Chapter 7
Risk, Return, and the
Capital Asset Pricing Model

ANSWERS TO END-OF-CHAPTER QUESTIONS

7-1 a. Stand-alone risk is only a part of total risk and pertains to the risk an investor takes by holding only one asset. Risk is the chance that some unfavourable event will occur. For instance, the risk of an asset is essentially the chance that the asset's cash flows will be unfavourable or less than expected. A probability distribution is a listing, chart, or graph of all possible outcomes, such as expected rates of return, with a probability assigned to each outcome. When in graph form, the tighter the probability distribution, the less uncertain the outcome.

b. The expected rate of return ($\hat{r}$) is the expected value of a probability distribution of expected returns.

c. A continuous probability distribution contains an infinite number of outcomes and is graphed from $-\infty$ and $+\infty$.

d. The standard deviation (σ) is a statistical measure of the variability of a set of observations. The variance (σ^2) of the probability distribution is the sum of the squared deviations about the expected value adjusted for deviation. The coefficient of variation (CV) is equal to the standard deviation divided by the expected return; it is a standardized risk measure which allows comparisons between investments having different expected returns and standard deviations.

e. A risk-averse investor dislikes risk and requires a higher rate of return as an inducement to buy riskier securities. A realized return is the actual return an investor receives on their investment. It can be quite different than their expected return.

f. A risk premium is the difference between the rate of return on a risk-free asset and the expected return on Stock i which has higher risk. The market risk premium is the difference between the expected return on the market and the risk-free rate.

g. CAPM is a model based upon the proposition that any stock's required rate of return is equal to the risk-free rate of return plus a risk premium reflecting only the risk remaining after diversification.

h. The expected return on a portfolio. $\hat{r}_p$, is simply the weighted-average expected return of the individual stocks in the portfolio, with the weights being the fraction of total portfolio value invested in each stock. The market portfolio is a portfolio consisting of all stocks.

i. Correlation is the tendency of two variables to move together. A correlation coefficient (ρ) of +1.0 means that the two variables move up and down in perfect synchronization, while a coefficient of -1.0 means the variables always move in opposite directions. A correlation coefficient of zero suggests that the two variables are not related to one another; that is, they are independent.

j. Market risk is that part of a security's total risk that cannot be eliminated by diversification. It is measured by the beta coefficient. Diversifiable risk is also known as company specific risk, that part of a security's total risk associated with random events not affecting the market as a whole. This risk can be eliminated by proper diversification. The relevant risk of a stock is its contribution to the riskiness of a well-diversified portfolio.

k. The beta coefficient is a measure of a stock's market risk, or the extent to which the returns on a given stock move with the stock market. The average stock's beta would move on average with the market so it would have a beta of 1.0.

l. The security market line (SML) represents in a graphical form, the relationship between the risk of an asset as measured by its beta and the required rates of return for individual securities. The SML equation is essentially the CAPM, $r_i = r_{RF} + b_i(r_M - r_{RF})$.

m. The slope of the SML equation is $(r_M - r_{RF})$, the market risk premium. The slope of the SML reflects the degree of risk aversion in the economy. The greater the average investor's aversion to risk, then the steeper the slope, the higher the risk premium for all stocks, and the higher the required return.

7-2 a. The probability distribution for complete certainty is a vertical line.

b. The probability distribution for total uncertainty is the X axis from $-\infty$ to $+\infty$.

7-3 Security A is less risky if held in a diversified portfolio because of its lower beta and negative correlation with other stocks. In a single-asset portfolio, Security A would be more risky because $\sigma_A > \sigma_B$ and $CV_A > CV_B$.

7-4 a. No, it is not riskless. The portfolio would be free of default risk and liquidity risk, but inflation could erode the portfolio's purchasing power. If the actual inflation rate is greater than that expected, interest rates in general will rise to incorporate a larger inflation premium (IP) and the value of the portfolio would decline.

b. No, you would be subject to reinvestment rate risk. You might expect to "roll over" the Treasury bills at a constant (or even increasing) rate of interest, but if interest rates fall, your investment income will decrease.

c. A government of Canada bond that provided interest with constant purchasing power (that is, an indexed bond) would be close to riskless.

7-5 The risk premium on a high beta stock would increase more.

$$RP_j = \text{Risk Premium for Stock j} = (r_M - r_{RF})b_j.$$

If risk aversion increases, the slope of the SML will increase, and so will the market risk premium ($r_M – r_{RF}$). The product ($r_M – r_{RF}$)b_j is the risk premium of the jth stock. If b_j is low (say, 0.5), then the product will be small; RP_j will increase by only half the increase in RP_M. However, if b_j is large (say, 2.0), then its risk premium will rise by twice the increase in RP_M.

7-6 According to the Security Market Line (SML) equation, an increase in beta will increase a company's expected return by an amount equal to the market risk premium times the change in beta. For example, assume that the risk-free rate is 6 percent, and the market risk premium is 5 percent. If the company's beta doubles from 0.8 to 1.6, its expected return increases from 10 percent to 14 percent. Therefore, in general, a company's expected return will not double when its beta doubles.

7-7 Yes, if the portfolio's beta is equal to zero. In practice, however, it may be impossible to find individual stocks that have a nonpositive beta. In this case it would also be impossible to have a stock portfolio with a zero beta. Even if such a portfolio could be constructed, investors would probably be better off just purchasing Treasury bills, or other zero beta investments.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

7-1

	Investment	Beta
	$35,000	0.8
	40,000	1.4
Total	$75,000	

($35,000/$75,000)(0.8) + ($40,000/$75,000)(1.4) = 1.12.

7-2 r_{RF} = 4%; r_M = 9%; b = 1.3; r_s = ?

$r_s = r_{RF} + (r_M - r_{RF})b$
$= 4\% + (9\% - 4\%)1.3$
$= 10.5\%.$

7-3 r_{RF} = 5%; RP_M = 6%; r_M = ?

$r_M = 5\% + (6\%)1 = 11\%.$

r_s when b = 1.2 = ?

