MAT 1322 MIDTERM 1, VERSION A

1. [2 points, 5.10 #19] Determine if the integral $\int_2^\infty \frac{1}{x(\ln x)^2}dx$ is convergent or divergent and evaluate if it is convergent.

A.1 B.2.51 C.0.50 D.5.33 E.1.44 F.divergent

Solution.
$\int_2^\infty \frac{1}{x(\ln x)^2}dx = \int_{\ln 2}^\infty \frac{1}{u^2}du \qquad [u = \ln x]$
$= \left[-\frac{1}{u}\right]_{\ln 2}^\infty = 0 + \frac{1}{\ln 2} \doteq 1.44$

2. [2 points, 6.1 #11] Find the area of the region enclosed by the curves $y^2 = 2x$ and $2x - 2y = 3$.

A.5.33 B.4.77 C.3.55 D.6 E.9.01 F.2.22

Solution. Intersection $y^2 = 2x$ and $2x - 2y = 3 \Rightarrow y^2 - 2y - 3 = 0 \Rightarrow (y-3)(y+1) = 0 \Rightarrow y = -1, 3$.
$A = \int_{-1}^3 [(\frac{1}{2}(2y+3) - \frac{1}{2}y^2]dy = \frac{1}{2}\int_{-1}^3 (2y+3-y^2)dy = \frac{1}{2}\left[y^2 + 3y - \frac{1}{3}y^3\right]_{-1}^3 = \frac{1}{2}\frac{32}{3} = 5.33$

3. [2 points, 6.2 #13] The region enclosed by the curves $y = \sqrt[3]{x}, x = 1, y = 0$ is rotated about the line $x = 2$. Find the volume of the resulting solid.

A.11.44 B.7.17 C.5.10 D.12.57 E.10.11 F.6.73

Solution. $y = \sqrt[3]{x} \Rightarrow x = y^3$
$A = \int_0^1 \pi[(y^3 - 2)^2 - 1^2]dy = \frac{15}{7}\pi \doteq 6.73$

4. [2 points, 6.2 #31] The base of a solid is the region $\{(x, y) \mid x^2 \leq y \leq 2\}$. Cross-sections perpendicular to the y-axis are semicircles. Find the volume of the solid.

A.6 B.3.14 C.8.02 D.11.50 E.7.07 F.4.75

Solution. The semicircle with base though (x, y) on the curve $y = x^2$ has radius $r = x$. Thus $r^2 = x^2 = y$ and
$A = \int_0^2 \frac{1}{2}\pi r^2 dy = \int_0^2 \frac{1}{2}\pi y dy = \left[\frac{1}{4}\pi y^2\right]_0^2 = \pi = 3.14$

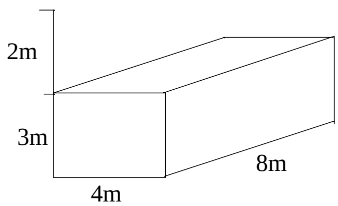
5. [2 points, 6.3 #7] Find the arclength of the curve $y = 1 + x^{3/2}$ from $x = 0$ to $x = 4$.

A.12.50 B.9.07 C.24.24 D.10.61 E.20 F.14.25

Solution. $L =_a^b \sqrt{1 + (\frac{dy}{dx})^2}\, dx$
$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$, $(\frac{dy}{dx})^2 = \frac{9}{4}x$
$L = \int_0^4 \sqrt{1 + \frac{9}{4}x}\, dx \qquad [u = 1 + \frac{9}{4}x,\ du = \frac{9}{4}dx]$
$= \int_1^{10} u^{1/2}\frac{4}{9}du = \frac{4}{9}\left[\frac{4}{2}u^{3/4}\right]_1^{10} \doteq 9.07$

6. [2 points, 6.5 #13] The rectangular tank shown is full of water. Find the work required to pump the water up 2m above its top. (The density of water is $1000\mathrm{kg/m}^3$ and the acceleration of gravity $9.8\mathrm{m}/\sec^2$. Work in joules J.)

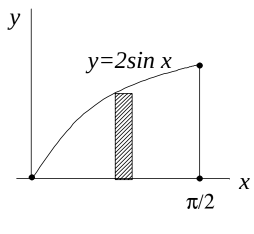
A.$3263,500$ B.$4939,200$ C.$4992,240$ D.$3292,800$ E.$5103,450$ F.$6485,200$



Solution. The work required to pump a slice at a distance x from the 2m above the tank is $(1000 \times 9.8)(4 \times 8)x\Delta x = 313,600$ J. The total work is
$W = \int_2^5 (1000 \times 9.8)(4 \times 8)x dx = 3292,800$ J

7. [3 points, 6.5 #29]Consider the region bounded by the curves $y = 2\sin x$, $y = 0$, $x = 0$, $x = \pi/2$.
(a)Sketch the region.
(b)Find the moment M_y about the y axis if the mass density is ρ (constant). (*Provide a detailed solution and draw a box around the final answer.*)

Solution.



