Chapter 6
Bonds, Bond Valuation, and Interest Rates

ANSWERS TO END-OF-CHAPTER QUESTIONS

6-1 a. A bond is a promissory note issued by a business or a governmental unit. Government bonds, are issued by the federal and provincial governments and (federal gov't bonds) are not exposed to default risk. Corporate bonds are issued by corporations and are exposed to default risk. Different corporate bonds have different levels of default risk, depending on the issuing company's characteristics and on the terms of the specific bond. Foreign bonds are issued by foreign governments or foreign corporations. These bonds are not only exposed to default risk, but are also exposed to an additional risk if the bonds are denominated in a currency other than that of the investor's home currency.

b. The par value is the nominal or face value of a stock or bond. The par value of a bond generally represents the amount of money that the firm borrows and promises to repay at some future date. The par value of a bond is often $1,000, but can be $5,000 or more. The maturity date is the date when the bond's par value is repaid to the bondholder. Maturity dates generally range from 10 to 40 years from the time of issue. A call provision may be written into a bond contract, giving the issuer the right to redeem the bonds under specific conditions prior to the normal maturity date. A bond's coupon, or coupon payment, is the dollar amount of interest paid to each bondholder on the interest payment dates. The coupon is so named because bonds used to have dated coupons attached to them which investors could tear off and redeem on the interest payment dates. The coupon interest rate is the stated rate of interest on a bond.

c. In some cases, a bond's coupon payment may vary over time. These bonds are called floating rate bonds. Floating rate debt is popular with investors because the market value of the debt is stabilized. It is advantageous to corporations because firms can issue long-term debt without committing themselves to paying a historically high interest rate for the entire life of the loan. Zero coupon bonds pay no coupons at all, but are offered at a substantial discount below their par values and hence provide capital appreciation rather than interest income.

d. Most bonds contain a call provision, which gives the issuing corporation the right to call the bonds for redemption. The call provision generally states that if the bonds are called, the company must pay the bondholders an amount greater than the par value, a call premium. Retractable bonds give investors the right to sell the bonds back to the corporation at a price that is usually close to the par value. If interest rates rise, investors can retract the bonds and reinvest at the higher rates. A sinking fund provision facilitates the orderly retirement of a bond issue. This can be achieved in one of two ways: The company can call in for redemption (at par value) a certain percentage of bonds each year. The company may buy the required amount of bonds on the open market.

e. Convertible bonds are securities that are convertible into shares of common stock, at a fixed price, at the option of the bondholder. Bonds issued with warrants are similar to convertibles. Warrants are options which permit the holder to buy stock for a stated price, thereby providing a capital gain if the stock price rises. Income bonds pay interest only if the interest is earned. These securities cannot bankrupt a company, but from an investor's standpoint they are riskier than “regular” bonds. The interest rate of a real return bond is based on an inflation index such as the consumer price index (CPI), so the interest paid rises automatically when the inflation rate rises, thus protecting the bondholders against inflation.

f. Bond prices and interest rates are inversely related; that is, they tend to move in the opposite direction from one another. A fixed-rate bond will sell at par when its coupon interest rate is equal to the going rate of interest, r_d. When the going rate of interest is above the coupon rate, a fixed-rate bond will sell at a “discount” below its par value. If current interest rates are below the coupon rate, a fixed-rate bond will sell at a “premium” above its par value.

g. The current yield on a bond is the annual coupon payment divided by the current market price. YTM, or yield to maturity, is the rate of interest earned on a bond if it is held to maturity. Yield to call (YTC) is the rate of interest earned on a bond if it is called. If current interest rates are well below an outstanding callable bond’s coupon rate, the YTC may be a more relevant estimate of expected return than the YTM, since the bond is likely to be called.

h. The shorter the maturity of the bond, the greater the risk of a decrease in interest rates. The risk of a decline in income due to a drop in interest rates is called reinvestment rate risk. Interest rates fluctuate over time, and people or firms who invest in bonds are exposed to risk from changing interest rates, or interest rate risk. The longer the maturity of the bond, the greater the exposure to interest rate risk. Interest rate risk relates to the value of the bonds in a portfolio, while reinvestment rate risk relates to the income the portfolio produces. No fixed-rate bond can be considered totally riskless. Bond portfolio managers try to balance these two risks, but some risk always exists in any bond. Another important risk associated with bonds is default risk. If the issuer defaults, investors receive less than the promised return on the bond. Default risk is influenced by both the financial strength of the issuer and the terms of the bond contract, especially whether collateral has been pledged to secure the bond. The greater the default risk, the higher the bond's yield to maturity.

i. Corporations can influence the default risk of their bonds by changing the type of bonds they issue. Under a mortgage bond, the corporation pledges certain assets as security for the bond. All such bonds are written subject to an indenture, which is a legal document that spells out in detail the rights of both the bondholders and the corporation. A debenture is an unsecured bond, and as such, it provides no lien against specific property as security for the obligation. Debenture holders are, therefore, general creditors whose claims are protected by property not otherwise pledged. Subordinated debentures have claims on assets, in the event of bankruptcy, only after senior debt as named in the subordinated debt's indenture has been paid off. Subordinated debentures may be subordinated to designated notes payable or to all other debt.

j. Agency bonds are issued by government agencies such as crown corporations. Agency bonds carry the same government guarantee as regular federal or provincial government bonds. Bond issues are normally assigned quality ratings by major rating agencies, such as Moody's Investors Service and Standard & Poor's Corporation. These ratings reflect the probability that a bond will go into default. Aaa (Moody's) and AAA (S&P) are the highest ratings. Rating assignments are based on qualitative and quantitative factors including the firm's debt/assets ratio, current ratio, and coverage ratios. Because a bond's rating is an indicator of its default risk, the rating has a direct, measurable influence on the bond's interest rate and the firm's cost of debt capital. Junk bonds are high-risk, high-yield bonds issued to finance leveraged buyouts, mergers, or troubled companies. Most bonds are purchased by institutional investors rather than individuals, and many institutions are restricted to investment grade bonds, securities with ratings of A or above.

k. The real risk-free rate is that interest rate which equalizes the aggregate supply of, and demand for, riskless securities in an economy with zero inflation. The real risk-free rate could also be called the pure rate of interest since it is the rate of interest that would exist on very short-term, default-free federal government securities if the expected rate of inflation were zero. It has been estimated that this rate of interest, denoted by r*, has fluctuated in recent years in Canada in the range of 2 to 4 percent. The nominal risk-free rate of interest, denoted by r_{RF}, is the real risk-free rate plus a premium for expected inflation. The short-term nominal risk-free rate is usually approximated by the government of Canada Treasury bill rate, while the long-term nominal risk-free rate is approximated by the rate on government of Canada bonds. Note that while bonds are free of default and liquidity risks, they are subject to risks due to changes in the general level of interest rates.

l. The inflation premium is the premium added to the real risk-free rate of interest to compensate for the expected loss of purchasing power. The inflation premium is the average rate of inflation expected over the life of the security. Default risk is the risk that a borrower will not pay the interest and/or principal on a loan as they become due. Thus, a default risk premium (DRP) is added to the real risk-free rate to compensate investors for bearing default risk. Liquidity refers to a firm's cash and marketable securities position, and to its ability to meet maturing obligations. A liquid asset is any asset that can be quickly sold and converted to cash at its "fair" value. Active markets provide liquidity. A liquidity premium is added to the real risk-free rate of interest, in addition to other premiums, if a security is not liquid.

m. Interest rate risk arises from the fact that bond prices decline when interest rates rise. Under these circumstances, selling a bond prior to maturity will result in a capital loss, and the longer the term to maturity, the larger the loss. Thus, a maturity risk premium must be added to the real risk-free rate of interest to compensate for interest rate risk. Reinvestment rate risk occurs when a short-term debt security must be "rolled over." If interest rates have fallen, the reinvestment of principal will be at a lower rate, with correspondingly lower interest payments and ending value. Note that long-term debt securities also have some reinvestment rate risk because their interest payments have to be reinvested at prevailing rates.

n. The term structure of interest rates is the relationship between yield to maturity and term to maturity for bonds of a single risk class. The yield curve is the curve that results when yield to maturity is plotted on the Y-axis with term to maturity on the X-axis.

o. When the yield curve slopes upward, it is said to be "normal," because it is like this most of the time. Conversely, a downward-sloping yield curve is termed "abnormal" or "inverted."

