Chapter 4
Time Value of Money

ANSWERS TO END-OF-CHAPTER QUESTIONS

4-1 a. PV (present value) is the value today of a future payment, or stream of payments, discounted at the appropriate rate of interest. PV is also the beginning amount that will grow to some future value. The parameter I is the periodic interest rate that an account pays. The parameter INT is the dollars of interest earned each period. FV_N (future value) is the ending amount in an account, where N is the number of periods the money is left in the account. PVA_N is the value today of a future stream of equal payments (an annuity) and FVA_N is the ending value of a stream of equal payments, where N is the number of payments of the annuity. PMT is equal to the dollar amount of an equal or constant cash flow (an annuity). In the EAR equation, M is used to denote the number of compounding periods per year, while I_{NOM} is the nominal, or quoted, interest rate.

b. The opportunity cost rate (I) of an investment is the rate of return available on the best alternative investment of similar risk.

c. An annuity is a series of payments of a fixed amount for a specified number of periods. A single sum, or lump sum payment, as opposed to an annuity, consists of one payment occurring now or at some future time. A cash flow can be an inflow (a receipt) or an outflow (a deposit, a cost, or an amount paid). We distinguish between the terms cash flow and PMT. We use the term cash flow for uneven streams, while we use the term PMT for annuities, or constant payment amounts. An uneven cash flow stream is a series of cash flows in which the amount varies from one period to the next. The PV (or FV_N) of an uneven payment stream is merely the sum of the present values (or future values) of each individual payment.

d. An ordinary annuity has payments occurring at the end of each period. A deferred annuity is just another name for an ordinary annuity. An annuity due has payments occurring at the beginning of each period. Most financial calculators will accommodate either type of annuity. The payment period must be equal to the compounding period.

e. A perpetuity is a series of payments of a fixed amount that last indefinitely. In other words, a perpetuity is an annuity where n equals infinity. Consol is another term for perpetuity. Consols were originally bonds issued by England in 1815 to consolidate past debt.

f. An outflow is a deposit, a cost, or an amount paid, while an inflow is a receipt. A time line is an important tool used in time value of money analysis; it is a graphical representation which is used to show the timing of cash flows. The terminal value is the future value of an uneven cash flow stream.

g. Compounding is the process of finding the future value of a single payment or series of payments. Discounting is the process of finding the present value of a single payment or series of payments; it is the reverse of compounding.

h. Annual compounding means that interest is paid once a year. In semiannual, quarterly, monthly, and daily compounding, interest is paid 2, 4, 12, and 365 times per year respectively. When compounding occurs more frequently than once a year, you earn interest on interest more often, thus increasing the future value. The more frequent the compounding, the higher the future value.

i. The effective annual rate is the rate that, under annual compounding, would have produced the same future value at the end of 1 year as was produced by more frequent compounding, say quarterly. The nominal (quoted) interest rate, I_{NOM}, is the rate of interest stated in a contract. If the compounding occurs annually, the effective annual rate and the nominal rate are the same. If compounding occurs more frequently, the effective annual rate is greater than the nominal rate. The nominal annual interest rate is also called the annual percentage rate, or APR. The periodic rate, I_{PER}, is the rate charged by a lender or paid by a borrower each period. It can be a rate per year, per 6-month period, per quarter, per month, per day, or per any other time interval (usually one year or less).

j. An amortization schedule is a table that breaks down the periodic fixed payment of an installment loan into its principal and interest components. The principal component of each payment reduces the remaining principal balance. The interest component is the interest payment on the beginning-of-period principal balance. An amortized loan is one that is repaid in equal periodic amounts (or "killed off" over time).

4-2 The opportunity cost rate is the rate of interest one could earn on an alternative investment with a risk equal to the risk of the investment in question. This is the value of I in the TVM equations, and it is shown on the top of a time line, between the first and second tick marks. It is <u>not</u> a single rate—the opportunity cost rate varies depending on the riskiness and maturity of an investment, and it also varies from year to year depending on inflationary expectations.

4-3 True. The second series is an uneven payment stream, but it contains an annuity of $400 for 8 years. The series could also be thought of as a $100 annuity for 10 years plus an additional payment of $100 in Year 2, plus additional payments of $300 in Years 3 through 10.

4-4 True, because of compounding effects—growth on growth. The following example demonstrates the point. The annual growth rate is I in the following equation:

$$\$1(1 + I)^{10} = \$2.$$

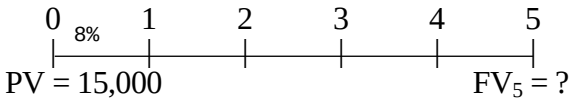
The term $(1 + I)^{10}$ is the FVIF for I percent, 10 years.

Using a financial calculator input N = 10, PV = -1, PMT = 0, FV = 2, and I = ?. Solving for I you obtain 7.18%.

4-5 For the same stated rate, daily compounding is best. You would earn more "interest on interest."

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

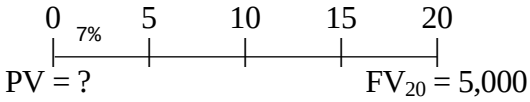
4-1



FV_5 = \$15,000(1.08)^5
= \$15,000(1.469328) = \$22,039.92.

Alternatively, with a financial calculator enter the following: N = 5, I = 8, PV = -15000, and PMT = 0. Solve for FV = \$22,039.92.

4-2



With a financial calculator enter the following: N = 20, I = 7, PMT = 0, and FV = 5000. Solve for PV = \$1,292.10.

4-3

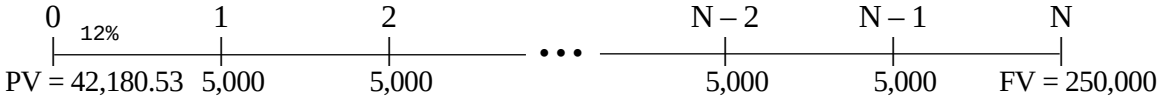
With a financial calculator enter the following: N = 15, PV = -150000, PMT = 0, and FV = 1000000. Solve for I = 13.48%.

4-4

\$3 = \$1(1.065)^N.

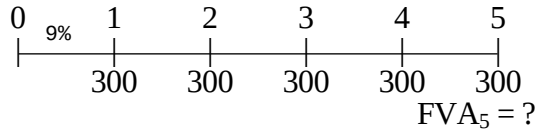
With a financial calculator enter the following: I = 6.5, PV = -1, PMT = 0, and FV = 3. Solve for N = 17.45 years.

4-5



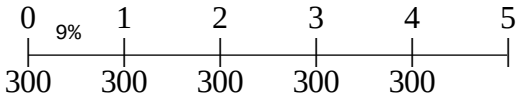
Using your financial calculator, enter the following data: I = 12; PV = -42180.53; PMT = -5000; FV = 250000; N = ? Solve for N = 11. It will take 11 years to accumulate \$250,000.

4-6 Ordinary annuity:



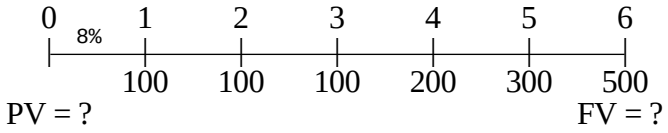
With a financial calculator enter the following: N = 5, I = 9, PV = 0, and PMT = 300 Solve for FV = $1,795.41.

Annuity due:



With a financial calculator, switch to "BEG" and enter the following: N = 5, I = 9, PV = 0, and PMT = 300. Solve for FV = $1,957. Don't forget to switch back to "END" mode.

4-7



Using a financial calculator, enter the following: $CF_0 = 0$; $CF_1 = 100$; $N_j = 3$; $CF_4 = 200$ (Note calculator will show CF_2 on screen.); $CF_5 = 300$ (Note calculator will show CF_3 on screen.); $CF_6 = 500$ (Note calculator will show CF_4 on screen.); and I = 8. Solve for NPV = $923.98.

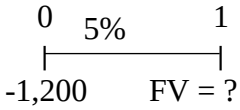
To solve for the FV of the cash flow stream with a calculator that doesn't have the NFV key, do the following: Enter N = 6, I = 8, PV = -923.98, and PMT = 0. Solve for FV = $1,466.24.

4-8 Using a financial calculator, enter the following: N = 60, I = 1, PV = -20000, and FV = 0. Solve for PMT = $444.89.

$$\begin{aligned} \text{EAR} &= \left(1 + \frac{I_{NOM}}{M}\right)^{M} - 1.0 \\ &= (1.01)^{12} - 1.0 \\ &= 12.68\%. \end{aligned}$$

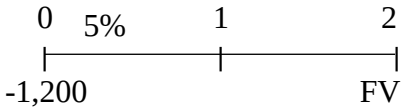
Alternatively, using a financial calculator, enter the following: NOM% = 12 and P/YR =12. Solve for EFF% = 12.6825%. Remember to change back to P/YR = 1 on your calculator.

4-9 a.



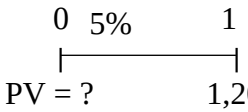
$1,200(1.05) = $1,260.

b.



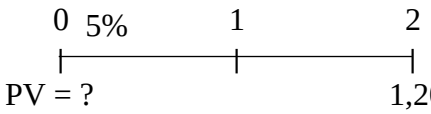
$1,200(1.05)^2 = $1,323.

c.