$r_s = 5\% + 6\%(1.2) = 12.2\%.$

7-4 $\hat{r} = (0.1)(-50\%) + (0.2)(-5\%) + (0.4)(16\%) + (0.2)(25\%) + (0.1)(60\%)$
$= 11.40\%.$

$\sigma^2 = (-50\% - 11.40\%)^2(0.1) + (-5\% - 11.40\%)^2(0.2) + (16\% - 11.40\%)^2(0.4) + (25\% - 11.40\%)^2(0.2) + (60\% - 11.40\%)^2(0.1)$
$\sigma^2 = 712.44; \sigma = 26.69\%.$

$$CV = \frac{26.69\%}{11.40\%} = 2.34.$$

7-5 a. $\hat{r}_m = (0.3)(12\%) + (0.4)(5\%) + (0.3)(15\%) = 10.1\%.$

$\hat{r}_j = (0.3)(13\%) + (0.4)(4\%) + (0.3)(12\%) = 9.1\%.$

b. $\sigma_M = [(0.3)(12\% - 10.1\%)^2 + (0.4)(5\% - 10.1\%)^2 + (0.3)(15\% - 10.1\%)^2]^{1/2}$
$= \sqrt{18.69\%} = 4.32\%.$

$\sigma_J = [(0.3)(13\% - 9.1\%)^2 + (0.4)(4\% - 9.1\%)^2 + (0.3)(12\% - 9.1\%)^2]^{1/2}$
$= \sqrt{17.49\%} = 4.18\%.$

c. $CV_M = \frac{4.32\%}{10.1\%} = 0.43.$

$CV_J = \frac{4.18\%}{9.1\%} = 0.46.$

7-6 a. $r_A = r_{RF} + (r_M - r_{RF})b_A$
$12\% = 5\% + (10\% - 5\%)b_A$
$12\% = 5\% + 5\%(b_A)$
$7\% = 5\%(b_A)$
$1.4 = b_A.$

b. $r_A = 5\% + 5\%(b_A)$
$r_A = 5\% + 5\%(2)$
$r_A = 15\%.$

7-7 a. $r_i = r_{RF} + (r_M - r_{RF})b_i$ = 9% + (14% - 9%)1.3 = 15.5%.

b. 1. r_{RF} increases to 10%:

r_M increases by 1 percentage point, from 14% to 15%.

$r_i = r_{RF} + (r_M - r_{RF})b_i$ = 10% + (15% - 10%)1.3 = 16.5%.

2. r_{RF} decreases to 8%:

r_M decreases by 1%, from 14% to 13%.

$r_i = r_{RF} + (r_M - r_{RF})b_i$ = 8% + (13% - 8%)1.3 = 14.5%.

c. 1. r_M increases to 16%:

$r_i = r_{RF} + (r_M - r_{RF})b_i$ = 9% + (16% - 9%)1.3 = 18.1%.

2. r_M decreases to 13%:

$r_i = r_{RF} + (r_M - r_{RF})b_i$ = 9% + (13% - 9%)1.3 = 14.2%.

7-8 Old portfolio beta = $\frac{\$142,500}{\$150,000}(b) + \frac{\$7,500}{\$150,000}(1.00)$

1.37 = 0.95b + 0.05
1.32 = 0.95b
1.39 = b.

New portfolio beta = 0.95(1.39) + 0.05(2.1) = 1.43.

Alternative Solutions:

1. Old portfolio beta = 1.37 = $(0.05)b_1 + (0.05)b_2 + ... + (0.05)b_{20}$

$1.37 = (\Sigma b_i)(0.05)$
$\Sigma b_i = 1.37/0.05 = 27.4$.

New portfolio beta = (27.4 - 1.0 + 2.1)(0.05) = 1.43.

2. Σb_i excluding the stock with the beta equal to 1.0 is 27.4 - 1.0 = 26.4, so the beta of the portfolio excluding this stock is b = 26.4/19 = 1.3895. The beta of the new portfolio is:

1.3895(0.95) + 2.1(0.05) = 1.42503 = 1.43.

7-9

a. Stock A: r_i = 2% + 5% × 1.4 = 9%
Stock B: r_i = 2% + 5% × 0.8 = 6%

b. An average stock would have a beta of 1, therefore the return would be:
r_i = 2% + 5% x 1 = 7%

c. A security with a 0 beta has a return not correlated with the market at all. If there is no market risk associated with a 0 beta security, its return should be risk-free, in this case 2%.

d. If the economy worsens, investors will become more pessimistic. They generally will revise their return requirements upwards for holding stocks. In other words, they will require greater returns, therefore the 6.5% market risk premium would be more suitable.

e. Stock A: r_i = 2% + 6.5% × 1.4 = 11.1%
Stock B: r_i = 2% + 6.5% × 0.8 = 7.2%. Stocks A and B would be expected to fall.

f. The required return on investments, including stocks would increase by 0.5%. The new required return on A would be: r_i = 2.5% + 5% × 1.4 = 9.5% and B: r_i = 2.5% + 5% × 0.8 = 6.5%. The stock prices of A and B would both fall if inflation expectations increased.

7-10 $\hat{r}_p = w_A\hat{r}_A + (1 - w_A)\,\hat{r}_B$
$= 0.30(12\%) + 0.70(18\%) = 16.20\%$

$$\sigma_p = \sqrt{w^2_A\sigma^2_A + (1\text{–}w_A)^2\sigma^2_B + 2w_A\,(1\text{–}w_A)\rho_{AB}\sigma_A\sigma_B}$$
$$= \sqrt{(0.3^2)(0.4^2) + (0.7^2)(0.6^2) + 2(0.3)(0.7)(0.2)(0.4)(0.6)}$$
$$= \sqrt{0.0144 + 0.1764 + 0.02016} = \sqrt{0.21096} = 0.459 = 45.9\%$$

7-11 Portfolio beta = $\frac{\$400,000}{\$4,000,000}(1.50) + \frac{\$600,000}{\$4,000,000}(-0.50)$

$+ \frac{\$1,000,000}{\$4,000,000}(1.25) + \frac{\$2,000,000}{\$4,000,000}(0.75)$

$= 0.1)(1.5) + (0.15)(-0.50) + (0.25)(1.25) + (0.5)(0.75)$

$= 0.15 - 0.075 + 0.3125 + 0.375 = 0.7625.$

$r_p = r_{RF} + (r_M - r_{RF})(b_p) = 6\% + (14\% - 6\%)(0.7625) = 12.1\%.$

Alternative solution: First compute the return for each stock using the CAPM equation $[r_{RF} + (r_M - r_{RF})b]$, and then compute the weighted average of these returns.