6-2 False. Short-term bond prices are less sensitive than long-term bond prices to interest rate changes because funds invested in short-term bonds can be reinvested at the new interest rate sooner than funds tied up in long-term bonds.

6-3 The price of the bond will fall and its YTM will rise if interest rates rise. If the bond still has a long term to maturity, its YTM will reflect long-term rates. Of course, the bond's *price* will be less affected by a change in interest rates if it has been outstanding a long time and matures shortly. While this is true, it should be noted that the YTM will increase only for buyers who purchase the bond after the change in interest rates and not for buyers who purchased previous to the change. If the bond is purchased and held to maturity, the bondholder's YTM will not change, regardless of what happens to interest rates.

6-4 If interest rates decline significantly, the values of callable bonds will not rise by as much as those of bonds without the call provision. It is likely that the bonds would be called by the issuer before maturity, so that the issuer can take advantage of the new, lower rates.

6-5 From the corporation's viewpoint, one important factor in establishing a sinking fund is that its own bonds generally have a higher yield than do government bonds; hence, the company saves more interest by retiring its own bonds than it could earn by buying government bonds. This factor causes firms to favor the second procedure. Investors also would prefer the annual retirement procedure if they thought that interest rates were more likely to rise than to fall, but they would prefer the government bond purchases program if they thought rates were likely to fall. In addition, bondholders recognize that, under the government bond purchase scheme, each bondholder would be entitled to a given amount of cash from the liquidation of the sinking fund if the firm should go into default, whereas under the annual retirement plan, some of the holders would receive a cash benefit while others would benefit only indirectly from the fact that there would be fewer bonds outstanding.

On balance, investors seem to have little reason for choosing one method over the other, while the annual retirement method is clearly more beneficial to the firm. The consequence has been a pronounced trend toward annual retirement and away from the accumulation scheme.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

6-1 With your financial calculator, enter the following:

N = 12; I/YR = YTM = 9%; PMT = 0.08 × 1,000 = 80; FV = 1000; PV = V_B = ?
PV = $928.39.

Alternatively,

$V_B = \$80((1- 1/1.09^{12})/0.09) + \$1,000(1/1.09^{12})$
$= \$928.39$

6-2 With your financial calculator, enter the following:

N = 12; PV = -850; PMT = 0.10 × 1,000 = 100; FV = 1000; I/YR = YTM = ?
YTM = 12.48%.

6-3 With your financial calculator, enter the following to find the current value of the bonds, so you can then calculate their current yield:

N = 12; I/YR = YTM = 8; PMT = 0.06 × 1,000 = 60; FV = 1000; PV = V_B = ?
PV = $849.28. Current yield = $60/$849.28 = 7.06%.

Alternatively,

$V_B = \$60((1- 1/1.08^{12})/0.08) + \$1,000(1/1.08^{12})$
$= \$849.28.$

Current yield = $60/$849.28 = 7.06%.

6-4 $r^* = 4\%$; $I_1 = 2\%$; $I_2 = 4\%$; $I_3 = 4\%$; MRP = 0; $r_{T\text{-}2} = ?$; $r_{T\text{-}3} = ?$

$r = r^* + IP + DRP + LP + MRP$.

Since these are government securities, DRP = LP = 0.

$r_{T\text{-}2} = r^* + IP_2$
$IP_2 = (2\% + 4\%)/2 = 3\%$
$r_{T\text{-}2} = 4\% + 3\% = 7\%$.

$r_{T\text{-}3} = r^* + IP_3$
$IP_3 = (2\% + 4\% + 4\%)/3 = 3.33\%$
$r_{T\text{-}3} = 4\% + 3.33\% = 7.33\%$.

6-5 $r_{T\text{-}10} = 6\%$; $r_{C\text{-}10} = 9\%$; LP = 0.5%; DRP = ?

$r = r^* + IP + DRP + LP + MRP$.

$r_{T\text{-}10} = 6\% = r^* + IP + MRP$; DRP = LP = 0.

$r_{C\text{-}10} = 8\% = r^* + IP + DRP + 0.5\% + MRP$.

Because both bonds are 10-year bonds the inflation premium and maturity risk premium on both bonds are equal. The only difference between them is the liquidity and default risk premiums.

$r_{C\text{-}10} = 9\% = r^* + IP + MRP + 0.5\% + DRP$. But we know from above that $r^* + IP + MRP = 6\%$; therefore,

$r_{C\text{-}10} = 9\% = 6\% + 0.5\% + DRP$
$2.5\% = DRP$.

6-6 $r^* = 3\%$; IP = 3%; $r_{T\text{-}2} = 6.3\%$; $MRP_2 = ?$

$r_{T\text{-}2} = r^* + IP + MRP = 6.3\%$
$r_{T\text{-}2} = 3\% + 3\% + MRP = 6.3\%$
MRP = 0.3%.

6-7 The problem asks you to find the price of a bond, given the following facts:

N = 12; I/YR = 5/2 = 2.5; PMT = 50; FV = 1000.

With a financial calculator, solve for PV = $1,256.44.

6-8 With your financial calculator, enter the following to find YTM:

$N = 10 \times 2 = 20$; PV = -1100; PMT = $0.08/2 \times 1{,}000 = 40$; FV = 1000; I/YR = YTM = ?
YTM = 3.31% × 2 = 6.62%.

With your financial calculator, enter the following to find YTC:

$N = 5 \times 2 = 10$; PV = -1100; PMT = $0.08/2 \times 1{,}000 = 40$; FV = 1050; I/YR = YTC = ?
YTC = 3.24% × 2 = 6.49%.

6-9

a. Input N=10, I=2.75, PMT=-30, FV=-1000, PV=? PV=$1,021.60.

   Input N=10, I=2.75, PMT=0, FV=-1000, PV=? PV=$762.40.

b. First bond with 10 year maturity (N=20) and all else the same, PV=$1,038.07.

   First bond with 15 year maturity (N=30) and all else the same, PV=$1,050.62.

   Second bond with 10 year maturity (N=20) and all else the same, PV=$581.25.

   Second bond with 15 year maturity (N=30) and all else the same, PV=$443.14.

c. There is greater discounting in the zero coupon bond price because there are no semi-annual payments provided. The return must be made up from the difference between the purchase price and maturity price.

6-10 a. Calculator solution:

1. Input N = 5, PV = -829, PMT = 90, FV = 1000, I/YR = ? I/YR = 13.98%.

2. Change PV = -1104, I/YR = ? I/YR = 6.50%.

b. Yes. At a price of $829, the yield to maturity, 13.98 percent, is greater than your required rate of return of 12 percent. If your required rate of return were 12 percent, you should be willing to buy the bond at any price below $891.86.

6-11 N = 4; PV = -1000; PMT = 70; FV = 1090; I/YR = ? Solve for I/YR = 8.97%.

6-12 a. Using a financial calculator, input the following:

N = 20, PV = -1100, PMT = 60, FV = 1000, and solve for I/YR = 5.1849%.

However, this is a periodic rate. The nominal annual rate = 5.1849%(2) = 10.3699% ≈ 10.37%.

b. The current yield = $120/$1,100 = 10.91%.

c. YTM = Current Yield + Capital Gains (Loss) Yield
10.37% = 10.91% + Capital Loss Yield
-0.54% = Capital Loss Yield.

d. Using a financial calculator, input the following:

N = 8, PV = -1100, PMT = 60, FV = 1060, and solve for I/YR = 5.0748%.

However, this is a periodic rate. The nominal annual rate = 5.0748%(2) = 10.1495% ≈ 10.15%.

6-13 The problem asks you to solve for the YTM, given the following facts:

N = 12, PMT = 60, and FV = 1000. In order to solve for I/YR we need PV.

However, you are also given that the current yield is equal to 6.4%. Given this information, we can find PV.

Current yield	= Annual interest/Current price
0.064	= $60/PV
PV	= $60/0.064 = $937.50.