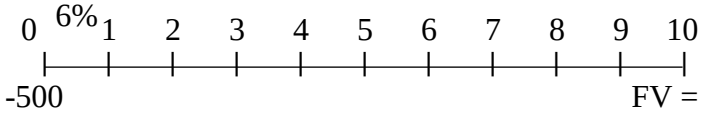
$1,200(1/1.05) = $1,142.86.

d.



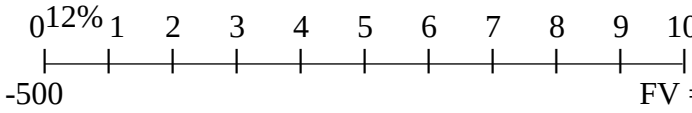
$1,200(1/1.05)^2 = $1,088.44.

4-10 a.



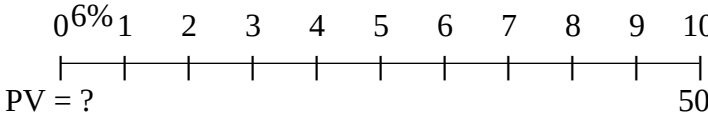
$500(1.06)^{10} = $895.42.

b.



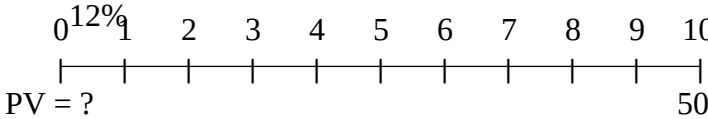
$500(1.12)^{10} = $1,552.92.

c.



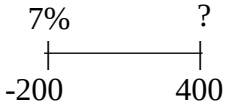
$500(1/1.06)^{10} = $279.20

d.



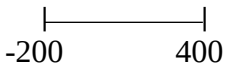
$500(1/1.12)^{10} = $160.99

4-11 a.



With a financial calculator, enter I = 7, PV = -200, PMT = 0, and FV = 400. Then press the N key to find N = 10.24 ≈ 10.

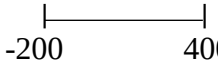
b. 10% ?



-200 400 .

With a financial calculator, enter I = 10, PV = -200, PMT = 0, and FV = 400. Then press the N key to find N = 7.27 ≈ 7.

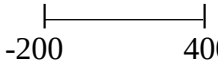
c. 18% ?



-200 400 .

With a financial calculator, enter I = 18, PV = -200, PMT = 0, and FV = 400. Then press the N key to find N = 4.19 ≈ 4.

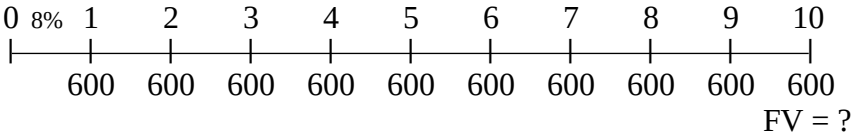
d. 100% ?



-200 400 .

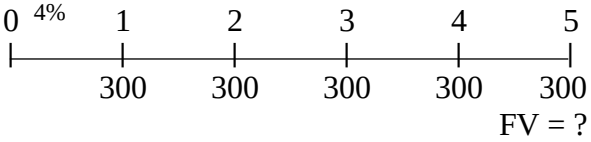
With a financial calculator, enter I = 100, PV = -200, PMT = 0, and FV = 400. Then press the N key to find N = 1.00 ≈ 1.

4-12 a.



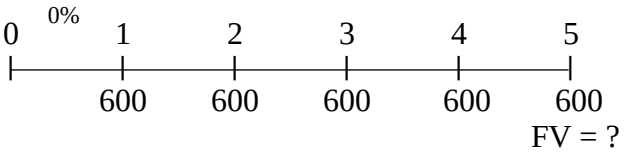
With a financial calculator, enter N = 10, I = 8, PV = 0, and PMT = -600. Then press the FV key to find FV = $8,691.94.

b.



With a financial calculator, enter N = 5, I = 4, PV = 0, and PMT = -300. Then press the FV key to find FV = $1,624.90.

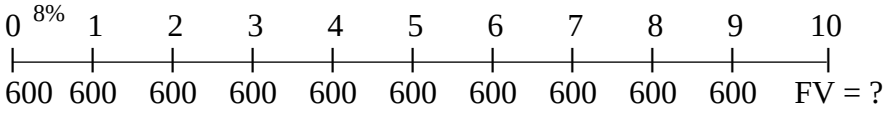
c.



With a financial calculator, enter N = 5, I = 0, PV = 0, and PMT = -600. Then press the FV key to find FV = $3,000.

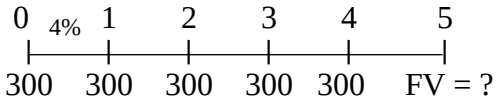
d. To solve Part d using a financial calculator, repeat the procedures discussed in Parts a, b, and c, but first switch the calculator to "BEG" mode. Make sure you switch the calculator back to "END" mode after working the problem.

(1)



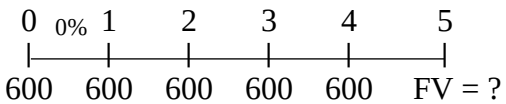
With a financial calculator set to "BEG" mode, enter N = 10, I = 8, PV = 0, and PMT = -600. Then press the FV key to find FV = $9,387.29.

(2)



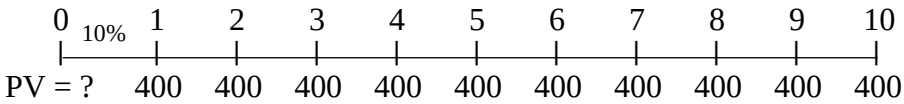
With a financial calculator set to "BEG" mode, enter N = 5, I = 4, PV = 0, and PMT = -300. Then press the FV key to find FV = $1,689.89.

(3)



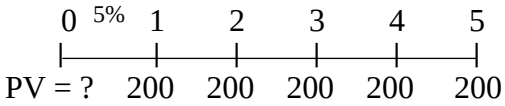
With a financial calculator set to "BEG" mode, enter N = 5, I = 0, PV = 0, and PMT = -600. Then press the FV key to find FV = $3,000.

4-13 a.



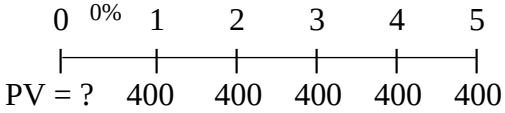
With a financial calculator, enter N = 10, I = 10, PMT = -400, and FV = 0. Then press the PV key to find PV = $2,457.83.

b.



With a financial calculator, enter N = 5, I = 5, PMT = -200, and FV = 0. Then press the PV key to find PV = $865.90.

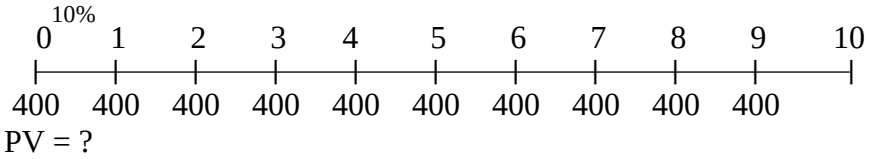
c.



$400(5) = $2,000.00.

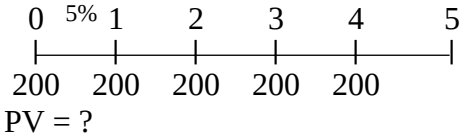
With a financial calculator, enter N = 5, I = 0, PMT = -400, and FV = 0. Then press the PV key to find PV = $2,000.

d. (1)



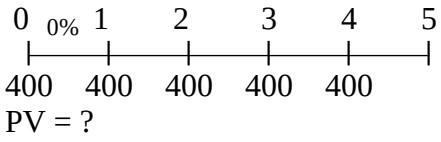
With a financial calculator set to "BEG" mode, enter N = 10, I = 10, PMT = -400, and FV = 0. Then press the PV key to find PV = $2,703.61.

(2)



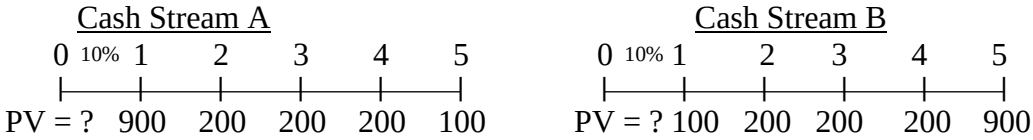
With a financial calculator set to "BEG" mode, enter N = 5, I = 5, PMT = -200, and FV = 0. Then press the PV key to find PV = $909.19.

(3)



With a financial calculator set to "BEG" mode, enter N = 5, I = 0, PMT = -400, and FV = 0. Then press the PV key to find PV = $2000.00.

4-14 a.



With a financial calculator, simply enter the cash flows (be sure to enter $CF_0 = 0$), enter I = 10, and press the NPV key to find NPV = PV = $1,332.43 for the first problem. Override I = 10 with I = 0 to find the next PV for Cash Stream A. Repeat for Cash Stream B to get NPV = PV = $1,101.89.

b. $PV_A = \$900 + \$200 + \$200 + \$200 + \$100 = \$1,600.$
$PV_B = \$100 + \$200 + \$200 + \$200 + \$900 = \$1,600.$

4-15 These problems can all be solved using a financial calculator by entering the known values shown on the time lines and then pressing the I button.

a.

With a financial calculator, enter N = 1, PV = 700, PMT = 0, and FV = -749. Then press the I key to find I = 7%.