$r_{RF} = 6\%$ and $r_M - r_{RF} = 8\%.$

Stock	Investment	Beta	$r = r_{RF} + (r_M - r_{RF})b$	Weight
A	$ 400,000	1.50	18%	0.10
B	600,000	(0.50)	2	0.15
C	1,000,000	1.25	16	0.25
D	2,000,000	0.75	12	0.50
Total	$4,000,000			1.00

$r_p = 18\%(0.10) + 2\%(0.15) + 16\%(0.25) + 12\%(0.50) = 12.1\%.$

7-12

a.

Total portfolio = \$75 + \$125 + \$200 = \$400

\$75/\$400 = 0.1875 x 12.5% =	2.34%
\$125/\$400 = 0.3125 x 6.75% =	2.11%
\$200/\$400 = 0.50 x 9% =	4.5%
Portfolio expected return =	8.95%

b.

\$75/\$400 = 0.1875 x 1.9 =	0.356
\$125/\$400 = 0.3125 x 0.75 =	0.234
\$200/\$400 = 0.50 x 1.2 =	0.6
Portfolio beta =	1.19

$\hat{r}_P = 3\% + 1.19(5\%) = 8.95\%$

Since $r_p = \hat{r}_P$ the stocks plot along the SML and are priced correctly.

c. $r_{\text{Off-shore}} = 3\% + 2.3(5\%) = 14.5\%$

Since the expected return of 14% < the required return of 14.5%, the investor should not purchase the stock.

d.

Total portfolio = \$75 + \$125 + \$200 +\$100 = \$500

\$75/\$500 x 12.5% =	1.88%
\$125/\$500 x 6.75% =	1.69%
\$200/\$500 x 9% =	3.6%
\$100/\$500 x 14.5% =	2.9%
Portfolio expected return =	10.1%

7-13 $\hat{r}_M = 0.1(7\%) + 0.2(9\%) + 0.4(11\%) + 0.2(13\%) + 0.1(15\%) = 11\%$.

$r_{RF} = 6\%$. (given)

Therefore, the SML equation is:

$r_i = r_{RF} + (r_M – r_{RF})b_i = 6\% + (11\% – 6\%)b_i = 6\% + (5\%)b_i$.

Next, determine the fund's beta, b_F. The weights are the percentage of funds invested in each stock:

A = \$160/\$500 = 0.32.
B = \$120/\$500 = 0.24.
C = \$80/\$500 = 0.16.
D = \$80/\$500 = 0.16.
E = \$60/\$500 = 0.12.

$b_F = 0.32(0.5) + 0.24(2.0) + 0.16(4.0) + 0.16(1.0) + 0.12(3.0)$
$= 0.16 + 0.48 + 0.64 + 0.16 + 0.36 = 1.8$.

Next, use $b_F = 1.8$ in the SML determined above.

$\hat{r}_F = 6\% + (11\% – 6\%)1.8 = 6\% + 9\% = 15\%$.

r_N = Required rate of return on new stock = 6% + (5%)2.0 = 16%.

An expected return of 15% on the new stock is below the 16% required rate of return on an investment with a risk of b = 2.0. Since $r_N = 16\% > \hat{r}_N = 15\%$, the new stock should not be purchased. The expected rate of return that would make the fund indifferent to purchasing the stock is 16%.

7-14 First, calculate the beta of what remains after selling the stock:

$b_p = 1.1 = (\$400,000/\$8,000,000)0.9 + (\$7,600,000/\$8,000,000)b_R$
$1.1 = 0.045 + (0.95)b_R$
$b_R = 1.1105$.

$b_N = (0.95)1.1105 + (0.05)1.9 = 1.15$.

7-15 We know that $b_R = 1.50$, $b_S = 0.75$, $r_M = 13\%$, $r_{RF} = 7\%$.

$r_i = r_{RF} + (r_M - r_{RF})b_i = 7\% + (13\% - 7\%)b_i$.

$r_R = 7\% + 6\%(1.50) = 16.0\%$
$r_S = 7\% + 6\%(0.75) = \underline{11.5}$
$\underline{\underline{4.5}}\%$

7-16 The answers to a, b, c, and d are given below:

		r_A	r_B	Portfolio
	2008	(18.00%)	(14.50%)	(16.25%)
	2009	33.00	21.80	27.40
	2010	15.00	30.50	22.75
	2011	(0.50)	(7.60)	(4.05)
	2012	27.00	26.30	26.65
Mean		11.30	11.30	11.30
Std Dev		20.79	20.78	20.13
CV		1.84	1.84	1.78

e. A risk-averse investor would choose the portfolio over either Stock A or Stock B alone, since the portfolio offers the same expected return but with less risk. This result occurs because returns on A and B are not perfectly positively correlated (ρ_{AB} = 0.88).

7-17 a. $r_X = 4\% + (6\%)1.65 = 13.9\%$.
$r_Y = 4\% + (6\%)0.22 = 5.3\%$.

b. $b_p = 0.6(1.65) + 0.4(0.22) = 1.08$.
$r_p = 4\% + (6\%)1.08 = 10.5\%$.
Alternatively,
$r_p = 0.6(13.9\%) + 0.4(5.3\%) = 10.5\%$.

c. Stock X is undervalued, because its expected return exceeds its required rate of return.

7-18 a. The arithmetic average return for Stock X is calculated as follows:

$$\bar{r}_{Avg} = \frac{(-14.0 + 23.0 + ... + 18.2)}{7} = 10.6\%.$$

The arithmetic average rate of return on the market portfolio, determined similarly, is 12.1%.