Now, solve for the YTM with a financial calculator:

N = 12, PV = -937.5, PMT = 60, and FV = 1000. Solve for I/YR = YTM = 6.78%.

6-14 The problem asks you to solve for the current yield, given the following facts: N = 14, I/YR = 10.5883/2 = 5.2942, PV = -1020, and FV = 1000. In order to solve for the current yield we need to find PMT. With a financial calculator, we find PMT = $55.00. However, because the bond is a semiannual coupon bond this amount needs to be multiplied by 2 to obtain the annual interest payment: $55.00(2) = $110.00. Finally, find the current yield as follows:

Current yield = Annual interest/Current Price = $110/$1,020 = 10.78%.

6-15 The bond is selling at a large premium, which means that its coupon rate is much higher than the going rate of interest. Therefore, the bond is likely to be called—it is more likely to be called than to remain outstanding until it matures. Thus, it will probably provide a return equal to the YTC rather than the YTM. So, there is no point in calculating the YTM—just calculate the YTC:

N = 8, PV = -1410, PMT = 70, FV = 1090, and then solve for I/YR.

The periodic rate is 2.35 percent, so the nominal YTC is 2 x 2.35% = 4.7%. This would be close to the going rate, and it is about what the firm would have to pay on new bonds.

6-16

	Price at 8%	Price at 7%	% change
10-year, 10% coupon	$1,135.90	$1,213.19	6.8%
10-year zero	456.39	502.57	10.1
30-year zero	95.06	126.93	33.5

6-17

t	Price of Bond C	Price of Bond Z
0	$1,121.56	$ 774.25
1	1,094.02	825.39
2	1,064.66	879.91
3	1,033.36	938.04
4	1,000.00	1,000.00

6-18 r = r* + IP + MRP + DRP + LP.
r* = 0.02.
IP = [0.03 + 0.04 + (5)(0.035)]/7 = 0.035.
MRP = 0.0005(6) = 0.003.
DRP = 0.
LP = 0.

r = 0.02 + 0.035 + 0.003 = 0.058 = 5.8%.

6-19 First, note that we will use the equation $r_t = 3\% + IP_t + MRP_t$. We have the data needed to find the IPs:

$$IP_5 = \frac{8\% + 5\% + 4\% + 4\% + 4\%}{5} = \frac{25\%}{5} = 5\%.$$

$$IP_2 = \frac{8\% + 5\%}{2} = 6.5\%.$$

Now we can substitute into the equation:

$r_2 = 3\% + 6.5\% + MRP_2 = 10\%$. $r_5 = 3\% + 5\% + MRP_5 = 10\%$.

Now we can solve for the MRPs, and find the difference:

$MRP_5 = 10\% - 8\% = 2\%$. $MRP_2 = 10\% - 9.5\% = 0.5\%$.

Difference = (2% - 0.5%) = 1.5%.

6-20 To find the purchase price, using a financial calculator input N=26, I=3.5, PMT=-30, FV=-1000 and PV=? PV=$915.55.

To find the selling price one year later, input N=24, I=4, PMT=-30, FV=-1000 and PV=? PV=$847.53.

Capital loss = $847.53 - $915.55 = -$68.02

6-21 To find the total cost to Harbour today, using a financial calculator input N=14, I=3, PMT=0, FV=11000000, PV=? PV= $7,272,296.

6-22 In order to solve for the price you are willing to pay today (time 0), you need the required rate of return, 10%, the coupon payment, $90, number of years you'll hold the bond, 5, and the price you'll sell it for in 5 years = ?.

You can calculate the price at in year 5, since you know the required return from years 6-20, 8.5%, number of years, 15, coupon payment, $90, and face value in year 20, $1,000

Input FV = 1000, N = 15, I/YR = 8.5%, PMT = 90, PV = ? = 1,041.52

Now you can calculate what you would pay at time = 0.

Input FV = 1,041.52, N = 5, I/Y = 10, PMT = 90, PV = ? = $987.87

6-23

a. 7% coupon bond: FV=1000, N=5, PMT=70, I/Y=7.5, PV=?=979.77
$50,000,000/0.97977 = $51,033,000.

Zero coupon bond: FV=1000, N=5, PMT=0, I/Y=7.5, PV=?=696.56
$50,000,000/0.69656 = $71,782,000.

Floating rate bond: $50,000,000

b. C_0 = -1,000
C_1 = 45
C_2 = 55
C_3 = 60
C_4 = 65
C_5 = 1,065; IRR = 5.74% = YTM

c. The yield to maturity of the 7% coupon and zero coupon bonds reflects a higher maturity risk premium as compensation for greater interest rate risk. The floating rate bond has little interest rate risk because LIBOR changes along with market interest rates. This results in a lower maturity risk premium and a lower required rate of return.

d.

	7% coupon	Zero coupon	Floating rate
Benefits	-Fixed borrowing cost. (no cost if rates rise).	-No annual interest payments to make. (Preserves cash flow.)	-Lower cost of financing (assuming normal yield curve).
Drawbacks	-Fixed borrowing cost (no benefit it rates drop).	-Large 'bullet' payment at maturity. -Potentially more difficult to sell.	-Interest rate risk now borne by the company. -Uncertain cash flows.

6-24 a. The bonds now have a 6-year, or a 12-semiannual period, maturity, and their value is calculated as follows:

Calculator solution: Input N = 12, I/YR = 2.5, PMT =35, FV = 1000,
PV = ? PV = $1,102.58.

b. Calculator solution: Change inputs from Part a to I/YR = 4.5, PV = ? PV= $908.81.

c. The price of the bond will decline toward $1,000, hitting $1,000 (plus accrued interest) at the maturity date in 6 years (12 six-month periods).

6-25 a. Find the YTM as follows:

N = 10, PV = -1200, PMT = 110, FV = 1000, I/YR = YTM = 8.02%.

b. Find the YTC, if called in Year 5 as follows:

N = 5, PV = -1200, PMT = 110, FV = 1090, I/YR = YTC = 7.59%.

c. The bonds are selling at a premium which indicates that interest rates have fallen since the bonds were originally issued. Assuming that interest rates do not change from the present level, investors would expect to earn the yield to call. (Note that the YTC is less than the YTM.)

d. Similarly from above, YTC can be found, if called in each subsequent year.

If called in Year 6:

N = 6, PV = -1200, PMT = 110, FV = 1080, I/YR = YTC = 7.80%.

If called in Year 7:

N = 7, PV = -1200, PMT = 110, FV = 1070, I/YR = YTC = 7.95%.

If called in Year 8:

N = 8, PV = -1200, PMT = 110, FV = 1060, I/YR = YTC = 8.07%.

If called in Year 9:

N = 9, PV = -1200, PMT = 110, FV = 1050, I/YR = YTC = 8.17%.

According to these calculations, the latest investors might expect a call of the bonds is in Year 7. This is the last year that the expected YTC will be less than the expected YTM. At this time, the firm still finds an advantage to calling the bonds, rather than seeing them to maturity.

6-26 a.