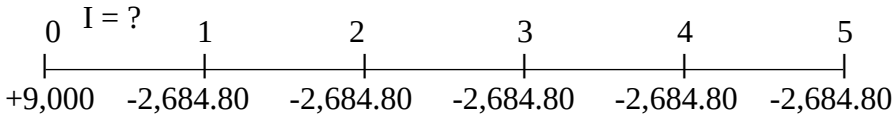
b.

With a financial calculator, enter N = 1, PV = -700, PMT = 0, and FV = 749. Then press the I key to find I = 7%.

c.

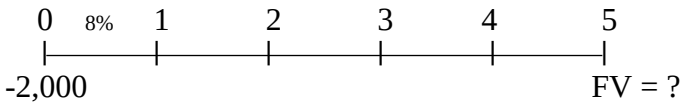
With a financial calculator, enter N = 10, PV = 85,000, PMT = 0, and FV = -201,229. Then press the I key to find I = 9%.

d.



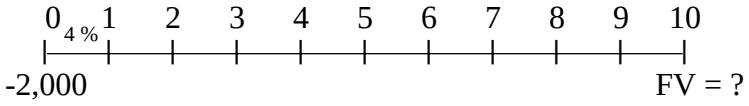
With a financial calculator, enter N = 5, PV = 9,000, PMT = -2,684.8, and FV = 0. Then press the I key to find I = 15%.

4-16 a.



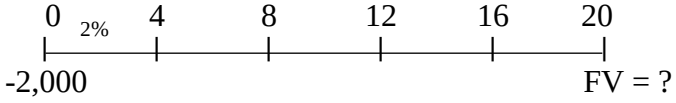
With a financial calculator, enter N = 5, I = 8, PV = -2,000, and PMT = 0, and then press FV to obtain FV = $2,938.66.

b.



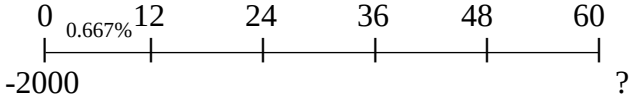
With a financial calculator, enter N = 10, I = 4, PV = -2,000, and PMT = 0, and then press FV to obtain FV = $2,960.49.

c.



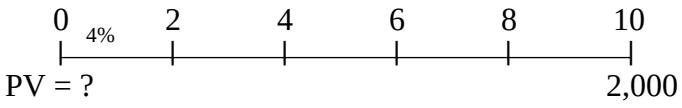
With a financial calculator, enter N = 20, I = 2, PV = -2,000, and PMT = 0, and then press FV to obtain FV = $2,971.89.

d.



With a financial calculator, enter N = 60, I = .666667, PV = -2000, and PMT = 0, and then press FV to obtain FV = $2,979.69.

4-17 a.

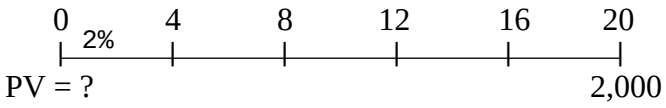


With a financial calculator, enter N = 10, I = 4, PMT = 0, and FV = -2,000. Then press the PV key to find PV = $1,351.13. Alternatively,

$$PV = FV_N\left(\frac{1}{1+\frac{I}{M}}\right)^{MN} = \$2{,}000\left(\frac{1}{1+\frac{0.08}{2}}\right)^{2(5)}$$

$$= \$2{,}000\left(\frac{1}{1.04}\right)^{10} = \$2{,}000(PVIF_{4\%,\,10}) = \$2{,}000(0.675564) = \$1{,}351.13.$$

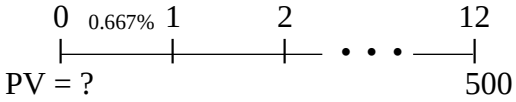
b.



With a financial calculator, enter N = 20, I = 2, PMT = 0, and FV = -2,000. Then press the PV key to find PV = $1,345.94, or

$$PV = \$2{,}000\left(\frac{1}{1+\frac{0.08}{4}}\right)^{4(5)} = \$2{,}000\left(\frac{1}{1.02}\right)^{20} = \$1{,}345.94.$$

c.

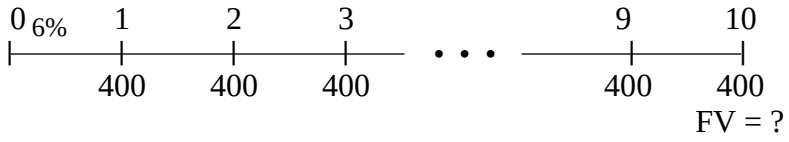


With a financial calculator, enter N = 12, I = .66667, PMT = 0, and FV = -2,000. Then press the PV key to find PV = $1,846.72, or

$$PV = \$2{,}000\left(\frac{1}{1+\frac{0.08}{12}}\right)^{12(1)}$$

$$= \$2{,}000\left(\frac{1}{1.006667}\right)^{12} = \$1{,}846.72.$$

4-18 a.



Enter N = 5 × 2 = 10, I = 12/2 = 6, PV = 0, PMT = -400, and then press FV to get FV = $5,272.32.

b. Now the number of periods is calculated as N = 5 x 4 = 20, I = 12/4 = 3, PV = 0, and PMT = -200. The calculator solution is $5,374.07.

Note that the solution assumes that the nominal interest rate is compounded at the annuity period.

c. The annuity in Part b earns more because some of the money is on deposit for a longer period of time and thus earns more interest. Also, because compounding is more frequent, more interest is earned on interest.

4-19 a. Universal Bank: Effective rate = 7%.

Regional Bank:

$$\text{Effective rate} = \left(1+\frac{0.06}{4}\right)^4 - 1.0 = (1.015)^4 - 1.0$$
$$= 1.0614 - 1.0 = 0.0614 = 6.14\%.$$

With a financial calculator, you can use the interest rate conversion feature to obtain the same answer. You would choose Universal Bank.

b. If funds must be left on deposit until the end of the compounding period (1 year for Universal and 1 quarter for Regional), and you think there is a high probability that you will make a withdrawal during the year, the Regional account might be preferable. For example, if the withdrawal is made after 10 months, you would earn nothing on the Universal account but $(1.015)^3 - 1.0 = 4.57\%$ on the Regional account.

4-20 a. With a financial calculator, enter N = 4, I = 10, PV = -15000, and FV = 0, and then press the PMT key to get PMT = $4,732.06. Then go through the amortization procedure as described in your calculator manual to get the entries for the amortization table.

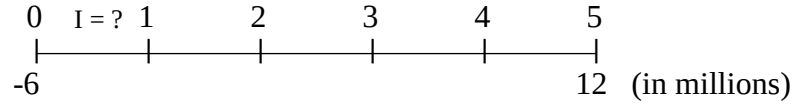
Year	Payment	Interest	Repayment of Principal	Remaining Balance
1	$ 4,732.06	$1,500.00	$ 3,232.06	$11,767.94
2	4,732.06	1,176.79	3,555.27	8,212.67
3	4,732.06	821.27	3,910.79	4,301.88
4	4,732.07*	430.19	4,301.88	0
	$18,928.25	$3,928.25	$15,000.00	

*The last payment must be larger to force the ending balance to zero.

b. Here the loan size is doubled, so the payments also double in size to $9,464.12: enter N = 4, I = 10, PV = -30000, and FV = 0, and then press the PMT key to get PMT = $9,464.12.

c. The annual payment on a $30,000, 8-year loan at 10 percent interest would be $5,623.32: enter N = 8, I = 10, PV = -30000, and FV = 0, and then press the PMT key to get PMT = $5,623.32. Because the payments are spread out over a longer time period, more interest must be paid on the loan. The total interest paid on the 8-year loan is 8($5,623.32) - $30,000 = $14,986.56 versus interest of 4($9,464.12) - $30,000 = $7,856.48 on the 4-year loan.

4-21 a.



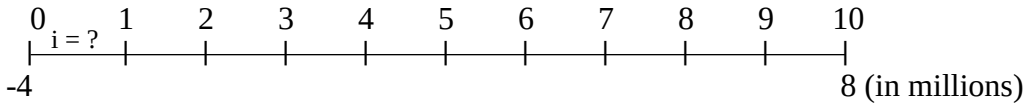
With a calculator, enter N = 5, PV = -6, PMT = 0, FV = 12, and then solve for I = 14.87% ≈ 15%.

b. The calculation described in the quotation fails to take account of the compounding effect. It can be demonstrated to be incorrect as follows:

$$\$6{,}000{,}000(1.20)^5 = \$6{,}000{,}000(2.4883) = \$14{,}929{,}800,$$

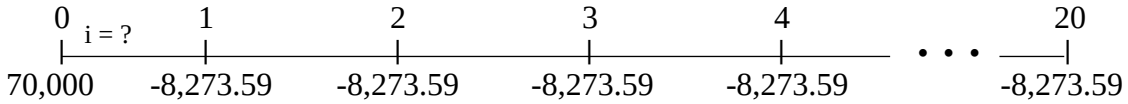
which is greater than $12 million. Thus, the annual growth rate is less than 20%; in fact, it is about 15%, as shown in Part a.

4-22



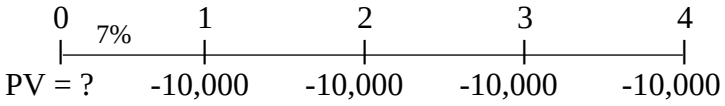
With a calculator, enter N = 10, PV = -4, PMT = 0, FV = 8, and then solve for I = 7.18%.

