0.0524

n=1016

We can use the Normal distribution to test the hypothesis that the sample is consistent with 50% of Canadian adults supporting scrapping the penny subject to the following conditions:

$np_0 = 508 > 10$

$nq_0 = 508 > 10$

H_0: $p = p_0$

H_A: $p > p_0$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q}{n}}} = 3.19$$

$P = 0.9993$

The statement is statistically valid at the 99.9% significance level.

b)

The statement is ethical since it is true at a very high level of statistical significance. The statement could be improved by stating the significance level.

79. The Canadian penny again.

H_0: $p_1 = p_2$

H_A: $p_1 < p_2$

Since the null hypothesis asserts that the two proportions are the same, we use a pooled estimate of the proportion, $\overline{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{0.62*1016*0.133+0.55*1016*0.388}{1016*(0.133+0.388)} = 0.5679$.

$$SD(\hat{p}_1 - \hat{p}_2) = \sqrt{\overline{pq}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{0.5679 * 0.4321\left(\frac{1}{1016 * 0.133} + \frac{1}{1016 * 0.388}\right)}$$
$$= 0.01358$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{SD(\hat{p}_1 - \hat{p}_2)} = -1.418$$

$P = 0.078 > 0.05$. So we cannot conclude a greater percentage in B.C. at the 95% level.

80. The Canadian penny, third time.

a)

H_0: $p_1 = p_2$

H_A: $p_1 < p_2$

Since the null hypothesis asserts that the two proportions are the same, we use a pooled estimate of the proportion,

$$\overline{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{0.65 * 508 + 0.45 * 508}{1016} = 0.55.$$

$$SD(\hat{p}_1 - \hat{p}_2) = \sqrt{\overline{pq}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{0.55 * 0.55\left(\frac{2}{508}\right)} = 0.03122$$

$$z = \frac{\hat{P}_1 - \hat{P}_2}{SD(\hat{P}_1 - \hat{P}_2)} = 6.407$$

The significance level is P = $7.42 * 10^{-11}$.

b)

Repeating the calculation in part a) for a range of sample sizes with the aim of $z = 1.645$ for a 95% significance level, we find $n = 34$.

81. Canadian Senate again.

H_0: $p_1 = p_2$
H_A: $p_1 < p_2$

Since the null hypothesis asserts that the two proportions are the same, we use a pooled estimate of the proportion,

$$\overline{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{0.32 * 1000 * 0.384 + 0.43 * 1000 * 0.236}{1000 * (0.384 + 0.236)} = 0.3619$$

$$SD(\hat{p}_1 - \hat{p}_2) = \sqrt{\overline{pq}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{0.3619 * 0.6381\left(\frac{1}{384} + \frac{1}{236}\right)} = 0.03975$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{SD(\hat{p}_1 - \hat{p}_2)} = -2.767$$

The significance level is P=0.997>0.99. More people in Quebec support abolishing the Senate than in Ontario at the 99% significance level.

82. Single Parent Families in Canada.

H_0: $p_1 = p_2$
H_A: $p_1 < p_2$

Since the null hypothesis asserts that the two proportions are the same, we use a pooled estimate of the proportion,

$$\overline{p} = \frac{x_1 + x_2}{n_2 + n_2} = \frac{0.14 * 1000 + 0.17 * 1000}{1000 + 7000} = 0.1524$$

$$SD(\hat{p}_1 - \hat{p}_2) = \sqrt{\overline{pq}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{0.1524 * 0.8476\left(\frac{1}{1000} + \frac{1}{700}\right)} = 0.01771$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{SD(\hat{p}_1 - \hat{p}_2)} = -1.694$$

The significance level is P=0.045<0.05. There is a lower percentage of single parent families in Alberta than in Nova Scotia at the 95% significance level.

83. Adults Living with Parents in Canada.

H_0: $p_1 = p_2$
H_A: $p_1 \neq p_2$

Since the null hypothesis asserts that the two proportions are the same, we use a pooled estimate of the proportion,

$$\overline{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{0.614*1000 + 0.653*1000}{2000} = 0.6335$$

$$SD(\hat{p}_1 - \hat{p}_2) = \sqrt{\overline{pq}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{0.6335*0.3665\left(\frac{2}{1000}\right)} = 0.02055$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{SD(\hat{p}_1 - \hat{p}_2)} = -1.898$$

P=0.0288. Since this is a two-tailed test, P is doubled to give 0.0577 so that there is not a significant difference at the 95% level.

84. Mature Middle Class in India.
H_0: $p_1 = p_2$
H_A: $p_1 < p_2$
Since the null hypothesis asserts that the two proportions are the same, we use a pooled estimate of the proportion,

$$\overline{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{0.27n + 0.5n}{2n}$$ since the sample size is the same each year.

$$SD(\hat{p}_1 - \hat{p}_2) = \sqrt{\overline{pq}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{\frac{2\overline{pq}}{n}}$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{SD(\hat{p}_1 - \hat{p}_2)} = -2.33$$ since we want a P-value of 0.01.

Trial and error with different values of n leads to n=49.

Mini Case Study: Common-Law Couples in Quebec

PLAN	Setup State the objective.	 We want to estimate confidence intervals for the proportion of common-law couples in Montreal and test whether it is > 29%.
DO	Mechanics Check the conditions. We will use alpha = 0.05 and 90% confidence intervals with z = 1.645	 Independence Assumption: People surveyed are unlikely to even know each other, let alone influence each other. Randomization Condition: Our survey used a random sample. We are not told about the other one, but since the survey company is reputable, we can assume it did a good job on randomization. 10% Condition: Our samples are very small compared to the population of Montreal. Success/Failure Condition: The number of respondents that were (were not) common-law couples was >10 in each survey. Survey #1: $n = 100,\ \hat{p} = 0.35$ Confidence interval: $SE(\hat{p}) = \sqrt{0.35*0.65/100} = 0.0477$ 90% confidence interval is $0.35 \pm 0.0477*1.645$ = [0.272 , 0.428]