Years to Maturity	Real Risk-Free Rate (r*)	IP**	MRP	$r_T = r^* + IP + MRP$
1	2%	7.00%	0.2%	9.20%
2	2	6.00	0.4	8.40
3	2	5.00	0.6	7.60
4	2	4.50	0.8	7.30
5	2	4.20	1.0	7.20
10	2	3.60	1.0	6.60
20	2	3.30	1.0	6.30

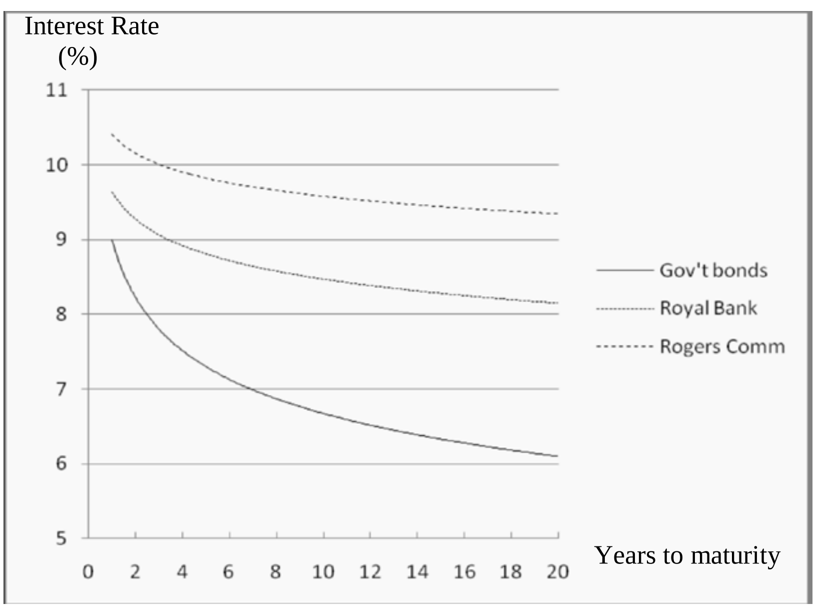
**The computation of the inflation premium is as follows:

Year	Expected Inflation	Average Expected Inflation
1	7%	7.00%
2	5	6.00
3	3	5.00
4	3	4.50
5	3	4.20
10	3	3.60
20	3	3.30

For example, the calculation for 3 years is as follows:

$$\frac{7\% + 5\% + 3\%}{3} = 5.00\%.$$

Thus, the yield curve would be as follows:



b. The interest rate on the Royal Bank bonds has the same components as the government bonds, except that the Royal Bank bonds have default risk, so a default risk premium must be included. Therefore:

$$r_{Royal} = r^{*} + IP + MRP + DRP$$

For a strong company such as Royal Bank, the default risk premium is fairly small for short-term bonds. However, as time to maturity increases, the probability of default, although still small, is sufficient to warrant a default premium. Thus, the yield risk curve for the Royal Bank bonds will rise above the yield curve for the government bonds. In the graph, the default risk premium was assumed to be about 2 percentage points on the 20-year Royal Bank bonds. The return should equal 6.3% + 2% = 8.3%.

c. Rogers Communications bonds would have significantly more default risk than either the government securities or Royal Bank bonds, and the risk of default would increase over time due to possible financial deterioration. In this example, the default risk premium was assumed to be 1.2 percentage point on the 1-year Rogers bonds and 3 percentage points on the 20-year bonds. The 20-year return should equal 6.3% + 3% = 9.3.

SOLUTION TO SPREADSHEET PROBLEM

6-27 The detailed solution for the spreadsheet problem is in the file ***Ch 06 Build a Model Solution.xlsx*** and is available on the textbook's website.

MINI CASE

Sam Strother and Shawna Tibbs are Vice Presidents of Great White North Investment Management and Co-directors of the company's pension fund management division. An important new client, has requested that Great White North present an investment seminar to its executive committee. Strother and Tibbs, who will make the actual presentation, have asked you to help them by answering the following questions.

a. What are the key features of a bond?

Answer:

1. Par or face value. We generally assume a $1,000 par value, but par can be anything, and often $5,000 or more is used. With registered bonds, which is what are issued today, if you bought $50,000 worth, that amount would appear on the certificate.

2. Coupon rate. The dollar coupon is the "rent" on the money borrowed, which is generally the par value of the bond. The coupon rate is the annual interest payment divided by the par value, and it is generally set at the value of r on the day the bond is issued.

3. Maturity. This is the number of years until the bond matures and the issuer must repay the loan (return the par value).

4. Issue date. This is the date the bonds were issued.

5. Default risk is inherent in all bonds except treasury bonds—will the issuer have the cash to make the promised payments? Bonds are rated from AAA to D, and the lower the rating the riskier the bond, the higher its default risk premium, and, consequently, the higher its required rate of return, r.

b.	**What are call provisions and sinking fund provisions? Do these provisions make bonds more or less risky?**

Answer: A call provision is a provision in a bond contract that gives the issuing corporation the right to redeem the bonds under specified terms prior to the normal maturity date. The call provision generally states that the company must pay the bondholders an amount greater than the par value if they are called. The additional sum, which is called a call premium, is typically set equal to one year's interest if the bonds are called during the first year, and the premium declines at a constant rate of INT/n each year thereafter.

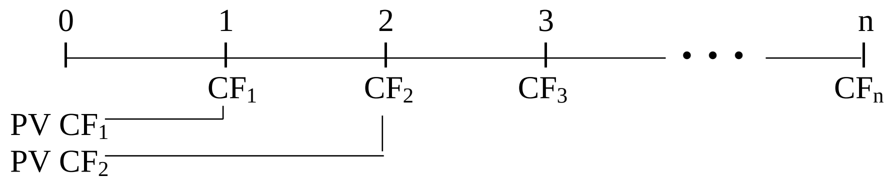
A sinking fund provision is a provision in a bond contract that requires the issuer to retire a portion of the bond issue each year. A sinking fund provision facilitates the orderly retirement of the bond issue.

The call privilege is valuable to the firm but potentially detrimental to the investor, especially if the bonds were issued in a period when interest rates were cyclically high. Therefore, bonds with a call provision are riskier than those without a call provision. Accordingly, the interest rate on a new issue of callable bonds will exceed that on a new issue of noncallable bonds.

Although sinking funds are designed to protect bondholders by ensuring that an issue is retired in an orderly fashion, it must be recognized that sinking funds will at times work to the detriment of bondholders. On balance, however, bonds that provide for a sinking fund are regarded as being safer than those without such a provision, so at the time they are issued sinking fund bonds have lower coupon rates than otherwise similar bonds without sinking funds.

c. How is the value of any asset whose value is based on expected future cash flows determined?

Answer:



The value of an asset is merely the present value of its expected future cash flows:

$$\text{VALUE} = \text{PV} = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \ldots + \frac{CF_n}{(1+r)^n} = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t}.$$

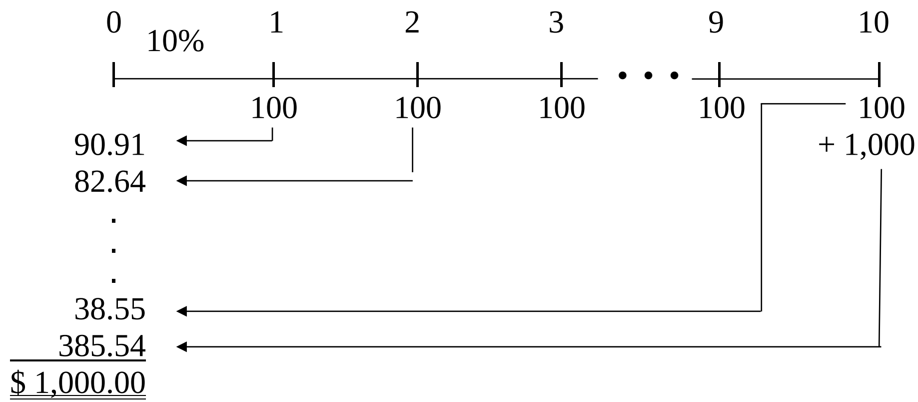
If the cash flows have widely varying risk, or if the yield curve is not horizontal, which signifies that interest rates are expected to change over the life of the cash flows, it would be logical for each period's cash flow to have a different discount rate. However, it is very difficult to make such adjustments; hence it is common practice to use a single discount rate for all cash flows.

The discount rate is the opportunity cost of capital; that is, it is the rate of return that could be obtained on alternative investments of similar risk. For a bond, the discount rate is r_d.

d.	**How is the value of a bond determined? What is the value of a 10-year, $1,000 par value bond with a 10 percent annual coupon if its required rate of return is 10 percent?**

Answer: A bond has a specific cash flow pattern consisting of a stream of constant interest payments plus the return of par at maturity. The annual coupon payment is the cash flow: PMT = (coupon rate) × (par value) = 0.1($1,000) = $100.