4-23



With a calculator, enter N = 20, PV = 70000, PMT = -8273.59, FV = 0, and then solve for I = 10.1%.

4-24 a.



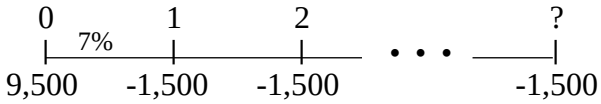
With a calculator, enter N = 4, I = 7, PMT = -10000, and FV = 0. Then press PV to get PV = $33,872.11.

b. (1) At this point, we have a 3-year, 7% annuity of $10,000 whose present value is $26,243.16: N = 3, I = 7, PMT = -10000, and FV = 0. Then press PV to get PV = $26,243.16. You can also think of the problem as follows:

(Beginning balance)(1+I) – PMT = Ending balance
$33,872.11 (1.07) ─ $10,000 = $26,243.16.

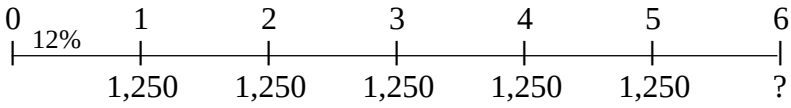
(2) Zero after the last withdrawal.

4-25



With a calculator, enter I = 7, PV = 9500, PMT = -1500, and FV = 0. Press N to get N = 8.9 ≈ 9 years. Therefore, it will take approximately 9 years to pay back the loan.

4-26



FV = 10,000

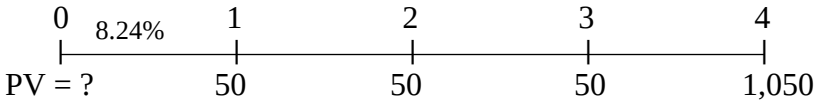
With a financial calculator, get a "ballpark" estimate of the years by entering I = 12, PV = 0, PMT = -1250, and FV = 10000, and then pressing the N key to find N = 5.94 years. This answer assumes that a payment of $1,250 will be made 94/100th of the way through Year 5.

Now find the FV of $1,250 for 5 years at 12%; N = 5, I = 12, PV = 0, and PMT = -1250. Press FV to get FV = $7,941.06. Compound this value for 1 year at 12% to obtain the value in the account after 6 years and before the last payment is made; it is $7,941.06(1.12) = $8,893.99. Thus, you will have to make a payment of $10,000 - $8,893.99 = $1,106.01 at Year 6, so the answer is: it will take 6 years, and $1,106.01 is the amount of the last payment.

4-27 PV = $100/0.07 = $1,428.57. PV = $100/0.14 = $714.29.

When the interest rate is doubled, the PV of the perpetuity is halved.

4-28



Discount rate: Effective rate on bank deposit:

$$EAR = (1 + 0.08/4)^4 - 1 = 8.24\%.$$

Find PV of above stream at 8.24%: with a financial calculator, N = 4, I = 8.24, PMT = 50, and FV = 1000, then press the PV key to get PV = $893.26

4-29 This can be done with a calculator by specifying an interest rate of 5% per period for 20 periods with 1 payment per period to get the payment each 6 months: N = 10 × 2 = 20, I = 10%/2 = 5, PV = -10000. FV = 0. Solve for PMT = $802.43. Set up amortization table as below:

Period	Beg Bal	Payment	Pmt of Interest	Pmt of Principal	End Bal
1	$10,000.00	$802.43	$500.00	$302.43	$9,697.57
2	9,697.57	802.43	484.88		
			$984.88		

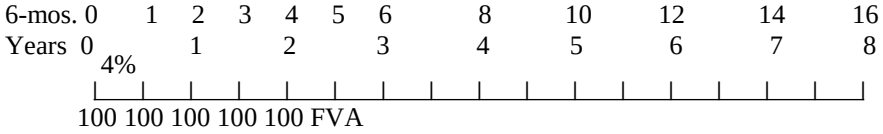
You can also work the problem with a calculator having an amortization function. Find the interest in each 6-month period, sum them, and you have the answer. Even simpler, some calculators such as the HP-10B, allow you to find the interest paid during a particular period of time.

4-30 First, find PMT by using a financial calculator: N = 5, I = 15, PV = -1000000, and FV = 0. Solve for PMT = $298,315.55. Then set up the amortization table:

Year	Beginning Balance	Payment	Interest	Principal	Ending Balance
1	$1,000,000.00	$298,315.55	$150,000.00	$148,315.55	$851,684.45
2	851,684.45	298,315.55	127,752.67	170,562.88	681,121.57

Fraction that is principal = $170,562.88/$298,315.55 = 0.5718 = 57.18%.

4-31 a. Begin with a time line:

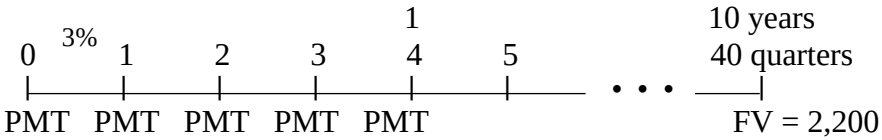


Since the first payment is made today, we have a 5-period annuity due. The applicable interest rate is I = 8/2 = 4 per period, N = 5, PV = 0, and PMT = -100. Setting the calculator on "BEG," we find FVA (Annuity due) = $563.30. That will be the value at the 5th 6-month period, which is t = 2.5. Now we must compound out to t = 8, or for 5.5 years at an EAR of 8.16%, or 11 semiannual periods at 4%.

$563.30 → 16 - 5 = 11 periods @ 4% → $867.17,

or $563.30 → 8 - 2.5 = 5.5 years @ 8.16% → $867.17.

b.



The time line depicting the problem is shown above. Because the payments only occur for 5 periods throughout the 40 quarters, this problem cannot be immediately solved as an annuity problem. The problem can be solved in two steps:

(1) Discount the $2,200 back to the end of Quarter 5 to obtain the PV of that future amount at Quarter 5.

(2) Then solve for PMT using the value solved in Step 1 as the FV of the five-period annuity due.

Step 1: Input the following into your calculator: N = 35, I = 3, PMT = 0, FV = 2200, and solve for PV at Quarter 5. PV_5 = $781.84.

Step 2: The PV found in Step 1 is now the FV for the calculations in this step. Change your calculator to the BEGIN mode. Input the following into your calculator: N = 5, I = 3, PV = 0, FV = 781.84, and solve for PMT = $147.26.

4-32 Here we want to have the same effective annual rate on the credit extended as on the bank loan that will be used to finance the credit extension.

We need to find the effective annual rate (EAR) the bank is charging first. Then, we can use this EAR to calculate the nominal rate that should be quoted to the customers.

Bank EAR: $EAR = (1 + I_{NOM}/M)^M - 1 = (1 + 0.10/12)^{12} - 1 = 10.47\%$.

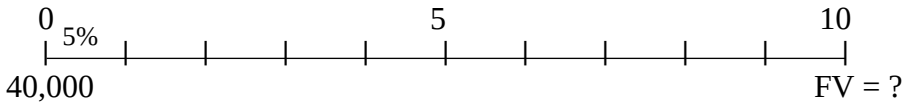
Nominal rate that should be quoted to customers:

$$\begin{aligned} 10.47\% &= (1 + I_{NOM}/4)^4 - 1 \\ 1.1047 &= (1 + I_{NOM}/4)^4 \\ 1.0252 &= 1 + I_{NOM}/4 \\ I_{NOM} &= 0.0252(4) = 10.08\%. \end{aligned}$$

4-33 Information given:

1. Will save for 10 years, then receive payments for 25 years.

2. Wants payments of $40,000 per year in today's dollars for first payment only. Real income will decline. Inflation will be 5%. Therefore, to find the inflated fixed payments, we have this time line:



   Enter N = 10, I = 5, PV = -40000, PMT = 0, and press FV to get FV = $65,155.79.

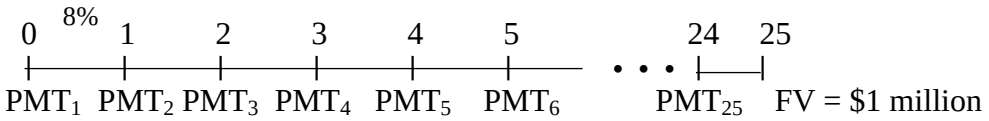
3. He now has $100,000 in an account which pays 8%, annual compounding. We need to find the FV of the $100,000 after 10 years. Enter N = 10, I = 8, PV = -100000, PMT = 0, and press FV to get FV = $215,892.50.

4. He wants to withdraw, or have payments of, $65,155.79 per year for 25 years, with the first payment made at the beginning of the first retirement year. So, we have a 25-year annuity due with PMT = 65,155.79, at an interest rate of 8%. (The interest rate is 8% annually, so no adjustment is required.) Set the calculator to "BEG" mode, then enter N = 25, I = 8, PMT = 65155.79, FV = 0, and press PV to get PV = $751,165.35. This amount must be on hand to make the 25 payments.

5. Since the original $100,000, which grows to $215,892.50, will be available, we must save enough to accumulate $751,165.35 - $215,892.50 = $535,272.85.