For Stock X, the estimated standard deviation is 13.1 percent:

$$\sigma_X = \sqrt{\frac{(-14.0 - 10.6)^2 + (23.0 - 10.6)^2 + ... + (18.2 - 10.6)^2}{7 - 1}} = 13.1\%.$$

The standard deviation of returns for the market portfolio is similarly determined to be 22.6 percent. The results are summarized below:

	Stock X	Market Portfolio
Average return, $\bar{r}_{Avg}$	10.6%	12.1%
Standard deviation, σ	13.1	22.6

Several points should be noted: (1) σ_M over this particular period is higher than the historic average σ_M of about 15 percent, indicating that the stock market was relatively volatile during this period; (2) Stock X, with σ_X = 13.1%, has much less total risk than an average stock, with σ_{Avg} = 22.6%; and (3) this example demonstrates that it is possible for a very low-risk single stock to have less risk than a portfolio of average stocks, since $\sigma_X < \sigma_M$.

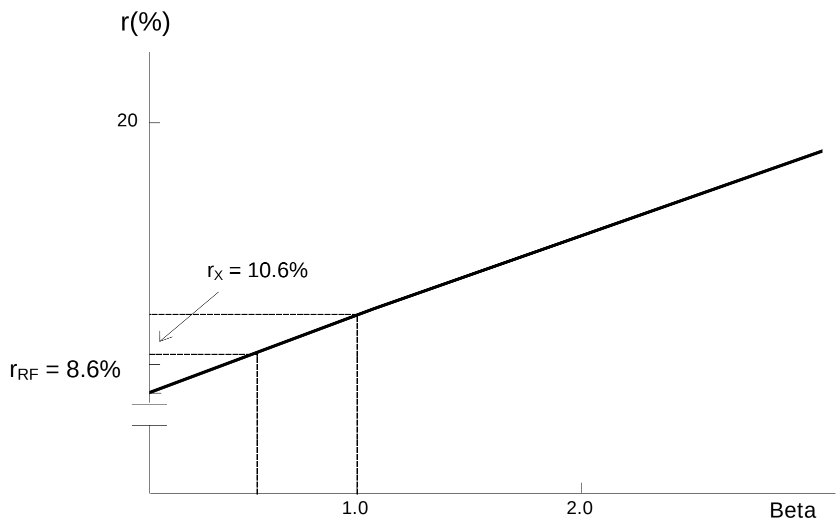
b. Since Stock X is in equilibrium and plots on the Security Market Line (SML), and given the further assumption that $\hat{r}_X = \bar{r}_X$ and $\hat{r}_M = \bar{r}_M$—and this assumption often does *not* hold—then this equation must hold:

$$\bar{r}_X = r_{RF} + (\bar{r} - r_{RF})b_X.$$

This equation can be solved for the risk-free rate, r_{RF}, which is the only unknown:

$$\begin{aligned} 10.6 &= r_{RF} + (12.1 - r_{RF})0.56 \\ 10.6 &= r_{RF} + 6.8 - 0.56r_{RF} \\ 0.44r_{RF} &= 10.6 - 6.8 \\ r_{RF} &= 3.8/0.44 = 8.6\%. \end{aligned}$$

c. The SML is plotted below. Data on the risk-free security (b_{RF} = 0, r_{RF} = 8.6%) and Security X (b_X = 0.56, $\bar{r}_X$ = 10.6%) provide the two points through which the SML can be drawn. r_M provides a third point.



d. In theory, you would be indifferent between the two stocks. Since they have the same beta, their relevant risks are identical, and in equilibrium they should provide the same returns. The two stocks would be represented by a single point on the SML. Stock Y, with the higher standard deviation, has more diversifiable risk, but this risk will be eliminated in a well-diversified portfolio, so the market will compensate the investor only for bearing market or relevant risk. In practice, it is possible that Stock Y would have a slightly higher required return, but this premium for diversifiable risk would be small.

SOLUTION TO SPREADSHEET PROBLEM

7-19 The detailed solution for the spreadsheet problem is in the file ***Ch 07 Build a Model Solution.xlsx*** and is available on the textbook's website.

MINI CASE

Assume that you recently graduated with a major in finance, and you just landed a job as a financial planner with Barney Smith Inc., a large financial services corporation. Your first assignment is to invest $100,000 for a client. Because the funds are to be invested in a business at the end of 1 year, you have been instructed to plan for a 1-year holding period. Further, your boss has restricted you to the investment alternatives shown in the table with their probabilities and associated outcomes. The relatively high T-bill rate reflects significant inflationary expectations.

Returns on Alternative Investments

		Estimated Rate of Return					
State of the Economy	**prob.**	**T-Bills**	**Alta Inds**	**Repo Men**	**Cdn. Foam**	**Market portfolio**	**2-stock portfolio**
Recession	**0.1**	**8.0%**	**-22.0%**	**28.0%**	**10.0%***	**-13.0%**	**3.0%**
Below avg	**0.2**	**8.0**	**-2.0**	**14.7**	**-10.0**	**1.0**	
Average	**0.4**	**8.0**	**20.0**	**0.0**	**7.0**	**15.0**	**10.0**
Above avg	**0.2**	**8.0**	**35.0**	**-10.0**	**45.0**	**29.0**	
Boom	**0.1**	**8.0**	**50.0**	**-20.0**	**30.0**	**43.0**	**15.0**
r-hat ($\hat{r}$)				**1.7%**	**13.8%**	**15.0%**	
Std dev (σ)		**0.0**		**13.4**	**18.8**	**15.3**	
Coef of var (cv)				**7.9**	**1.4**	**1.0**	
beta (b)				**-0.86**	**0.68**		

***Note that the estimated returns of Canadian Foam do not always move in the same direction as the overall economy. For example, when the economy is below average, consumers purchase fewer mattresses than they would if the economy was stronger. However, if the economy is in a flat-out recession, a large number of consumers who were planning to purchase a more expensive inner spring mattress may purchase, instead, a cheaper foam mattress. Under these circumstances, we would expect Canadian Foam's stock price to be higher if there is a recession than if the economy was just below average.**

Barney Smith's economic forecasting staff have developed probability estimates for the state of the economy, and its security analysts have developed a sophisticated computer program that was used to estimate the rate of return on each alternative under each state of the economy. Alta Industries is an electronics firm; Repo Men Inc. collects past-due debts; and Canadian Foam manufactures mattresses and other foam products. Barney Smith also maintains an "index fund" that owns a market-weighted fraction of all publicly traded stocks; you can invest in that fund, and thus obtain average stock market results. Given the situation as described, answer the following questions.

a. What are investment returns? What is the return on an investment that costs $1,000 and is sold after 1 year for $1,100?