For a 10-year, 10 percent annual coupon bond, the bond's value is found as follows:



Expressed as an equation, we have:

$$V_B = \frac{\$100}{(1+r_d)^1} + \ldots + \frac{\$100}{(1+r_d)^{10}} + \frac{\$1,000}{(1+r_d)^{10}}$$
$$= \$90.91 + \ldots + \$38.55 + \$385.54 = \$1,000.$$

The bond consists of a 10-year, 10% annuity of $100 per year plus a $1,000 lump sum payment at t = 10:

PV Annuity =	$ 614.46
PV Maturity Value =	385.54
Value Of Bond =	$1,000.00

The mathematics of bond valuation is programmed into financial calculators which do the operation in one step, so the easy way to solve bond valuation problems is with a financial calculator. Input N = 10, r_d = I/YR = 10, PMT = 100, and FV = 1000, and then press PV to find the bond's value, $1,000. Then change n from 10 to 1 and press PV to get the value of the 1-year bond, which is also $1,000.

e. 1. What would be the value of the bond described in part d if, just after it had been issued, the expected inflation rate rose by 3 percentage points, causing investors to require a 13 percent return? Would we now have a discount or a premium bond?

Answer: With a financial calculator, just change the value of r_d = I/YR from 10% to 13%, and press the PV button to determine the value of the bond:

10-year = $837.21.

Using the formulas, we would have, at r = 13 percent,

$$V_{B(10\text{-}YR)} = \$100\,((1- 1/(1+0.13)^{10})/0.13) + \$1{,}000\,(1/(1+0.13)^{10})$$
$$= \$542.62 + \$294.59 = \$837.21.$$

In a situation like this, where the required rate of return, r, rises above the coupon rate, the bonds' values fall below par, so they sell at a discount.

e. 2. What would happen to the bonds' value if inflation fell, and r_d declined to 7 percent? Would we now have a premium or a discount bond?

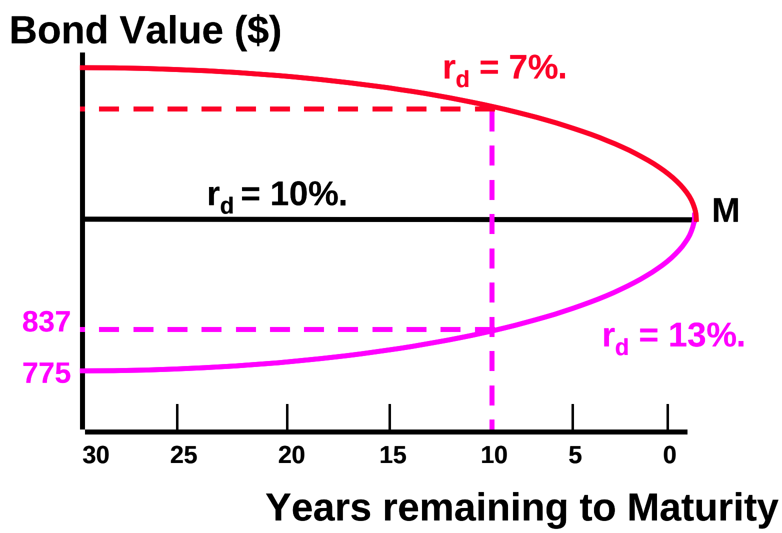
Answer: In the second situation, where r_d falls to 7 percent, the price of the bond rises above par. Just change r_d from 13% to 7%. We see that the 10-year bond's value rises to $1,210.71.

$$V_{B(10\text{-}YR)} = \$100\,((1- 1/(1+0.07)^{10})/0.07) + \$1{,}000\,(1/(1+0.07)^{10})$$
$$= \$702.36 + \$508.35 = \$1{,}210.71.$$

Thus, when the required rate of return falls below the coupon rate, the bonds' value rises above par, or to a premium. Further, the longer the maturity, the greater the price effect of any given interest rate change.

e.	**3.**	**What would happen to the value of the 10-year bond over time if the required rate of return remained at 13 percent, or if it remained at 7 percent? (Hint: with a financial calculator, enter PMT, I/YR, FV, and N, and then change (override) n to see what happens to the PV as the bond approaches maturity.)**

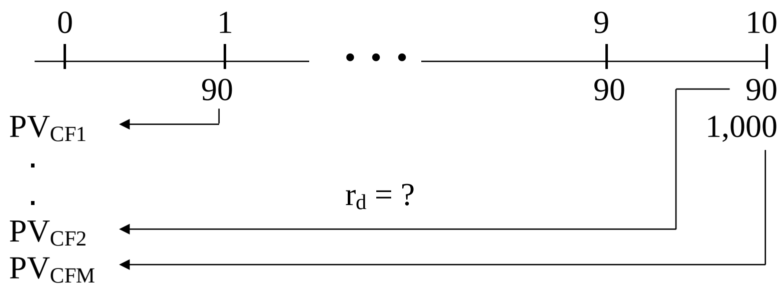
Answer: Assuming that interest rates remain at the new levels (either 7% or 13%), we could find the bond's value as time passes, and as the maturity date approaches. If we then plotted the data, we would find the situation shown below:



At maturity, the value of any bond must equal its par value (plus accrued interest). Therefore, if interest rates, hence the required rate of return, remain constant over time, then a bond's value must move toward its par value as the maturity date approaches, so the value of a premium bond decreases to $1,000, and the value of a discount bond increases to $1,000 (barring default).

f.	**1. What is the yield to maturity on a 10-year, 9 percent annual coupon, $1,000 par value bond that sells for $887.00? That sells for $1,134.20? What does the fact that a bond sells at a discount or at a premium tell you about the relationship between r_d and the bond's coupon rate?**

Answer: The yield to maturity (YTM) is that discount rate which equates the present value of a bond's cash flows to its price. In other words, it is the promised rate of return on the bond. (Note that the expected rate of return is less than the YTM if some probability of default exists.) On a time line, we have the following situation when the bond sells for $887:



SUM = PV = 887

We want to find r in this equation:

$$V_B = PV = \frac{INT}{(1+r)^1} + \ldots + \frac{INT}{(1+r)^N} + \frac{M}{(1+r)^N}.$$

We know N = 10, PV = -887, PMT = 90, and FV = 1000, so we have an equation with one unknown, r_d. We can solve for r_d by entering the known data into a financial calculator and then pressing the I/YR = r_d button. The YTM is found to be 10.91%.

Alternatively, we could use present value interest factors:

We can tell from the bond's price, even before we begin the calculations, that the YTM must be above the 9% coupon rate. We know this because the bond is selling at a discount, and discount bonds always have r > coupon rate.

If the bond were priced at $1,134.20, then it would be selling at a premium. In that case, it must have a YTM that is below the 9 percent coupon rate, because all premium bonds must have coupons which exceed the going interest rate. Going through the same procedures as before—plugging the appropriate values into a financial calculator and then pressing the r = I button, we find that at a price of $1,134.20, r = YTM = 7.08%.

f. 2. What are the total return, the current yield, and the capital gains yield for the discount bond? (Assume the bond is held to maturity and the company does not default on the bond.)

Answer: The current yield is defined as follows:

$$\text{Current Yield} = \frac{\text{Annual coupon interest payment}}{\text{Current price of the bond}}.$$

The capital gains yield is defined as follows:

$$\text{Capital gains yield} = \frac{\text{Expected Change in bond's price}}{\text{Beginning - of - year price}}.$$

The total expected return is the sum of the current yield and the expected capital gains yield:

$$\begin{matrix}\text{Expected} \\ \text{Total Return}\end{matrix} = \begin{matrix}\text{Expected} \\ \text{current yield}\end{matrix} + \begin{matrix}\text{Expected capital} \\ \text{gains yield}\end{matrix}.$$

For our 9% coupon, 10-year bond selling at a price of $887 with a YTM of 10.91%, the current yield is:

$$\text{Current yield} = \frac{\$90}{\$887} = 0.1015 = 10.15\%.$$

Knowing the current yield and the total return, we can find the capital gains yield:

$$\text{YTM} = \text{current yield} + \text{capital gains yield}$$

And

$$\text{Capital gains yield} = \text{YTM} - \text{current yield} = 10.91\% - 10.15\% = 0.76\%.$$

