

60. Amendment.

- a) This poll is inconclusive because the confidence interval, $52\% \pm 3\%$, contains 50%. The true proportion of stock owners in favour of the amendment is estimated to be between 49% (minority) to 55% (majority). We can't be sure whether or not the majority of stock owners support the amendment or not.

b)

$$\begin{aligned} \text{ME} &= z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} \\ 0.03 &= z^* \sqrt{\frac{(0.52)(0.48)}{1505}} \\ z^* &= \frac{0.03}{\sqrt{\frac{(0.52)(0.48)}{1505}}} \\ z^* &\approx 2.3295 \end{aligned}$$

Since $z^* \approx 2.3295$, which is close to 2.326, the board was probably using 98% confidence. The slight difference in the z^* values is due to rounding of the amendment's approval rating.

61. Customer spending.

This was a random sample of less than 10% of all customers. There were 67 successes and 433 failures, both at least 10. From the data set, $\hat{p} = 67/500 = 0.134$.

The 95% confidence interval:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.134 \pm 1.96 \sqrt{\frac{(0.134)(0.866)}{(500)}} = 0.134 \pm 0.030 = (0.104, 0.164) = (10.4\%, 16.4\%)$$

We are 95% confident that the true proportion of customers who spend \$1000 per month or more is between 10.4% and 16.4%.

62. Advertising.

This was a random sample of less than 10% of all donors. There were 365 successes and 135 failures, both at least 10. From the data set, $\hat{p} = 365/500 = 0.73$. The 95% confidence interval:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.73 \pm 1.96 \sqrt{\frac{(0.73)(0.27)}{(500)}} = 0.73 \pm 0.039 = (0.691, 0.769) = (69.1\%, 76.9\%)$$

We are 95% confident that the true proportion of donors who are 50 years old or older is between 69.1% and 76.9%.

63. Health insurance.

- a) This was a random sample of less than 10% of all MA males. There were 2662 successes and 398 failures, both at least 10.

b) The 95% confidence interval:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.87 \pm 1.96 \sqrt{\frac{(0.87)(0.13)}{(3060)}} = 0.87 \pm 0.012 = (0.858, 0.882) = (85.8\%, 88.2\%)$$

- c) We are 95% confident that between 85.8% and 88.2% of MA males have health insurance.

64. Health insurance, part 2.

- a) This was a random sample of less than 10% of all MA Blacks/African-Americans. There were 370 successes and 70 failures, both at least 10.
- b) The 95% confidence interval:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.84 \pm 1.96 \sqrt{\frac{(0.84)(0.16)}{(440)}} = 0.84 \pm 0.034 = (0.806, 0.874) = (80.6\%, 87.4\%)$$
- c) We are 95% confident that between 80.6% and 87.4% of MA Blacks/African-Americans have health insurance.

65. Abortion in Canada.

Plausible Independence Condition: There is no reason to believe that the people polled would have influenced each other.

Randomization Condition: We are told that the sample was randomly selected.

10% Condition: 1002 is certainly less than 10% of all adults.

Success/Failure Condition:

$$n\hat{p} = 1002 \times 0.43 = 431 > 10$$

$$n\hat{q} = 1002 \times 0.57 = 571 > 10$$

$$n\hat{p} = 1002 \times 0.41 = 411 > 10$$

$$n\hat{q} = 1002 \times 0.59 = 591 > 10$$

So the sample is large enough for each question.

The 95% confidence intervals are:

$$0.43 \pm 1.96 \times \text{sqrt}(0.43 \cdot 0.57 / 1002) = \text{from } 40\% \text{ to } 46\%$$

$$0.41 \pm 1.96 \times \text{sqrt}(0.41 \cdot 0.59 / 1002) = \text{from } 38\% \text{ to } 44\%$$

We are 95% confident that between 40% and 46% of Canadian adults think that the health care system should fund abortions whenever they are requested.

We are 95% confident that between 38% and 44% of Canadian adults think that the health care system should only fund abortions in the event of medical emergencies.

66. Climate change opinion in Canada.

Plausible Independence Condition: There is no reason to believe that the people polled would have influenced each other

Randomization Condition: We are told that the sample was randomly selected.

10% Condition: 1013 is certainly less than 10% of all adults.

Success/Failure Condition:

$$n\hat{p} = 1013 \times 0.12 = 122 > 10$$

$$n\hat{q} = 1013 \times 0.88 = 891 > 10$$

So the sample is large enough

The 90% confidence interval is: $0.12 \pm 1.64 \times \text{sqrt}(0.12 \cdot 0.88 / 1013) = \text{from } 10\% \text{ to } 14\%$.

We are 90% confident that between 10% and 14% of Canadian adults think that the Copenhagen Accord will become a legally binding treaty in the future in Canada.

67. Same-sex marriage in Canada

Plausible Independence Condition: There is no reason to believe that the people polled would have influenced each other.