Answer: Investment return measures the financial results of an investment. They may be expressed in either dollar terms or percentage terms.

The dollar return is $1,100 - $1,000 = $100. The percentage return is $100/$1,000 = 0.10 = 10%.

b. 1. Why is the T-bill's return independent of the state of the economy? Do T-bills promise a completely risk-free return?

Answer: The 8 percent T-bill return does not depend on the state of the economy because the treasury must (and will) redeem the bills at par regardless of the state of the economy.

The T-bills are risk-free in the default risk sense because the 8 percent return will be realized in all possible economic states. However, remember that this return is composed of the real risk-free rate, say 3 percent, plus an inflation premium, say 5 percent. Since there is uncertainty about inflation, it is unlikely that the realized real rate of return would equal the expected 3 percent. For example, if inflation averaged 6 percent over the year, then the realized real return would only be 8% - 6% = 2%, not the expected 3%. Thus, in terms of purchasing power, T-bills are not riskless.

Also, if you invested in a portfolio of T-bills, and rates then declined, your nominal income would fall; that is, T-bills are exposed to reinvestment rate risk. So, we conclude that there are no truly risk-free securities in Canada. If the treasury sold inflation-indexed, tax-exempt bonds, they would be truly riskless, but all actual securities are exposed to some type of risk.

b. 2. Why are Alta Industries' returns expected to move with the economy whereas Repo Men's are expected to move counter to the economy?

Answer: Alta Industries' returns move with, hence are positively correlated with, the economy, because the firm's sales, and hence profits, will generally experience the same type of ups and downs as the economy. If the economy is booming, so will Alta. On the other hand, Repo Men is considered by many investors to be a hedge against both bad times and high inflation, so if the stock market crashes, investors in this stock should do relatively well. Stocks such as Repo Men are thus negatively correlated with (move counter to) the economy. (note: in actuality, it is almost impossible to find stocks that are expected to move counter to the economy. Even Repo Men shares have positive (but low) correlation with the market.)

c. Calculate the expected rate of return on each alternative and fill in the blanks in the row for $\hat{r}$ in the table.

Answer: The expected rate of return, $\hat{r}$, is expressed as follows:

$$\hat{r} = \sum_{i=1}^{n} P_i r_i .$$

Here P_i is the probability of occurrence of the ith state, r_i is the estimated rate of return for that state, and n is the number of states. Here is the calculation for Alta Inds.:

$$\hat{r}_{\text{Alta Inds}} = 0.1(-22.0\%) + 0.2(-2.0\%) + 0.4(20.0\%) + 0.2(35.0\%) + 0.1(50.0\%)$$
$$= 17.4\%.$$

We use the same formula to calculate r's for the other alternatives:

$$\hat{r}_{\text{T-bills}} = 8.0\%.$$

$$\hat{r}_{\text{Repo Men}} = 1.7\%.$$

$$\hat{r}_{\text{Cdn Foam}} = 13.8\%.$$

$$\hat{r}_{M} = 15.0\%.$$

d. **You should recognize that basing a decision solely on expected returns is only appropriate only for risk-neutral individuals. Because your client, like virtually everyone, is risk averse, the riskiness of each alternative is an important aspect of the decision. One possible measure of risk is the standard deviation of returns.**

1. Calculate this value for each alternative, and fill in the blank on the row for σ in the table.

Answer: The standard deviation is calculated as follows:

$$\sigma = \sqrt{\sum_{i=1}^{n} (r_i - \hat{r}_i)^2 P_i}\,.$$

$$\sigma_{Alta} = [(-22.0 - 17.4)^2(0.1) + (-2.0 - 17.4)^2(0.2) + (20.0 - 17.4)^2(0.4) + (35.0 - 17.4)^2(0.2) + (50.0 - 17.4)^2(0.1)]^{0.5}$$
$$= \sqrt{401.4} = 20.0\%.$$

Here are the standard deviations for the other alternatives:

$$\sigma_{\text{T-bills}} = 0.0\%.$$

$$\sigma_{\text{Repo}} = 13.4\%.$$

$$\sigma_{\text{Cdn Foam}} = 18.8\%.$$

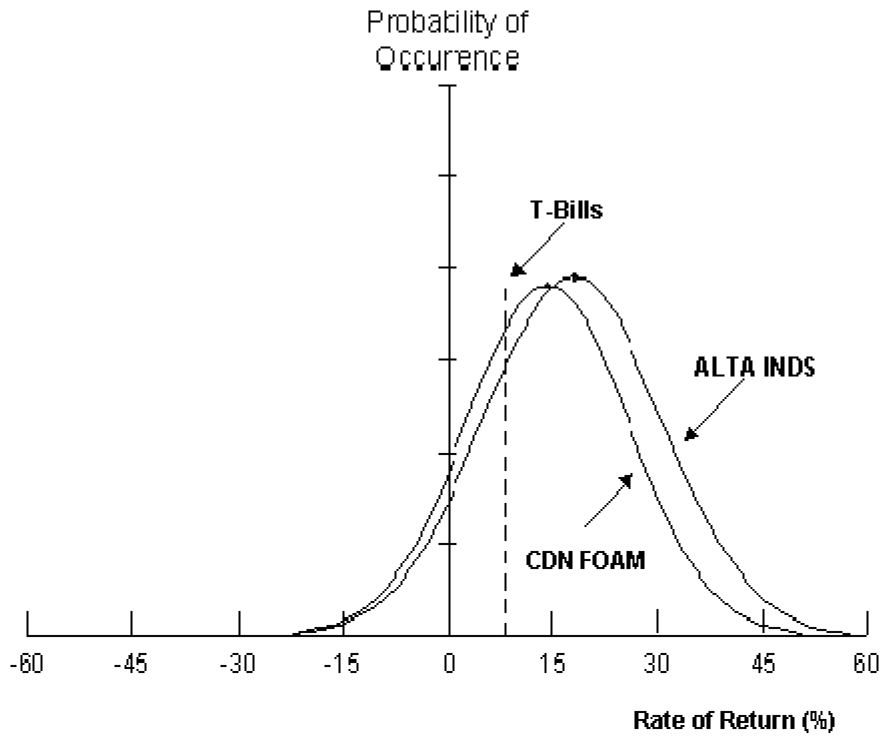
$$\sigma_{M} = 15.3\%.$$

d. 2. What type of risk is measured by the standard deviation?