The capital gains yield calculation can be checked by asking this question: "What is the expected value of the bond 1 year from now, assuming that interest rates remain at current levels?" This is the same as asking, "What is the value of a 9-year, 9 percent annual coupon bond if its YTM (its required rate of return) is 10.91 percent?" The answer, using the bond valuation function of a calculator, is $893.87. With this data, we can now calculate the bond's capital gains yield as follows:

$$\begin{aligned}\text{Capital Gains Yield} &= (V_{B_1} - V_{B_0})/V_{B_0} \\ &= (\$893.87 - \$887)/\$887 = 0.0077 = 0.77\%,\end{aligned}$$

This agrees with our earlier calculation (except for rounding). When the bond is selling for $1,134.20 and providing a total return of r_d = YTM = 7.08%, we have this situation:

Current Yield = $90/$1,134.20 = 7.94%

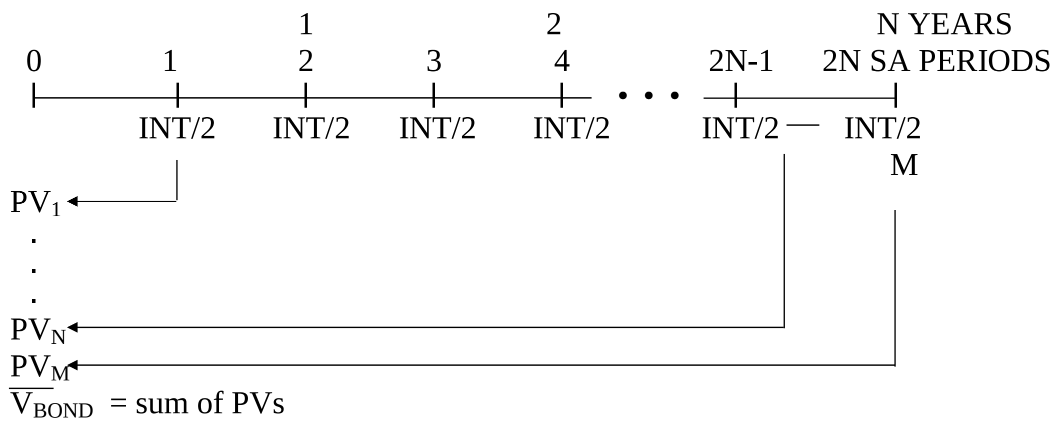
and

Capital Gains Yield = 7.08% - 7.94% = -0.86%.

The bond provides a current yield that exceeds the total return, but a purchaser would incur a small capital loss each year, and this loss would exactly offset the excess current yield and force the total return to equal the required rate.

g.	**How does the equation for valuing a bond change if semiannual payments are made? Find the value of a 10-year, semiannual payment, 10 percent coupon bond if nominal r_d = 13%.**

Answer: In reality, virtually all bonds issued in Canada have semiannual coupons and are valued using the setup shown below:



We would use this equation to find the bond's value:

$$V_B = \sum_{t=1}^{2N} \frac{INT/2}{(1 + r_d/2)^t} + \frac{M}{(1 + r_d/2)^{2N}}.$$

The payment stream consists of an annuity of 2N payments plus a lump sum equal to the maturity value.

To find the value of the 10-year, semiannual payment bond, semiannual interest = annual coupon/2 = \$100/2 = \$50 and N = 2 (years to maturity) = 2(10) = 20. To find the value of the bond with a financial calculator, enter n = 20, $r_d/2$ = I/YR = 5, PMT = 50, FV = 1000, and then press PV to determine the value of the bond. Its value is \$1,000.

You could then change r_d = I/YR to see what happens to the bond's value as r changes, and plot the values—the graph would look like the one we developed earlier.

For example, if r_d rose to 13%, we would input I/YR= 6.5 rather than 5%, and find the 10-year bond's value to be \$834.72. If r_d fell to 7%, then input I/YR = 3.5 and press PV to find the bond's new value, \$1,213.19.

We would find the values with a financial calculator, but they could also be found with formulas. Thus:

$$V_{10\text{-YEAR}} = \$50\ ((1-\ 1/(1+0.05)^{20})/0.065) + \$1{,}000\ (1/(1+0.05)^{20})$$
$$= \$50(12.4622) + \$1{,}000(0.37689) = \$623.11 + \$376.89 = \$1{,}000.00.$$

At a 13 percent required return:

$$V_{10\text{-YEAR}} = \$50\ ((1- \ 1/(1+0.065)^{20})/0.065) + \$1{,}000\ (1/(1+0.065)^{20})$$
$$= \$834.72.$$

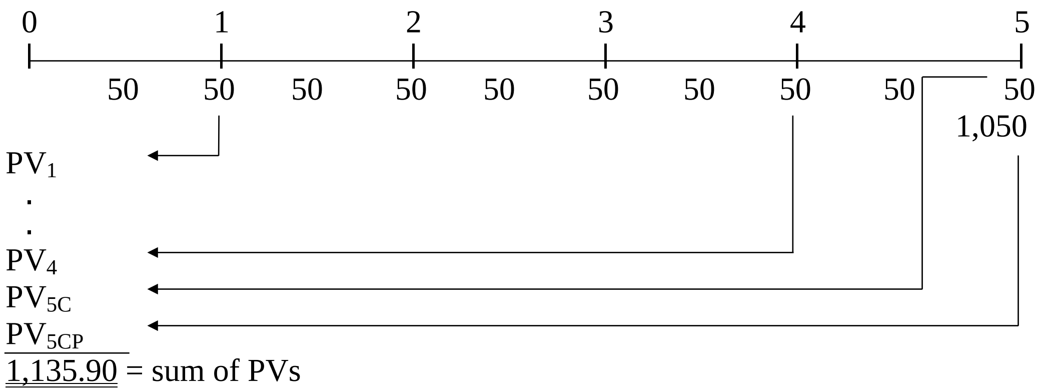
At a 7 percent required return:

$$V_{10\text{-YEAR}} = \$50\ ((1- \ 1/(1+0.035)^{20})/0.035) + \$1{,}000\ (1/(1+0.035)^{20})$$
$$= \$1{,}213.19.$$

h.		**Suppose a 10-year, 10 percent, semiannual coupon bond with a par value of \$1,000 is currently selling for \$1,135.90, producing a nominal yield to maturity of 8 percent. However, the bond can be called after 5 years for a price of \$1,050.**

h.	**1.**	**What is the bond's *nominal yield to call (YTC)*?**

Answer: If the bond were called, bondholders would receive \$1,050 at the end of year 5. Thus, the time line would look like this:



The easiest way to find the YTC on this bond is to input values into your calculator: n = 10; PV = -1135.90; PMT = 50; and FV = 1050, which is the par value plus a call premium of \$50; and then press the r_d = I/YR button to find I/YR = 3.765%. However, this is the 6-month rate, so we would find the nominal rate on the bond as follows:

$$r_{NOM} = 2(3.765\%) = 7.5301\% \approx 7.5\%.$$

This 7.5% is the rate brokers would quote if you asked about buying the bond.

You could also calculate the EAR on the bond:

$$EAR = (1.03765)^2 - 1 = 7.672\%.$$

Usually, people in the bond business just talk about nominal rates, which is OK so long as <u>all</u> the bonds being compared are on a semiannual payment basis. When you start making comparisons among investments with different payment patterns, though, it is important to convert to EARs.

h. 2. If you bought this bond, do you think you would be more likely to earn the YTM or the YTC? Why?

Answer: Since the coupon rate is 10% versus YTC = r_d = 7.53%, it would pay the company to call the bond, get rid of the obligation to pay $100 per year in interest, and sell replacement bonds whose interest would be only $75.30 per year. Therefore, if interest rates remain at the current level until the call date, the bond will surely be called, so investors should expect to earn 7.53%. In general, investors should expect to earn the YTC on premium bonds, but to earn the YTM on par and discount bonds. (Bond brokers publish lists of the bonds they have for sale; they quote YTM or YTC depending on whether the bond sells at a premium or a discount.)

i. Write a general expression for the yield on any debt security (r_d) and define these terms: real risk-free rate of interest (r*), inflation premium (IP), default risk premium (DRP), liquidity premium (LP), and maturity risk premium (MRP).