$\Delta M = \rho y \Delta x$,
$M_y = \int_0^{\pi/2} x\rho y dx = \rho \int_0^{\pi/2} x 2\sin x\, dx$
$= 2\int_0^{\pi/2} x \sin x\, dx \qquad [u = x, v' = \sin x,\ u' = 1, v = -\cos x]$
$= \rho 2 \left[-x\cos x + \int \cos x\right]_0^{\pi/2} = \rho 2 \left[-x\cos x + \sin x\right]_0^{\pi/2} = 2\rho$

MAT 1322   MIDTERM 1, VERSION B

1. [2 points, 5.10 #19] Determine if the integral $\int_1^2 \frac{1}{x(\ln x)^2}dx$ is convergent or divergent and evaluate if it is convergent.
A.1 B.2.51 C.0.50 D.5.33 E.0.69 F.divergent

Solution.
$\int_1^2 \frac{1}{x(\ln x)^2}dx = \int_0^2 \frac{1}{u^2}du \qquad [u = \ln x]$
$= \left[-\frac{1}{u}\right]_0^2 = \infty$ [divergent]

2. [2 points, 6.1 #11] Find the area of the region enclosed by the curves $y^2 = 3x$ and $3x - 2y = 3$.
A.5.33 B.4.77 C.3.55 D.6 E.9.01 F.2.22

Solution. Intersection $y^2 = 3x$ and $3x - 2y = 3 \Rightarrow y^2 - 2y - 3 = 0 \Rightarrow (y-3)(y+1) = 0 \Rightarrow y = -1, 3$.
$A = \int_{-1}^3 [(\frac{1}{3}(2y+3) - \frac{1}{3}y^2]dy = \frac{1}{3}\int_{-1}^3 (2y+3-y^2)dy = \frac{1}{3}\left[y^2 + 3y - \frac{1}{3}y^3\right]_{-1}^3$
$= \frac{1}{3}\frac{32}{3} \doteq 3.55$

3. [2 points, 6.2 #13] The region enclosed by the curves $y = \sqrt[3]{x}, x = 1, y = 0$ is rotated about the line $x = 3$. Find the volume of the resulting solid.
A.11.44 B.7.17 C.5.10 D.12.57 E.10.11 F.6.73

Solution. $y = \sqrt[3]{x} \Rightarrow x = y^3$
$A = \int_0^1 \pi[(y^3 - 3)^2 - 2^2]dy = 11.44$
A.11.44 B.7.17 C.5.10 D. E.10.11 F.6.73

4. [2 points, 6.2 #31] The base of a solid is the region $\{(x, y) \mid x^2 \le y \le 3\}$. Cross-sections perpendicular to the y-axis are semicircles. Find the volume of the solid.
A.6 B.3.14 C.8.02 D.11.50 E.7.07 F.4.75

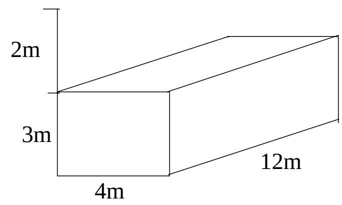
Solution. The semicircle with base though (x, y) on the curve $y = x^2$ has radius $r = x$. Thus $r^2 = x^2 = y$ and
$A = \int_0^3 \frac{1}{2}\pi r^2 dy = \int_0^3 \frac{1}{2}\pi y dy = \left[\frac{1}{4}\pi y^2\right]_0^3 = \frac{9}{4}\pi \doteq 7.07$

5. [2 points, 6.3 #7] Find the arclength of the curve $y = 1 + x^{3/2}$ from $x = 0$ to $x = 8$.
A.12.50 B.9.07 C.24.24 D.10.61 E.20 F.14.25

Solution. $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}\, dx$
$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$, $(\frac{dy}{dx})^2 = \frac{9}{4}x$
$L = \int_0^8 \sqrt{1 + \frac{9}{4}x}\, dx \qquad [u = 1 + \frac{9}{4}x,\ du = \frac{9}{4}dx]$
$= \int_1^{18} u^{1/2}\,\frac{4}{9}du = \frac{4}{9}\left[\frac{4}{2}u^{3/4}\right]_1^{18} \doteq 24.24$

6. [2 points, 6.5 #13] The rectangular tank shown is full of water. Find the work required to pump the water up 2m above its top. (The density of water is 1000kg/m^3 and the acceleration of gravity 9.8m/ sec^2. Work in joules J.)

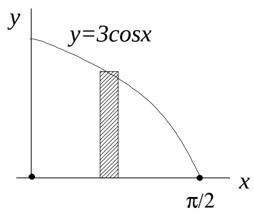
A.3263, 500 B.4939, 200 C.4992, 240 D.3292, 800 E.5103, 450 F.6485, 200



Solution. The work required to pump a slice at a distance x from the 2m above the tank is $(1000 \times 9.8)(4 \times 12)x\Delta x =$J. The total work is
$W = \int_2^5 (1000 \times 9.8)(4 \times 12)x dx = 4939, 200$J

7. [3 points, 6.5 #29]Consider the region bounded by the curves $y = 3\cos x,\ y = 0,\ x = 0,\ x = \pi/2$.
(a)Sketch the region.
(b)Find the moment M_y about the y axis if the mass density is ρ (constant). (*Provide a detailed solution and draw a box around the final answer.*)

Solution.



$\Delta M = \rho y \Delta x,$
$M_y = \int_0^{\pi/2} x\rho y\, dx =_y= \rho \int_0^{\pi/2} x 3\cos x\, dx$
$= 3\int_0^{\pi/2} x\cos x\, dx \qquad [u = x, v' = \cos x,\ u' = 1, v = \sin x]$
$= \rho 3 \left[x\sin x - \int \sin x\right]_0^{\pi/2} = \rho 3 \left[x\sin x + \cos x\right]_0^{\pi/2} = 3(\frac{\pi}{2} - 1)\rho \doteq 1.17\rho$