Answer: The standard deviation is a measure of a security's (or a portfolio's) stand-alone risk. The larger the standard deviation, the higher the probability that actual realized returns will fall far below the expected return, and that losses rather than profits will be incurred.

d.	**3. Draw a graph that shows *roughly* the shape of the probability distributions for Alta Industries, Canadian Foam, and T-bills.**

Answer:



Based on these data, Alta Industries is the most risky investment, T-bills the least risky.

e. Suppose you suddenly remembered that the coefficient of variation (CV) is generally regarded as being a better measure of stand-alone risk than the standard deviation when the alternatives being considered have widely differing expected returns. Calculate the missing CVs, and fill in the blanks in the row for CV in the table. Does the CV produce the same risk rankings as the standard deviation?

Answer: The coefficient of variation (CV) is a standardized measure of dispersion about the expected value; it shows the amount of risk per unit of return.

$$CV = \frac{\sigma}{\hat{r}}.$$

$$CV_{T\text{-bills}} = 0.0\%/8.0\% = 0.0.$$

$$CV_{Alta\ Inds} = 20.0\%/17.4\% = 1.1.$$

$$CV_{Repo\ Men} = 13.4\%/1.7\% = 7.9.$$

$$CV_{Cdn\ Foam} = 18.8\%/13.8\% = 1.4.$$

$$CV_{M} = 15.3\%/15.0\% = 1.0.$$

When we measure risk per unit of return, Repo Men, with its low expected return, becomes the most risky stock. The CV is a better measure of an asset's stand-alone risk than σ because CV considers both the expected value and the dispersion of a distribution—a security with a low expected return and a low standard deviation could have a higher chance of a loss than one with a high σ but a high $\hat{r}$.

f. Suppose you created a 2-stock portfolio by investing $50,000 in Alta Industries and $50,000 in Repo Men.

1. Calculate the expected return ($\hat{r}_p$), the standard deviation (σ_p), and the coefficient of variation (cv_p) for this portfolio and fill in the appropriate blanks in the table.

Answer: To find the expected rate of return on the two-stock portfolio, we first calculate the rate of return on the portfolio in each state of the economy. Since we have half of our money in each stock, the portfolio's return will be a weighted average in each type of economy. For a recession, we have: $r_p = 0.5(-22\%) + 0.5(28\%) = 3\%$. We would do similar calculations for the other states of the economy, and get these results:

State	Portfolio
Recession	3.0%
Below Average	6.4
Average	10.0
Above Average	12.5
Boom	15.0

Now we can multiply probabilities times outcomes in each state to get the expected return on this two-stock portfolio, 9.6%.

Alternatively, we could apply this formula,

$$R = w_i \text{ x } r_i = 0.5(17.4\%) + 0.5(1.7\%) = 9.6\%,$$

which finds r as the weighted average of the expected returns of the individual securities in the portfolio.

It is tempting to find the standard deviation of the portfolio as the weighted average of the standard deviations of the individual securities, as follows:

$$\sigma_p \neq w_i(\sigma_i) + w_j(\sigma_j) = 0.5(20\%) + 0.5(13.4\%) = 16.7\%.$$

However, this is **not** correct—it is necessary to use a different formula, the one for σ that we used earlier, applied to the two-stock portfolio's returns.

The portfolio's σ depends jointly on (1) each security's σ and (2) the correlation between the securities' returns. The best way to approach the problem is to estimate the portfolio's risk and return in each state of the economy, and then to estimate σ_p with the σ formula. Given the distribution of returns for the portfolio, we can calculate the portfolio's σ and CV as shown below:

$$\sigma_p = [(3.0 - 9.6)^2(0.1) + (6.4 - 9.6)^2(0.2) + (10.0 - 9.6)^2(0.4) + (12.5 - 9.6)^2(0.2) + (15.0 - 9.6)^2(0.1)]^{0.5} = 3.3\%.$$

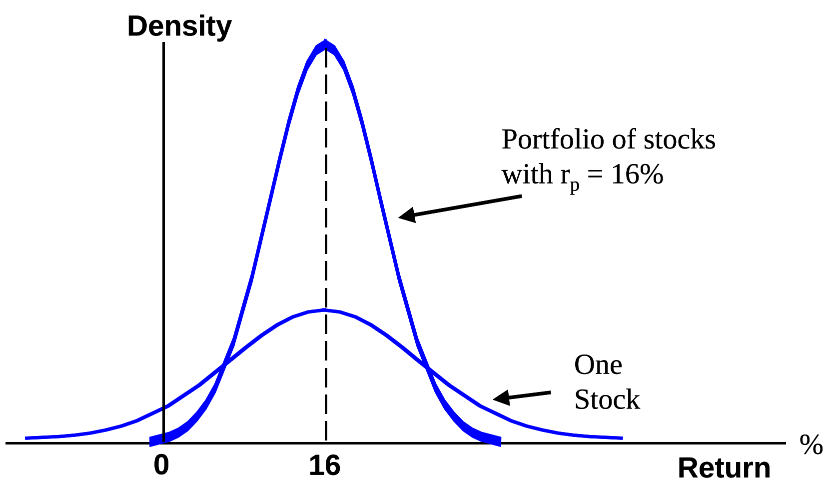
$$CV_p = 3.3\%/9.6\% = 0.3.$$

f. 2.	**How does the risk of this 2-stock portfolio compare with the risk of the individual stocks if they were held in isolation?**

Answer: Using either σ or CV as our stand-alone risk measure, the stand-alone risk of the portfolio is significantly less than the stand-alone risk of the individual stocks. This is because the two stocks are negatively correlated—when Alta Industries is doing poorly, Repo Men is doing well, and vice versa. Combining the two stocks diversifies away some of the risk inherent in each stock if it were held in isolation, i.e., in a 1-stock portfolio.

g.	**Suppose an investor starts with a portfolio consisting of one randomly selected stock. What would happen (1) to the risk and (2) to the expected return of the portfolio as more and more randomly selected stocks were added to the portfolio? What is the implication for investors? Draw a graph of the two portfolios to illustrate your answer.**

Answer:



The standard deviation gets smaller as more stocks are combined in the portfolio, while r_p (the portfolio's return) remains constant. Thus, by adding stocks to your portfolio, which initially started as a 1-stock portfolio, risk has been reduced.