6. The $535,272.85 is the FV of a 10-year ordinary annuity. The payments will be deposited in the bank and earn 8 percent interest. Therefore, set the calculator to "END" mode and enter N = 10, I = 8, PV = 0, FV = 535272.85, and press PMT to find PMT = $36,949.61.

4-34 Information given:
The nominal time line is shown below, with a different payment each period and a FV of a nominal $1 million:

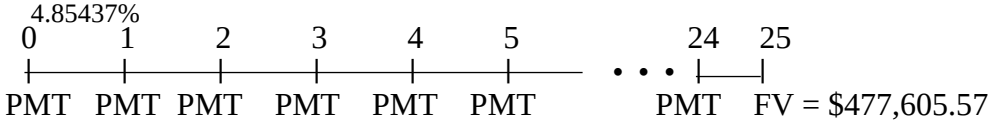


The key is to "rewrite" this as a real time line (i.e., a time line based on today's purchasing power). First, the purchasing power of $1 million in 25 years with an inflation rate of 3% per year is:

$$\$1{,}000{,}000 / (1 + \text{Inflation})^{N} = \$1{,}000{,}000 / (1 + 0.03)^{25} = \$477{,}605.57.$$

This is a growing annuity, with a nominal rate of 8% and an inflation rate of 3%. You should use the real rate in the calculator: $r_r = [(1 + r_{NOM})/(1 + \text{Inflation})] - 1.0 = [1.08/1.03] - 1.0 = .0485437 = 4.85437\%$.

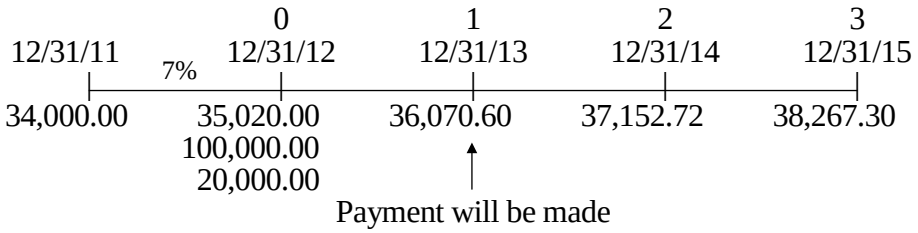
So the "real" time line expressed in today's purchasing power is:



Set the financial calculator to the "BEG" mode, N = 25, I = 4.85437, PV = 0, FV = 477,605.57, and press PMT for PMT = $9,736.96.

Thus, an initial payment of $9,736.96 that grows at 3% each year for 24 more payments, invested at a rate of 8% per year, will accumulate to $1 million at Year 25.

4-35



Step 1: Calculate salary amounts (2011-2015):

2011: $34,000
2012: $34,000(1.03) = $35,020.00
2013: $35,020(1.03) = $36,070.60
2014: $36,070.60(1.03) = $37,152.72
2015: $37,152.72(1.03) = $38,267.30

Step 2: Compound back pay, pain and suffering, and legal costs to 12/31/13 payment date:

$\$34,000(1.07)^2 + \$155,020(1.07)^1 =$
$\$38,926.60 + \$165,871.40 = \$204,798.00.$

Step 3: Discount future salary back to 12/31/13 payment date:

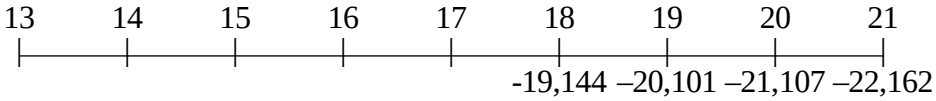
$\$36,070.60 + \$37,152.72/(1.07)^1 + \$38,267.30/(1.07)^2 =$
$\$36,070.60 + \$34,722.17 + \$33,424.14 = \$104,216.91.$

Step 4: City must write a cheque for $204,798.00 + $104,216.91 = $309,014.91.

4-36 Step 1: Determine the annual cost of university. The current cost is $15,000 per year, but that is escalating at a 5% inflation rate:

University Year	Current Cost	Years from Now	Inflation Adjustment	Cash Required
1	$15,000	5	$(1.05)^5$	$19,144.22
2	15,000	6	$(1.05)^6$	20,101.43
3	15,000	7	$(1.05)^7$	21,106.51
4	15,000	8	$(1.05)^8$	22,161.83

Now put these costs on a time line:



How much must be accumulated by age 18 to provide these payments at ages 18 through 21 if the funds are invested in an account paying 6%, compounded annually?

With a financial calculator enter: $CF_0 = 19144$, $CF_1 = 20101$, $CF_2 = 21107$, $CF_3 = 22162$, and I/YR = 6. Solve for NPV = $75,500.00.

Thus, the father must accumulate $75,500 by the time his daughter reaches age 18.

Step 2: The daughter has $7,500 now (age 13) to help achieve that goal. Five years hence, that $7,500, when invested at 6%, will be worth $10,037: $\$7{,}500(1.06)^5 = \$10{,}036.69 \approx \$10{,}037$.

Step 3: The father needs to accumulate only $75,500 – $10,037 = $65,463. The key to completing the problem at this point is to realize the series of deposits represent an ordinary annuity rather than an annuity due, despite the fact the first payment is made at the beginning of the first year. The reason it is not an annuity due is there is no interest paid on the last payment that occurs when the daughter is 18.

Using a financial calculator, N = 6, I/YR = 6, PV = 0, and FV = -65463. PMT = $9,385.

SOLUTION TO SPREADSHEET PROBLEM

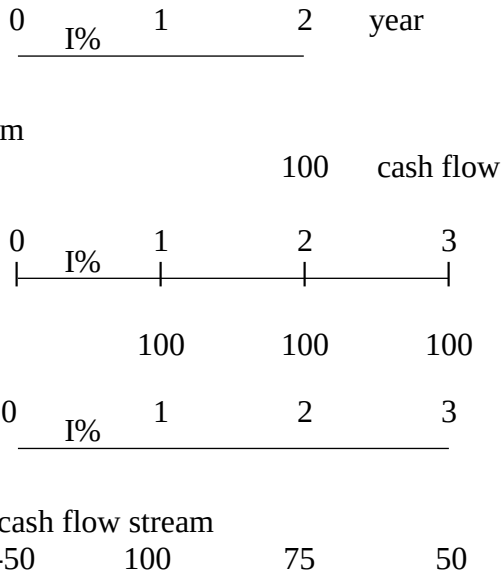
4-37 The detailed solution for the spreadsheet problem is in the file ***Ch 04 Build a Model Solution.xlsx*** and is available on the textbook's website.

MINI CASE

Assume that you are nearing graduation and that you have applied for a job with a local bank. As part of the bank's evaluation process, you have been asked to take an examination that covers several financial analysis techniques. The first section of the test addresses discounted cash flow analysis. See how you would do by answering the following questions.

a. Draw time lines for (1) a $100 lump sum cash flow at the end of Year 2, (2) an ordinary annuity of $100 per year for 3 years, and (3) an uneven cash flow stream of -$50, $100, $75, and $50 at the end of Years 0 through 3.

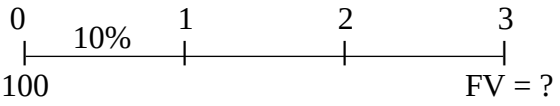
Answer: (Begin by discussing basic discounted cash flow concepts, terminology, and solution methods.) A time line is a graphical representation which is used to show the timing of cash flows. The tick marks represent end of periods (often years), so time 0 is today; time 1 is the end of the first year, or 1 year from today; and so on.



A lump sum is a single flow; for example, a $100 inflow in Year 2, as shown in the top time line. An annuity is a series of equal cash flows occurring over equal intervals, as illustrated in the middle time line. An uneven cash flow stream is an irregular series of cash flows that do not constitute an annuity, as in the lower time line. -50 represents a cash outflow rather than a receipt or inflow.

b. 1. What is the future value of an initial $100 after 3 years if it is invested in an account paying 10% annual interest?

Answer: Show dollars corresponding to question mark, calculated as follows:



After 1 year:

$$FV_1 = PV + I_1 = PV + PV(I) = PV(1 + I) = \$100(1.10) = \$110.00.$$

Similarly:

$$FV_2 = FV_1 + I_2 = FV_1 + FV_1(I) = FV_1(1 + I)$$
$$= \$110(1.10) = \$121.00 = PV(1 + i)(1 + i) = PV(1 + i)^2.$$

$$FV_3 = FV_2 + I_3 = FV_2 + FV_2(I) = FV_2(1 + I)$$
$$= \$121(1.10) = \$133.10 = PV(1 + I)^2(1 + I) = PV(1 + I)^3.$$

In general, we see that:

$$FV_N = PV(1 + I)^N,$$

SO $FV_3 = \$100(1.10)^3 = \$100(1.3310) = \$133.10.$

Note that this equation has 4 variables: FV_N, PV, I, and N. Here we know all except FV_N, so we solve for FV_N. We will, however, often solve for one of the other three variables. By far, the easiest way to work all time value problems is with a financial calculator. Just plug in any 3 of the four values and find the 4th.

Finding future values (moving to the right along the time line) is called compounding. Note that there are 3 ways of finding FV_3: using a regular calculator, financial calculator, or spreadsheets. For simple problems, we show only the regular calculator and financial calculator methods.