Randomization Condition: We are told that the sample was randomly selected.

10% Condition: 1006 is certainly less than 10% of all adults.

Success/Failure Condition:

$$n\hat{p} = 1006 \times 0.61 = 614 > 10$$

$$n\hat{q} = 1006 \times 0.39 = 392 > 10$$

So the sample is large enough

The 99% confidence interval is: $0.61 \pm 2.57 \times \text{sqrt}(0.61 \cdot 0.39 / 1006) = \text{from } 57\% \text{ to } 65\%$.

We are 99% confident that between 57% and 65% of Canadian adults think that same-sex couples should continue to be allowed to legally marry in Canada.

68. Carbon tax in British Columbia.

$$\text{a) } SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{0.54 * 0.46}{1023}} = 0.01558$$

The critical value for a 90% confidence interval: $z^*=1.645$. The confidence interval is:

$$\hat{p} \pm z^* SE(\hat{p}) = 0.54 \pm 1.645 * 0.01558 = 0.54 \pm 0.0256$$

i.e., between 0.51 and 0.57.

- b)** “54% of adult British Columbians are in favour of the carbon tax. This result is accurate to plus or minus 2.6% 9 times out of 10.”

69. Canadian Senate.

a)

$$SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{0.34 * 0.66}{1000}} = 0.01498$$

The critical value for a 95% confidence interval:

$$z^*=1.96. \text{ The confidence interval is: } \hat{p} \pm z^* SE(\hat{p}) = 0.34 \pm 1.96 * 0.01498 = 0.34 \pm 0.0294$$

i.e., between 0.31 and 0.37.

- b)** “34% of adult Canadians support abolishing the Senate. This result is accurate to plus or minus 2.9 19 times out of 20.”

70. Mature middle class in India.

$$SE(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} = \sqrt{\frac{0.27 * 0.73}{1500} + \frac{0.5 * 0.5}{1500}} = 0.01726$$

The critical value for a 90% confidence interval: $z^*=1.645$. The confidence interval for the difference between two proportions is:

$$(\hat{p}_1 - \hat{p}_2) \pm z^* SE(\hat{p}_1 - \hat{p}_2) = 0.27 - 0.5 \pm 1.645 * 0.01726 = -0.23 \pm 0.0284$$

i.e., between -0.25 and -0.20.

71. Canadian Senate again.

$$SE(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} = \sqrt{\frac{0.32 * 0.68}{384} + \frac{0.43 * 0.57}{236}} = 0.04006$$

The critical value for a 90% confidence interval: $z^*=1.645$. The confidence interval for the difference between two proportions is:

$$(\hat{p}_1 - \hat{p}_2) \pm z^* SE(\hat{p}_1 - \hat{p}_2) = 0.32 - 0.43 + 1.645 * 0.04006 = -0.11 \pm 0.0660$$

i.e., between -0.18 and -0.04.

72. The Canadian penny.

$$SE(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} = \sqrt{\frac{0.65 * 0.35}{508} + \frac{0.45 * 0.55}{508}} = 0.03058$$

- a) The critical value for a 90% confidence interval: $z^*=1.645$. The confidence interval for the difference between two proportions is:

$$(\hat{p}_1 - \hat{p}_2) \pm z^* SE(\hat{p}_1 - \hat{p}_2) = 0.65 - 0.45 \pm 1.645 * 0.03058 = 0.2 \pm 0.0503$$

i.e., between 0.15 and 0.25.

- b) The critical value for a 99% confidence interval: $z^*=2.58$. The confidence interval for the difference between two proportions is:

$$(\hat{p}_1 - \hat{p}_2) \pm z^* SE(\hat{p}_1 - \hat{p}_2) = 0.65 - 0.45 \pm 2.58 * 0.03058 = 0.2 \pm 0.0789$$

i.e., between 0.12 and 0.28.

- c) The 99% confidence interval is wider, since we need to be more certain that it includes the correct value.

73. Canadian Values.

The 95% confidence interval for a sample of 1006 adults is at most $\pm 1.96 * \sqrt{\frac{0.5 * 0.5}{1006}} = 0.031$

The statement “plus or minus 3.1% 19 times out of 20” is correct for the result of 37% for the whole country. However the sample size for the different age groups is smaller than 1006 and the confidence intervals for those groups are therefore wider than $\pm 3.1\%$.

The statement is non-professional and infringes on Item A of the ASA Ethical Guidelines

74. Same-sex marriage.

- a) No. The size of the population does not affect the results of a survey, since the standard error and hence the width of the confidence interval depends only on sample size and proportion of people answering yes to the survey question.
- b) No. The accuracy of the results depends on the width of the confidence interval, which depends only on sample size and proportion of people answering yes to the survey question. It does not depend on the population size.
- c) The advantage of surveying more people is to reduce the width of the confidence interval, and hence give a more accurate result.