Answer: $r_d = r^* + IP + DRP + LP + MRP$.

r* is the real risk-free interest rate. It is the rate you see on a riskless security if there were no inflation.

The inflation premium (IP) is a premium added to the real risk-free rate of interest to compensate for expected inflation.

The default risk premium (DRP) is a premium based on the probability that the issuer will default on the loan, and it is measured by the difference between the interest rate on a Government of Canada bond and a corporate bond of equal maturity and marketability.

A liquid asset is one that can be sold at a predictable price on short notice; a liquidity premium is added to the rate of interest on securities that are not liquid.

The maturity risk premium (MRP) is a premium which reflects interest rate risk; longer-term securities have more interest rate risk (the risk of capital loss due to rising interest rates) than do shorter-term securities, and the MRP is added to reflect this risk.

j.	**Define the nominal risk-free rate (r_{RF}). What security can be used as an estimate of r_{RF}?**

Answer: The real risk-free rate, r*, is the rate that would exist on default-free securities in the absence of inflation:

$$r_{RF} = r^* + IP.$$

The real risk-free rate, r*, is the rate that would exist on default-free securities in the absence of inflation.

The nominal risk-free rate, r_{RF}, is equal to the real risk-free rate plus an inflation premium which is equal to the average rate of inflation expected over the life of the security.

There is no truly riskless security, but the closest thing is a short-term Canadian Treasury bill (t-bill), which is free of most risks. The real risk-free rate, r*, is estimated by subtracting the expected rate of inflation from the rate on short-term treasury securities. It is generally assumed that r* is in the range of 1 to 4 percentage points. The long-term government bond rate is used as a proxy for the long-term risk-free rate. However, we know that all long-term bonds contain interest rate risk, so the long-term government bond rate is not really riskless. It is, however, free of default risk.

k. **What is a bond spread and how is it related to the default risk premium? How are bond ratings related to default risk? What factors affect a company's bond rating?**

Answer: A "bond spread" is often calculated as the difference between a corporate bond's yield and a Treasury security's yield of the same maturity. Therefore:

$$\text{Spread} = \text{DRP} + \text{LP}.$$

Bonds of large, strong companies often have very small LPs. Bonds of small companies often have LPs as high as 2%.

Bond ratings are based upon a company's default risk. They are based on both qualitative and quantitative factors, some of which are listed below.

1. Financial performance—determined by ratios such as the debt, TIE, FCC, and current ratios.

2. Provisions in the bond contract:
   A. Secured vs. Unsecured debt
   B. Senior vs. Subordinated debt
   C. Guarantee provisions
   D. Sinking fund provisions
   E. Debt maturity

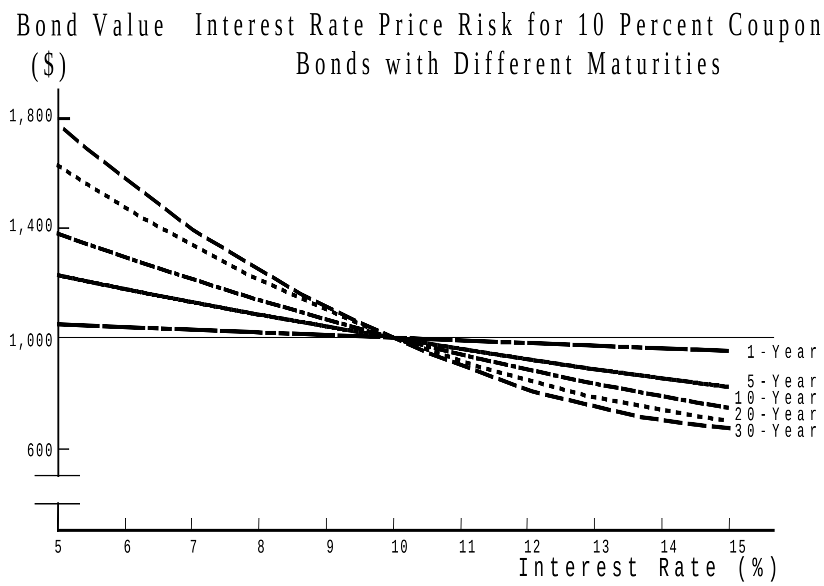
3. Other factors:
   A. Earnings stability
   B. Regulatory environment
   C. Potential product liability
   D. Accounting policy

l.	**What is *interest rate (or price) risk*? Which bond has more interest rate risk, an annual payment 1-year bond or a 10-year bond? Why?**

Answer: Interest rate risk, which is often just called price risk, is the risk that a bond will lose value as the result of an increase in interest rates. Earlier, we developed the following values for a 10 percent, annual coupon bond:

		Maturity		
r_d	1-Year	Change	10-Year	Change
5%	$1,048	4.8%	$1,386	38.6%
10	1,000	4.4%	1,000	25.1%
15	956		749	

A 5 percentage point increase in r causes the value of the 1-year bond to decline by only 4.8 percent, but the 10-year bond declines in value by more than 38 percent. Thus, the 10-year bond has more interest rate price risk.



The graph above shows the relationship between bond values and interest rates for a 10 percent, annual coupon bond with different maturities. The longer the maturity, the greater the change in value for a given change in interest rates, r_d.

m.	**What is reinvestment rate risk? Which has more reinvestment rate risk, a 1-year bond or a 10-year bond?**

Answer: Investment rate risk is defined as the risk that cash flows (interest plus principal repayments) will have to be reinvested in the future at rates lower than today's rate. To illustrate, suppose you just won the lottery and now have $500,000. You plan to invest the money and then live on the income from your investments. Suppose you buy a 1-year bond with a YTM of 10 percent. Your income will be $50,000 during the first year. Then, after 1 year, you will receive your $500,000 when the bond matures, and you will then have to reinvest this amount. If rates have fallen to 3 percent, then your income will fall from $50,000 to $15,000. On the other hand, had you bought 30-year bonds that yielded 10%, your income would have remained constant at $50,000 per year. Clearly, buying bonds that have short maturities carries reinvestment rate risk. Note that long maturity bonds also have reinvestment rate risk, but the risk applies only to the coupon payments, and not to the principal amount. Since the coupon payments are significantly less than the principal amount, the reinvestment rate risk on a long-term bond is significantly less than on a short-term bond.

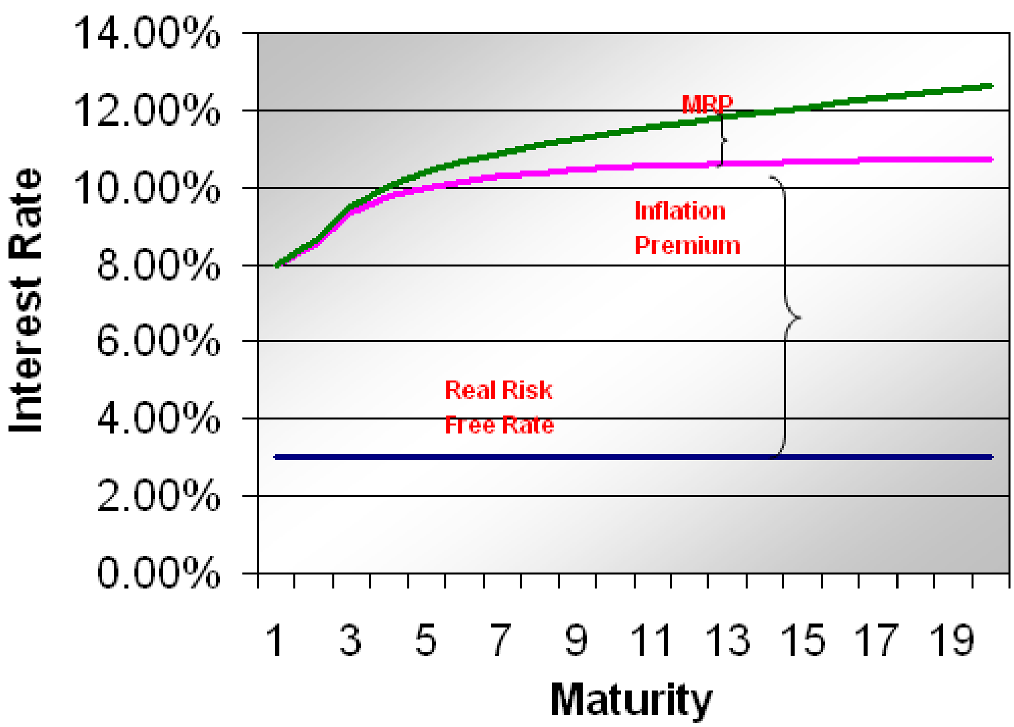
n.	**How are interest rate risk and reinvestment rate risk related to the maturity risk premium?**