(1) regular calculator:

1. $\$100(1.10)(1.10)(1.10) = \$133.10.$

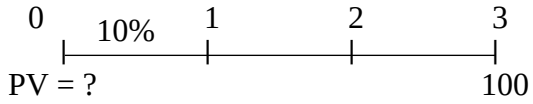
2. $\$100(1.10)^3 = \$133.10.$

(2) financial calculator:

This is especially efficient for more complex problems, including exam problems. Input the following values: N = 3, I = 10, PV = -100, PMT = 0, and solve for FV = $133.10.

b. 2. What is the present value of $100 to be received in 3 years if the appropriate interest rate is 10%?

Answer: Finding present values, or <u>discounting</u> (moving to the left along the time line), is the reverse of compounding, and the basic present value equation is the reciprocal of the compounding equation:



$FV_N = PV(1 + I)^N$ transforms to:

$$PV = \frac{FV_N}{(1+I)^N} = FV_N\left(\frac{1}{1+I}\right)^N = FV_N(1 + I)^{-N}$$

thus:

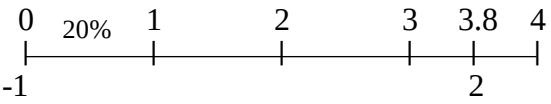
$$PV = \$100\left(\frac{1}{1.10}\right)^3 = \$100(PVIF_{I,N}) = (0.7513) = \$75.13.$$

The same methods used for finding future values are also used to find present values.

Using a financial calculator input N = 3, I = 10, PMT = 0, FV = -100, and then solve for PV = $75.13.

c.	**We sometimes need to find how long it will take a sum of money (or anything else) to grow to some specified amount. For example, if a company's sales are growing at a rate of 20% per year, how long will it take sales to double?**

Answer: We have this situation in time line format:

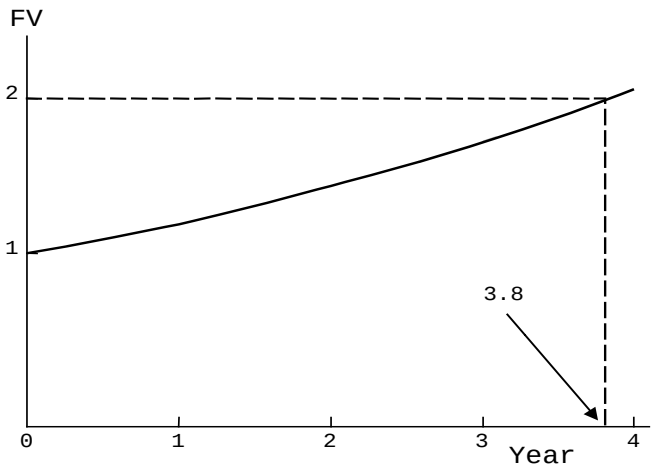


Say we want to find out how long it will take us to double our money at an interest rate of 20%. We can use any numbers, say \$1 and \$2, with this equation:

$$FV_N = \$2 = \$1(1 + I)^N = \$1(1.20)^N.$$

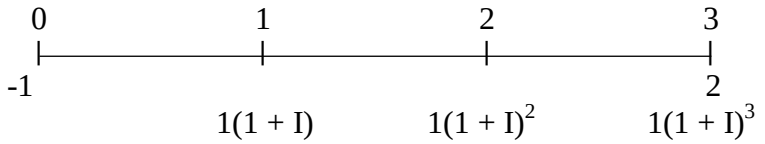
$$\begin{aligned} (1.2)^N &= \$2/\$1 = 2 \\ N\ LN(1.2) &= LN(2) \\ N &= LN(2)/LN(1.2) \\ N &= 0.693/0.182 = 3.8. \end{aligned}$$

Alternatively, we could use a financial calculator. We would plug I = 20, PV = -1, PMT = 0, and FV = 2 into our calculator, and then press the N button to find the number of years it would take 1 (or any other beginning amount) to double when growth occurs at a 20% rate. The answer is 3.8 years, but some calculators will round this value up to the next highest whole number. The graph also shows what is happening.



d. If you want an investment to double in 3 years, what interest rate must it earn?

Answer:



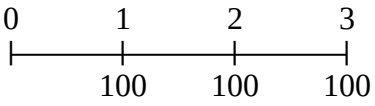
$$FV = \$1(1 + I)^3 = \$2.$$

$$\begin{aligned} \$1(1 + I)^3 &= \$2. \\ (1 + I)^3 &= \$2/\$1 = 2. \\ 1 + I &= (2)^{1/3} \\ 1 + I &= 1.2599 \\ I &= 25.99\%. \end{aligned}$$

Use a financial calculator to solve: enter N = 3, PV = -1, PMT = 0, FV = 2, then press the I button to find I = 25.99%.

Calculators can find interest rates quite easily, even when periods and/or interest rates are not even numbers, and when uneven cash flow streams are involved. (With uneven cash flows, we must use the "CFLO" function, and the interest rate is called the IRR, or "internal rate of return;" we will use this feature in capital budgeting.)

e. What is the difference between an ordinary annuity and an annuity due? What type of annuity is shown below? How would you change it to the other type of annuity?

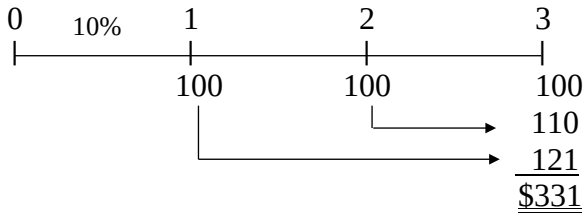


Answer: This is an ordinary annuity—it has its payments at the end of each period; that is, the first payment is made 1 period from today. Conversely, an annuity due has its first payment today. In other words, an ordinary annuity has end-of-period payments, while an annuity due has beginning-of-period payments.

The annuity shown above is an ordinary annuity. To convert it to an annuity due, shift each payment to the left, so you end up with a payment under the 0 but none under the 3.

f. 1. What is the future value of a 3-year ordinary annuity of $100 if the appropriate interest rate is 10%?

Answer:



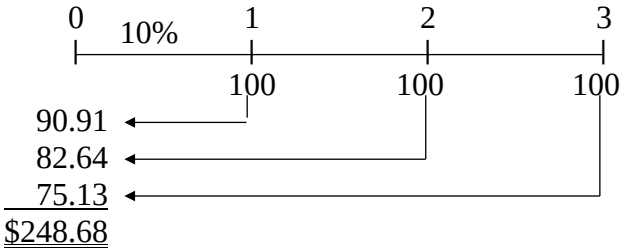
Go through the following discussion. One approach would be to treat each annuity flow as a lump sum. Here we have

$$FVA_N = \$100(1) + \$100(1.10) + \$100(1.10)^2$$
$$= \$100[1 + (1.10) + (1.10)^2] = \$100(3.3100) = \$331.00.$$

Using a financial calculator, N = 3, I = 10, PV = 0, PMT = -100. This gives FV = $331.00.

f. 2. What is the present value of the annuity?

Answer:



The present value of the annuity is $248.68. Using a financial calculator, input N = 3, I = 10, PMT = 100, FV = 0, and press the PV button.

Spreadsheets are useful for time lines with multiple cash flows.

The following spreadsheet shows this problem:

	A	B	C	D
1	0	1	2	3
2		100	100	100
3	248.69			

The excel formula in cell A3 is = NPV(10%,B2:D2). This gives a result of 248.69. Note that the interest rate can be either 10% or 0.10, not just 10. Also, note that the range does not include any cash flow at time zero.

Excel also has special functions for annuities. For ordinary annuities, the Excel formula is = PV(interest rate, number of periods, payment). In this problem, = PV(10%,3,-100), gives a result of 248.69. For the future value, it would be = FV(10%,3,-100), with a result of 331.

f. 3. What would the future and present values be if the annuity were an annuity due?

Answer: If the annuity were an annuity due, each payment would be shifted to the left, so each payment is compounded over an additional period or discounted back over one less period.

To find the future value of an annuity due use the following formula:

$$FVA_N(\text{Annuity Due}) = FVA_N(1 + I).$$

In our situation, the future value of the annuity due is $364.10:

$$FVA_3(\text{Annuity Due}) = \$331.00(1.10)^1 = \$364.10.$$

This same result could be obtained by using the time line: $133.10 + $121.00 + $110.00 = $364.10.

The best way to work annuity due problems is to switch your calculator to "beg" or beginning or "due" mode, and go through the normal process. Note that it's critical to remember to change back to "end" mode after working an annuity due problem with your calculator.

This formula could be used to find the present value of an annuity due:

$$PVA_N(\text{Annuity Due}) = PVA_N(1 + I) = PMT(PVIFA_{I,N})(1 + I).$$

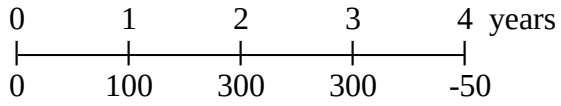
In our situation, the present value of the annuity due is $273.56:

$$PVA_3(\text{Annuity Due}) = \$248.69(1.10)^1 = \$273.56.$$

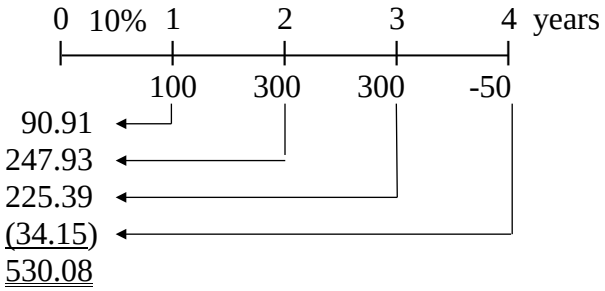
The Excel function is = PV(10%,3,-100,0,1). The fourth term, 0, tells Excel there are no additional cash flows. The fifth term, 1, tells Excel it is an annuity due. The result is $273.56.