In the real world, stocks are positively correlated with one another—if the economy does well, so do stocks in general, and vice versa. Correlation coefficients between stocks generally range from +0.5 to +0.7. The average correlation between stocks is about 0.35. A single stock selected at random would on average have a standard deviation of about 35 percent. As additional stocks are added to the portfolio, the portfolio's standard deviation decreases because the added stocks are not perfectly positively correlated. However, as more and more stocks are added, each new stock has less of a risk-reducing impact, and eventually adding additional stocks has virtually no effect on the portfolio's risk as measured by σ. In fact, σ stabilizes at about 20 percent when 40 or more randomly selected stocks are added. Thus, by combining stocks into well-diversified portfolios, investors can eliminate almost one-half the riskiness of holding individual stocks. (Note: it is not completely costless to diversify, so even the largest institutional investors hold less than all stocks. Even index funds generally hold a smaller portfolio that is highly correlated with an index such as the S&P 500 rather than hold all the stocks in the index.)

The implication is clear: investors should hold well-diversified portfolios of stocks rather than individual stocks. (In fact, individuals can hold diversified portfolios through mutual fund investments.) By doing so, they can eliminate about half of the riskiness inherent in individual stocks.

h. 1. Should portfolio effects impact the way investors think about the risk of individual stocks?

Answer: Portfolio diversification does affect investors' views of risk. A stock's stand-alone risk as measured by its σ or CV, may be important to an undiversified investor, but it is not relevant to a well-diversified investor. A rational, risk-averse investor is more interested in the impact that the stock has on the riskiness of his or her portfolio than on the stock's stand-alone risk. Stand-alone risk is composed of diversifiable risk, which can be eliminated by holding the stock in a well-diversified portfolio, and the risk that remains is called market risk because it is present even when the entire market portfolio is held.

h. 2. If you decided to hold a 1-stock portfolio, and consequently were exposed to more risk than diversified investors, could you expect to be compensated for all of your risk? That is, could you earn a risk premium on that part of your risk that you could have eliminated by diversifying?

Answer: If you hold a one-stock portfolio, you will be exposed to a high degree of risk, but you won't be compensated for it. If the return were high enough to compensate you for your high risk, it would be a bargain for more rational, diversified investors. They would start buying it, and these buy orders would drive the price up and the return down. Thus, you simply could not find stocks in the market with returns high enough to compensate you for the stock's diversifiable risk.

i.	**How is market risk measured for individual securities? How are beta coefficients calculated?**

Answer: Market risk, which is relevant for stocks held in well-diversified portfolios, is defined as the contribution of a security to the overall riskiness of the portfolio. It is measured by a stock's beta coefficient, which measures the stock's volatility relative to the market.

Run a regression with returns on the stock in question plotted on the y axis and returns on the market portfolio plotted on the x axis.

The slope of the regression line, which measures relative volatility, is defined as the stock's beta coefficient, or b.

j. **The expected rates of return and the beta coefficients of the alternatives as supplied by Barney Smith's computer program are as follows:**

Security	**Return ($\hat{r}$)**	**Risk (Beta)**
Alta Inds	**17.4%**	**1.29**
Market	**15.0**	**1.00**
Cdn Foam	**13.8**	**0.68**
T-Bills	**8.0**	**0.00**
Repo Men	**1.7**	**(0.86)**

j. **1. Do the expected returns appear to be related to each alternative's market risk?**
2. Is it possible to choose among the alternatives on the basis of the information developed thus far?

Answer: The expected returns are related to each alternative's market risk—that is, the higher the alternative's rate of return the higher its beta. Also, note that T-bills have 0 risk.

We do not yet have enough information to choose among the various alternatives. We need to know the required rates of return on these alternatives and compare them with their expected returns.

k. 1. Write out the security market line (SML) equation, use it to calculate the required rate of return on each alternative, and then graph the relationship between the expected and required rates of return.

Answer: Here is the SML equation:

$$r_i = r_{rf} + (r_m - r_{rf})b_i.$$

If we use the T-bill yield as a proxy for the risk-free rate, then r_{RF} = 8%. Further, our estimate of $r_m = \hat{r}_m$ is 15%. Thus, the required rates of return for the alternatives are as follows:

Alta Inds: 8% + (15% - 8%)1.29 = 17.03% ≈ 17.0%.

Market: 8% + (15% - 8%)1.00 = 15.0%.

Cdn Foam : 8% +(15% - 8%)0.68 = 12.76% ≈ 12.8%.

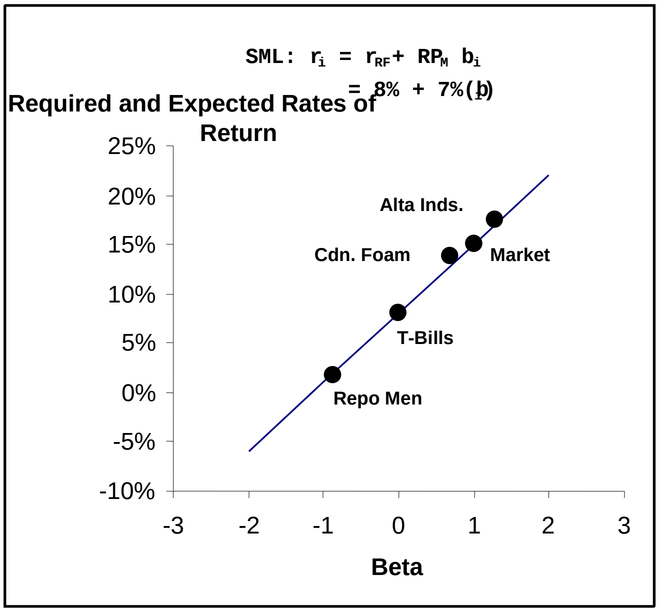
T-Bills: 8% + (15% - 8%)0.0 = 8.0%.