Answer: Long-term bonds have high interest rate risk but low reinvestment rate risk. Short-term bonds have low interest rate risk but high reinvestment rate risk. Nothing is riskless! Yields on longer term bonds usually are greater than on shorter term bonds, so the MRP is more affected by interest rate risk than by reinvestment rate risk.

o.	**What is the term structure of interest rates? What is a yield curve?**

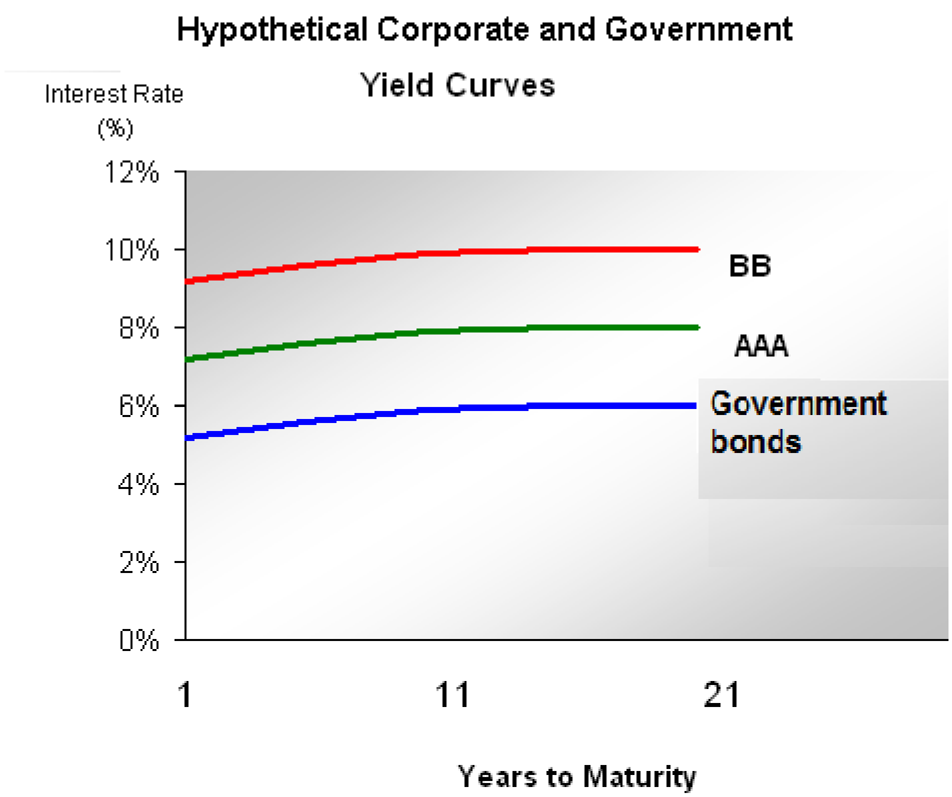
Answer: The term structure of interest rates is the relationship between interest rates, or yields, and maturities of securities. When this relationship is graphed, the resulting curve is called a yield curve.

Hypothetical Government Bond Yield Curve



p.	**At any given time, how would the yield curve facing an AA-rated company compare with the yield curve for Government of Canada bonds? At any given time, how would the yield curve facing a BB-rated company compare with the yield curve for Government of Canada bonds?**

Answer: The yield curve normally slopes upward, indicating that short-term interest rates are lower than long-term interest rates. Yield curves can be drawn for government securities or for the securities of any corporation, but corporate yield curves will always lie above government yield curves, and the riskier the corporation, the higher its yield curve. The spread between a corporate yield curve and the government curve widens as the corporate bond rating decreases.



q. **Briefly describe bankruptcy law. If a firm were to default on the bonds, would the company be immediately liquidated? Would the bondholders be assured of receiving all of their promised payments?**

Answer: When a business becomes insolvent, it does not have enough cash to meet scheduled interest and principal payments. A decision must then be made whether to dissolve the firm through liquidation or to permit it to reorganize and thus stay alive.

The decision to force a firm to liquidate or to permit it to reorganize depends on whether the value of the reorganized firm is likely to be greater than the value of the firm's assets if they were sold off piecemeal. In a reorganization, management along with a court appointed trustee creates a proposal which unsecured creditors vote on. The proposal may call for a restructuring of the firm's debt, in which case the interest rate may be reduced, the term to maturity lengthened, or some of the debt may be exchanged for equity. The point of the restructuring is to reduce the financial charges to a level that the firm's cash flows can support.

If the firm is deemed to be too far gone to be saved, it will be liquidated and the priority of claims would be as follows:

- Costs for environmental damage
- Unremitted payroll taxes and deductions
- Secured creditors whose loans are secured by some specific asset, mortgage, lien, etc.
- Trustee expenses
- Bankruptcy administration costs
- Unpaid wages, up to $2,000 per worker
- Municipal taxes
- Claims for rent, up to 3 months prior to bankruptcy
- Creditor costs who first filed a claim
- Injury claim costs to employees not covered under Workers' Compensation
- Other unsecured creditors
- Preferred shareholders
- Common shareholders

If the firm's assets are worth more "alive" than "dead," the company would be reorganized. Its creditors, however, would expect to take a "hit." Thus, they would not expect to receive all their promised payments. If the firm is deemed to be too far gone to be saved, it would be liquidated.

WEB EXTENSION 6C: THE PURE EXPECTATIONS THEORY AND ESTIMATION OF FORWARD RATES

SOLUTIONS TO PROBLEMS

6C-1 $r_{T1} = 5\%$; ${}_1r_{T1} = 6\%$; $r_{T2} = ?$

$$(1 + r_{T2})^2 = (1.05)(1.06)$$
$$(1 + r_{T2})^2 = 1.113$$
$$1 + r_{T2} = 1.055$$
$$r_{T2} = 5.5\%.$$

6C-2 Let X equal the yield on 2-year securities 4 years from now:

$$(1.07)^4(1 + X)^2 = (1.075)^6$$
$$(1.3108)(1 + X)^2 = 1.5433$$
$$1 + X = \left(\frac{1.5433}{1.3108}\right)^{1/2}$$
$$X = 8.5\%.$$

6C-3 a. $(1.045)^2 = (1.03)(1 + X)$

$$1.092/1.03 = 1 + X$$
$$X = 6\%.$$

b. For riskless bonds under the expectations theory, the interest rate for a bond of any maturity is
$r_N = r^* +$ average inflation over N years. If $r^* = 1\%$, we can solve for IP_N:

Year 1: $r_1 = 1\% + I_1 = 3\%$;
I_1 = expected inflation = 3% – 1% = 2%.

Year 2: $r_1 = 1\% + I_2 = 6\%$;
I_2 = expected inflation = 6% – 1% = 5%.

Note also that the average inflation rate is (2% + 5%)/2 = 3.5%, which, when added to $r^* = 1\%$, produces the yield on a 2-year bond, 4.5%. Therefore, all of our results are consistent.

6C-4 $r^* = 2\%$; MRP = 0%; $r_1 = 5\%$; $r_2 = 7\%$; X = ?

X represents the one-year rate on a bond one year from now (Year 2).

$$(1.07)^2 = (1.05)(1 + X)$$
$$\frac{1.1449}{1.05} = 1 + X$$
$$X = 9\%.$$

$9\% = r^* + I_2$
$9\% = 2\% + I_2$
$7\% = I_2$.

The average interest rate during the 2-year period differs from the 1-year interest rate expected for Year 2 because of the inflation rate reflected in the two interest rates. The inflation rate reflected in the interest rate on any security is the average rate of inflation expected over the security's life.