A similar modification gives the future value: = FV(10%,3,-100,0,1), with a result of 364.10.

g. What is the present value of the following uneven cash flow stream? The appropriate interest rate is 10%, compounded annually.



Answer: Here we have an uneven cash flow stream. The most straightforward approach is to find the PVs of each cash flow and then sum them as shown below:



Note (1) that the $50 year 4 outflow remains an outflow even when discounted. There are numerous ways of finding the present value of an uneven cash flow stream. But by far the easiest way to deal with uneven cash flow streams is with a financial calculator or a spreadsheet. Calculators have a function which on the HP 17B is called "CFLO," for "cash flow." Other calculators could use other designations such as cf_0 and CF_i, but they explain how to use them in the manual. You would input the cash flows, so they are in the calculator's memory, then input the interest rate, I, and then press the NPV or PV button to find the present value.

Spreadsheets are especially useful for uneven cash flows. The following spreadsheet shows this problem:

	A	B	C	D	E
1	0	1	2	3	4
2		100	300	300	-50
3	530.09				

The Excel formula in cell A3 is = NPV(10%,B2:E2), with a result of 530.09.

h. 1. Define (a) the stated, or quoted, or nominal rate, (I_{NOM}), and (b) the periodic rate (I_{Per}).

Answer: The quoted, or nominal, rate is merely the quoted percentage rate of return. The periodic rate is the rate charged by a lender or paid by a borrower each period (periodic rate = I_{NOM}/M).

h. 2. Will the future value be larger or smaller if we compound an initial amount more often than annually, for example, every 6 months, or semiannually, holding the stated interest rate constant? Why?

Answer: Accounts that pay interest more frequently than once a year, for example, semiannually, quarterly, or daily, have future values that are higher because interest is earned on interest more often. Virtually all banks now pay interest daily on passbook and money fund accounts, so they use daily compounding.

h. 3. What is the future value of $100 after 5 years under 12% annual compounding? Semiannual compounding? Quarterly compounding? Monthly compounding? Daily compounding

Answer: Under annual compounding, the $100 is compounded over 5 annual periods at a 12.0 percent periodic rate:

I_{NOM}= 12%.

$$FV_N = PV\left(1+\frac{I_{NOM}}{M}\right)^{MN} = \$100\left(1+\frac{0.12}{1}\right)^{1*5} = \$100(1.12)^5 = \$176.23.$$

Under semiannual compounding, the $100 is compounded over 10 semiannual periods at a 6.0% periodic rate:

I_{NOM} = 12%.

$$FV_N = PV\left(1+\frac{I_{NOM}}{M}\right)^{MN} = \$100\left(1+\frac{0.12}{2}\right)^{2*5} = \$100(1.06)^{10} = \$179.08.$$

quarterly: $FV_N = \$100(1.03)^{20} = \$180.61$.

monthly: $FV_N = \$100(1.01)^{60} = \$181.67$.

daily: $FV_N = \$100(1+ 0.12/365)^{365*5} = \$182.19$.

h. 4. What is the effective annual rate (EFF%)? What is the EFF% for a nominal rate of 12%, compounded semiannually? Compounded quarterly? Compounded monthly? Compounded daily?

Answer: The effective annual rate is the annual rate that causes the PV to grow to the same FV as under multi-period compounding. For 12% semiannual compounding, the EFF is 12.36%:

$$\text{EAR} = \text{Effective Annual Rate} = \left(\frac{1+I_{NOM}}{M}\right)^{M} - 1.0.$$

IF I_{NOM} = 12% and interest is compounded semiannually, then:

$$\text{EAR} = \left(1+\frac{0.12}{2}\right)^{2} - 1.0 = (1.06)^2 - 1.0 = 1.1236 - 1.0 = 0.1236 = 12.36\%.$$

For quarterly compounding, the effective annual rate is:

$$(1.03)^4 - 1.0 = 12.55\%.$$

For monthly compounding, the effective annual rate is:

$$(1.01)^{12} - 1.0 = 12.68\%.$$

For daily compounding, the effective annual rate is:

$$(1 + 0.12/365)^{365} - 1.0 = 12.75\%.$$

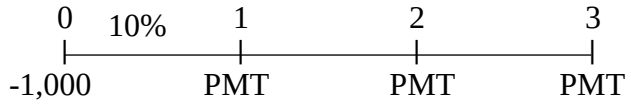
i. Will the effective annual rate ever be equal to the nominal (quoted) rate?

Answer: If annual compounding is used, then the nominal rate will be equal to the effective annual rate. If more frequent compounding is used, the effective annual rate will be above the nominal rate.

j. 1. Construct an amortization schedule for a $1,000, 10% annual rate loan with 3 equal installments.

2. What is the annual interest expense for the borrower, and the annual interest income for the lender, during Year 2?

Answer: To begin, note that the face amount of the loan, $1,000, is the present value of a 3-year annuity at a 10 percent rate:



$$PVA_3 = PMT\left(\frac{1}{1+I}\right)^1 + PMT\left(\frac{1}{1+I}\right)^2 + PMT\left(\frac{1}{1+I}\right)^3$$

$$\$1{,}000 = PMT(1 + I)^{-1} + PMT(1 + I)^{-2} + PMT(1 + I)^{-3}$$
$$= PMT(1.10)^{-1} + PMT(1.10)^{-2} + PMT(1.10)^{-3}.$$

We have an equation with only one unknown, so we can solve it to find PMT. The easy way is with a financial calculator. Input N = 3, I = 10, PV = -1,000, FV = 0, and then press the PMT button to get PMT = 402.1148036, rounded to $402.11.

Now make the following points regarding the amortization schedule:

- The $402.11 annual payment includes both interest and principal. Interest in the first year is calculated as follows:

  1st year interest = I × beginning balance = 0.1 × $1,000 = $100.

- The repayment of principal is the difference between the $402.11 annual payment and the interest payment:

  1st year principal repayment = $402.11 - $100 = $302.11.

- The loan balance at the end of the first year is:

  1st year ending balance = beginning balance – principal repayment
  = $1,000 - $302.11 = $697.89.

- We would continue these steps in the following years.

- Notice that the interest each year declines because the beginning loan balance is declining. Since the payment is constant, but the interest component is declining, the principal repayment portion is increasing each year.

- The interest component is an expense which is deductible to a business and it is taxable income to the lender. If you buy a house, you will get a schedule constructed similar to ours, but longer, with $25 \times 12 = 300$ monthly payments if you get a 25-year, fixed-rate mortgage. Also, mortgages in Canada are compounded semiannually.

- The payment may have to be adjusted by a few cents in the final year to take care of rounding errors and make the final payment produce a zero ending balance.

- The lender received a 10% rate of interest on the average amount of money that was invested each year, and the $1,000 loan was paid off. This is what amortization schedules are designed to do.

- Most financial calculators have amortization functions built in.

k. Suppose on January 1 you deposit $100 in an account that pays a nominal, or quoted, interest rate of 11.33463%, with interest added (compounded) daily. How much will you have in your account on October 1, or after 9 months?

Answer: The daily periodic interest rate is $r_{Per} = 11.3346\%/365 = 0.031054\%$. There are 273 days between January 1 and October 1. Calculate FV as follows:

$$\begin{aligned} FV_{273} &= \$100(1.00031054)^{273} \\ &= \$108.85. \end{aligned}$$

Using a financial calculator, input N = 273, I = 0.031054, PV = -100, and PMT = 0. Pressing FV gives $108.85.

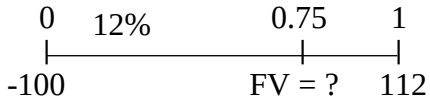
An alternative approach would be to first determine the effective annual rate of interest, with daily compounding, using the formula:

$$EAR = \left(1 + \frac{0.1133463}{365}\right)^{365} - 1 = 0.12 = 12.0\%.$$

(Some calculators, e.g., the hp 10b and 17b, have this equation built in under the ICNV [interest conversion] function.)

Thus, if you left your money on deposit for an entire year, you would earn $12 of interest, and you would end up with $112. The question, though, is this: how much will be in your account on October 1?

Here you will be leaving the money on deposit for 9/12 = 3/4 = 0.75 of a year.



You would use the regular set-up, but with a fractional exponent:

$$FV_{0.75} = \$100(1.12)^{0.75} = \$100(1.088713) = \$108.87.$$

This is slightly different from our earlier answer, because n is actually 273/365 = 0.7479 rather than 0.75.

Fractional time periods

Thus far all of our examples have dealt with full years. Now we are going to look at the situation when we are dealing with fractional years, such as 9 months, or 10 years. In these situations, proceed as follows:

- As always, start by drawing a time line so you can visualize the situation.