Repo Men: 8% + (15% - 8%)-0.86 = 1.98% ≈ 2%.

k. 2. How do the expected rates of return compare with the required rates of return?

Answer: We have the following relationships:

SECURITY	Expected Return ($\hat{r}$)	Required Return (r)	CONDITION
Alta Inds	17.4%	17.0%	Undervalued: $\hat{r}$ > R
Market	15.0	15.0	Fairly Valued (Market Equilibrium)
Cdn Foam	13.8	12.8	Undervalued: $\hat{r}$ > R
T-Bills	8.0	8.0	Fairly Valued
Repo Men	1.7	2.0	Overvalued: R > $\hat{r}$



(Note: the plot looks somewhat unusual in that the x axis extends to the left of zero. We have a negative beta stock, hence a required return that is less than the risk-free rate.) The T-bills and market portfolio plot on the SML, Alta Inds. and Cdn. Foam plot above it, and Repo Men plots below it. Thus, the T-bills and the market portfolio promise a fair return, Alta Inds and Cdn. Foam are good deals because they have expected returns above their required returns, and Repo Men has an expected return below its required return.

k. 3. Does the fact that Repo Men has an expected return that is less than the T-bill rate make any sense?

Answer: Repo Men is an interesting stock. Its negative beta indicates negative market risk—including it in a portfolio of "normal" stocks will lower the portfolio's risk. Therefore, its required rate of return is below the risk-free rate. Basically, this means that Repo Men is a valuable security to rational, well-diversified investors. To see why, consider this question: would any rational investor ever make an investment that has a <u>negative</u> expected return? The answer is "yes"—just think of the purchase of a life or fire insurance policy. The fire insurance policy has a negative expected return because of commissions and insurance company profits, but businesses buy fire insurance because they pay off at a time when normal operations are in bad shape. Life insurance is similar—it has a high return when work income ceases. A negative beta stock is conceptually similar to an insurance policy.

k. 4. What would be the market risk and the required return of a 50-50 portfolio of Alta Inds and Repo Men? Of Alta Inds and Cdn Foam?

Answer: Note that the beta of a portfolio is simply the weighted average of the betas of the stocks in the portfolio. Thus, the beta of a portfolio with 50 percent Alta Industries and 50 percent Repo Men is:

$$b_p = \sum_{i=1}^{n} w_i b_i .$$

$$b_p = 0.5(b_{Alta}) + 0.5(b_{Repo}) = 0.5(1.29) + 0.5(-0.86)$$
$$= 0.215,$$

$$r_p = r_{RF} + (r_M - r_{RF})b_p = 8.0\% + (15.0\% - 8.0\%)(0.215)$$
$$= 8.0\% + 7\%(0.215)$$
$$= 9.51\% \approx 9.5\%.$$

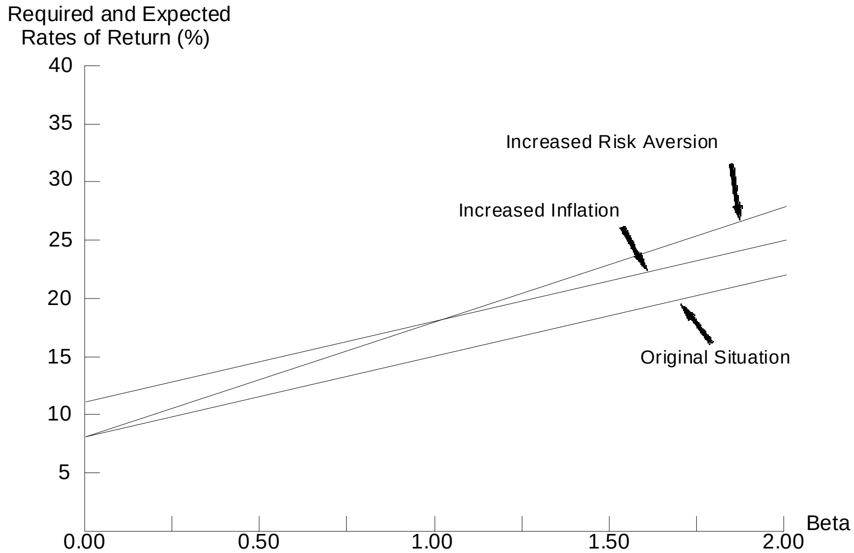
For a portfolio consisting of 50% Alta Industries plus 50% Canadian Foam, the required return would be 14.9%:

$$b_p = 0.5(1.29) + 0.5(0.68) = 0.985.$$

$$r_p = 8.0\% + 7\%(0.985) = 14.9\%.$$

l.	**1.**	**Suppose investors raised their inflation expectations by 3 percentage points over current estimates as reflected in the 8% T-bill rate. What effect would higher inflation have on the SML and on the returns required on high- and low-risk securities?**

Answer:



Here we have plotted the SML for betas ranging from 0 to 2.0. The base case SML is based on r_{RF} = 8% and r_M = 15%. If inflation expectations increase by 3 percentage points, with no change in risk aversion, then the entire SML is shifted upward (parallel to the base case SML) by 3 percentage points. Now, r_{RF} = 11%, r_M = 18%, and all securities' required returns rise by 3 percentage points. Note that the market risk premium, $r_m - r_{RF}$, remains at 7 percentage points.

l.	**2.**	**Suppose instead that investors' risk aversion increased enough to cause the market risk premium to increase by 3 percentage points. (Inflation remains constant.) What effect would this have on the SML and on returns of high- and low-risk securities?**

Answer: When investors' risk aversion increases, the SML is rotated upward about the y-intercept (r_{RF}). r_{RF} remains at 8%, but now r_M increases to 18%, so the market risk premium increases to 10 percent. The required rate of return will rise sharply on high-risk (high-beta) stocks, but not much on low-beta securities.