- Then think about the interest rate—the nominal rate, the compounding periods per year, and the effective annual rate. If you have been given a nominal rate, you may have to convert to the EAR, using this formula:

$$EAR = \left(1 + \frac{I_{NOM}}{M}\right)^{M} - 1.$$

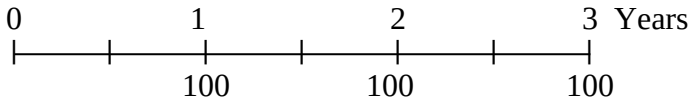
- If you have the effective annual rate—either because it was given to you or after you calculated it with the formula—then you can find the PV of a lump sum by applying this equation:

$$PV = FV_N\left(\frac{1}{1 + EAR}\right)^{N}.$$

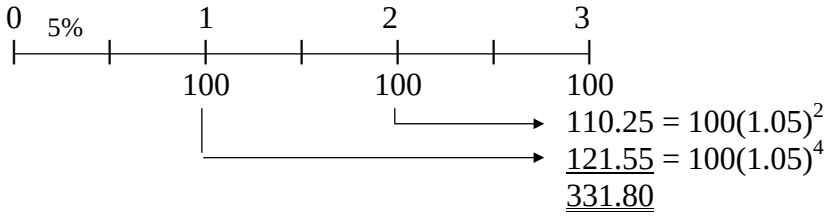
- Here t can be a fraction of a year, such as 0.75, if you need to find the PV of $1,000 due in 9 months, or 450/365 = 1.2328767 if the payment is due in 450 days.

- If you have an annuity with payments different from once a year, say every month, you can always work it out as a series of lump sums. That procedure always works. We can also use annuity formulas and calculator functions, but you have to be careful.

l. 1. What is the value at the end of Year 3 of the following cash flow stream if the quoted interest rate is 10%, compounded semiannually?



Answer:



Here we have a different situation. The payments occur annually, but compounding occurs each 6 months. Thus, we cannot use normal annuity valuation techniques. There are two approaches that can be applied: (1) treat the cash flows as lump sums, as was done above, or (2) treat the cash flows as an ordinary annuity, but use the effective annual rate:

$$EAR = \left(1 + \frac{I_{NOM}}{M}\right)^{M} - 1 = \left(1 + \frac{0.10}{2}\right)^{2} - 1 = 10.25\%.$$

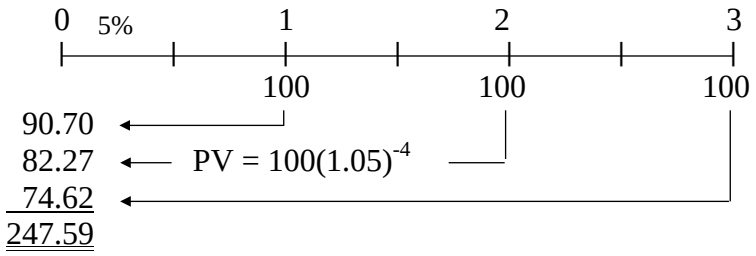
Now we have this 3-period annuity:

$$FVA_3 = \$100(1.1025)^2 + \$100(1.1025)^1 + \$100 = \$331.80.$$

You can plug in N = 3, I = 10.25, PV = 0, and PMT = -100, and then press the FV button to find FV = $331.80.

l. 2. What is the PV of the same stream?

Answer:



PV = \$100(2.4759) = \$247.59 AT 10.25%.

To use a financial calculator, input N = 3, I = 10.25, PMT = 100, FV = 0, and then press the PV key to find PV = \$247.59.

l. 3. Is the stream an annuity?

Answer: The payment stream is an annuity in the sense of constant amounts at regular intervals, but the intervals do not correspond with the compounding periods. This kind of situation occurs often. In this situation the interest is compounded semiannually, so with a quoted rate of 10%, the EAR will be 10.25%. Here we could find the effective rate and then treat it as an annuity. Enter N = 3, I = 10.25, PMT = 100, and FV = 0. Now press PV to get \$247.59.

l. 4. An important rule is that you should *never* show a nominal rate on a time line or use it in calculations unless what condition holds? (Hint: Think of annual compounding, when I_{Nom} = EFF% = I_{Per}.) What would be wrong with your answer to questions l(1) and l(2) if you used the nominal rate (10%) rather than the periodic rate (I_{NOM}/2 = 10%/2 = 5%)?

Answer: I_{Nom} can only be used in the calculations when annual compounding occurs. If the nominal rate of 10% was used to discount the payment stream the present value would be overstated by \$248.69 - \$247.59 = \$1.10.

m. **Suppose someone offered to sell you a note calling for the payment of $1,000 15 months from today. They offer to sell it to you for $850. You have $850 in a bank time deposit which pays a 6.76649% nominal rate with daily compounding, which is a 7% effective annual interest rate, and you plan to leave the money in the bank unless you buy the note. The note is not risky—you are sure it will be paid on schedule. Should you buy the note? Check the decision in three ways: (1) by comparing your future value if you buy the note versus leaving your money in the bank, (2) by comparing the PV of the note with your current bank account, and (3) by comparing the EFF% on the note versus that of the bank account.**

Answer: You can solve this problem in three ways—(1) by compounding the $850 now in the bank for 15 months and comparing that FV with the $1,000 the note will pay, (2) by finding the PV of the note and then comparing it with the $850 cost, and (3) finding the effective annual rate of return on the note and comparing that rate with the 7% you are now earning, which is your opportunity cost of capital. All three procedures lead to the same conclusion. Here is the time line:

(1) $FV = \$850(1.07)^{1.25} = \$925.01$ = amount in bank after 15 months versus $1,000 if you buy the note. (Again, you can find this value with a financial calculator. Note that certain calculators like the hp 12c perform a straight-line interpolation for values in a fractional time period analysis rather than an effective interest rate interpolation. The value that the hp 12c calculates is $925.42.) This procedure indicates that you should buy the note.

Alternatively, 15 months = (1.25 years)(365 days per year) = 456.25 ≈ 456 days.

$$FV_{456} = \$850(1.00018538)^{456}$$
$$= \$924.97.$$

The slight difference is due to using N = 456 rather than N = 456.25.

(2) $PV = \$1{,}000/(1.07)^{-1.25} = \$918.90$. Since the present value of the note is greater than the $850 cost, it is a good deal. You should buy it.

Alternatively, $PV = \$1000/(1.00018538)^{456} = \$918.95$.

(3) $FV_N = PV(1 + I)^N$, so \$1,000 = \$850$(1 + I)^{1.25}$ = \$1,000. Since we have an equation with one unknown, we can solve it for I. You will get a value of I = 13.88%. The easy way is to plug values into your calculator. Since this return is greater than your 7% opportunity cost, you should buy the note. This action will raise the rate of return on your asset portfolio.

Alternatively, we could solve the following equation:

$$\$1{,}000 = \$850(1 + I)^{456} \text{ for a daily } I = 0.00035646,$$

With a result of EAR = EFF% = $(1.00035646)^{365} - 1 = 13.89\%$.

WEB EXTENSION 4B: CONTINUOUS COMPOUNDING AND DISCOUNTING

SOLUTIONS TO PROBLEMS

4B-1 $FV_{15} = \$15{,}000e^{0.06(15)} = \$36{,}894.05.$

4B-2 $PV = FV_N/e^{IN} = \$200{,}000/e^{0.09(7)} = \$200{,}000/1.8776 = \$106{,}518.36.$

4B-3 Daily compounding:

$FV_2 = PV(1 + 0.07/365)^{365(2)} = \$1{,}000(1.15026) = \$1{,}150.26$

Continuous compounding:

$FV_2 = PVe^{IN} = \$1{,}000e^{0.069(2)} = \$1{,}000(1.14798) = \underline{\$1{,}147.98}$

Difference between accounts $\underline{\$ \quad 2.28}$

4B-4 Calculate the growth factor using PV and FV which are given:

$FV_N = PVe^{IN}$; $\$40{,}000 = \$20{,}000e^{I6}$

$e^{I6} = 2.0.$

Take the natural logarithm of both sides:

I(6)ln e = ln 2.00.
The natural log of e = 1.0.
Inputs: 2.0. Press LN key. Output: LN = 0.6931.

I(6)ln e = ln 2.0
I(6) = 0.6931
I = 0.1155 = 11.55%.

4B-5 Determine the effective annual rates and then choose c.

a. 10.25% annually = 10.25%.

b. 10.0% semiannually = $\left(1 + \frac{0.10}{2}\right)^2 - 1.0 = 0.1025 = 10.25\%.$

c. 9.8% continuously = $e^{0.098} - 1.0 = 0.10296 = 10.30\%.$

4B-6

$\$11,572.28 = PVe^{0.09(15)}$

$\$11,572.28 = PVe^{1.35}$

$PV = \$11,572.28/e^{1.35}$

$= \$11,572.28/3.85743 = \$3,000.$

4B-7 $e^{(0.03)(10)} = \left(1 + \frac{I_{NOM}}{2}\right)^{20}$

$e^{0.3} = \left(1 + \frac{I_{NOM}}{2}\right)^{20}$

$e^{0.3/20} = 1 + \frac{I_{NOM}}{2}$

$1.01511 = 1 + \frac{I_{NOM}}{2}$

$\frac{I_{NOM}}{2} = 0.01511$

$I_{NOM} = 0.0302 = 3.02\%.$

4B-8 Step 1: Calculate the FV of the $2,000 deposit at 8% with continuous compounding:

Using e^x key:

Inputs: X = 0.24; press e^x key.
Output: $e^x = 1.27125$.

$FV_N = \$2,000e^{0.08(3)} = \$2,000(1.27125) = \$2,542.50.$

Step 2: Calculate the PV or initial deposit:

Input the following data into your calculator: N = 6; I/YR = 4.5; PMT = 0; FV = 2542.50; and then solve for PV = $1,952.37.