Combined Stresses

Q1. Determine the principal stresses and their directions at the surface of point H of the hollow bronze shaft (outside diameter = 80 mm and wall thickness = 5 mm) shown in Figure 1 subjected to a torque of T = 620 N-m and an axial load of P = 9,500 N.

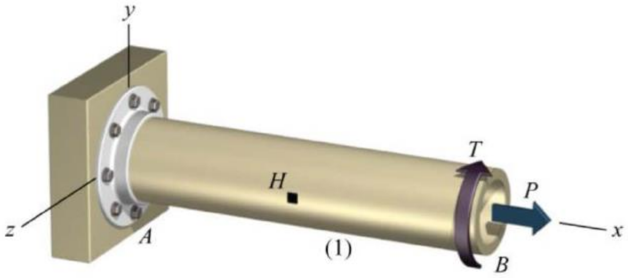


Figure 1

<u>Solution</u>:

Section Properties:

$$A = \frac{\pi}{4}\left(80^2 - 70^2\right) = 1{,}178.097\ \text{mm}^2$$

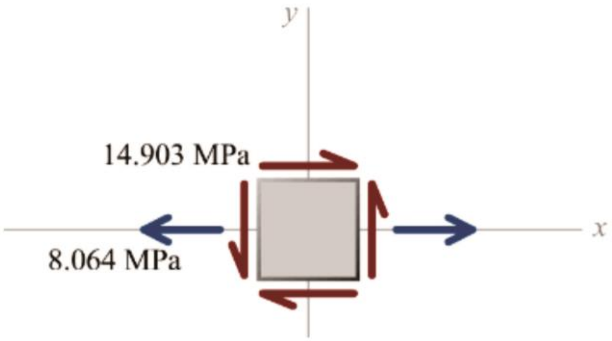
$$J = \frac{\pi}{32}\left(80^4 - 70^4\right) = 1{,}664{,}062.359\ \text{mm}^4$$

Normal and shear stress magnitude

$$\sigma = \frac{P}{A} = \frac{9{,}500}{1{,}178.079} = 8.064\ (\text{T})$$

$$\tau = \frac{Tr}{J} = \frac{(620\ )(80/2)(1{,}000\ \ mm/m)}{1{,}664{,}062.359} = 14.903\ \text{MPa}$$

(sense of shear stress determined by inspection)



Principal stress calculations:

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{(8.064)+(0)}{2} \pm \sqrt{\left(\frac{(8.064)-(0)}{2}\right)^2 + (14.903)^2}$$

$$= 4.032 \pm 15.439$$

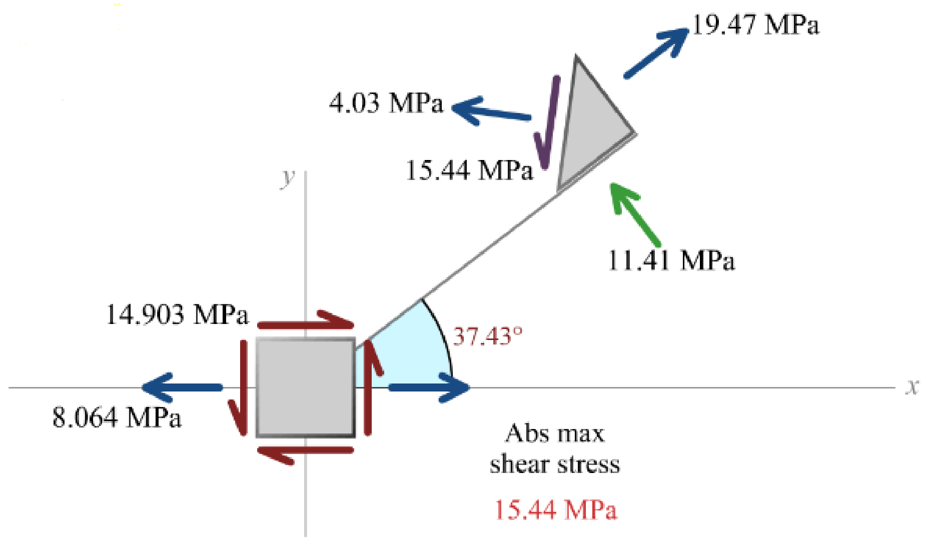
$\sigma_{p1} =$ 19.47 MPa and $\sigma_{p2} =$ −11.41 MPa

$t_{\max} =$ 15.44 MPa (maximum in-plane shear stress)

$\sigma_{avg} =$ 4,03 MPa (T) (normal stress on planes of maximum in-plane shear stress)

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left[(\sigma_x - \sigma_y)/2\right]} = \frac{14.903}{\left[(8.064-0)/2\right]} = \frac{14.903}{4.032} = 3.6953 \text{ Rad}$$

$\therefore \theta_p =$ 37.43° (counter clockwise from the *x* axis to the direction of σ_{p1})



Q2. The solid shaft is subjected to a torque, bending moment, and shear force as shown. Determine the principal stresses acting at points A and B.

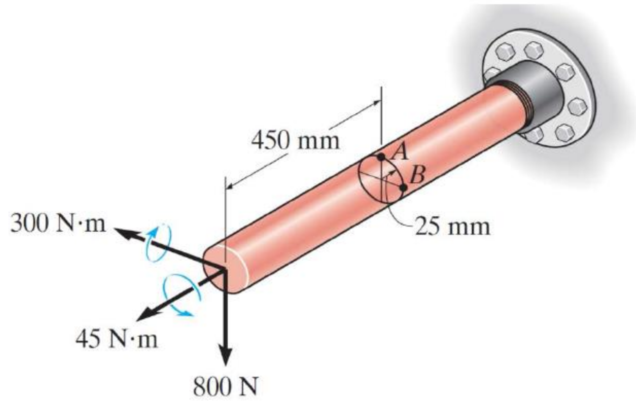


Figure 2

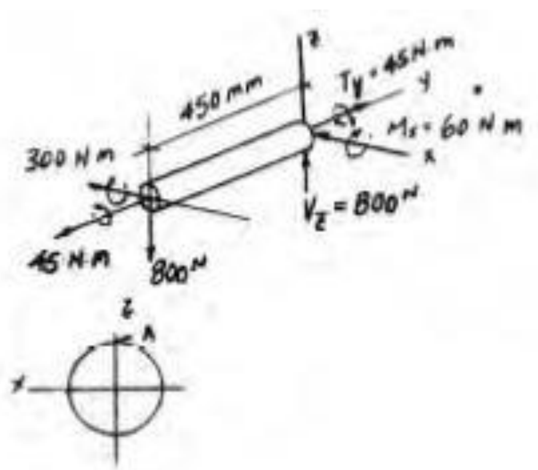
Solution:

For Point *A*

$$I_x = I_y = \frac{\pi}{4}(0.025)^4 = 0.306796\left(10^{-6}\right)\text{ m}^4$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.613592\left(10^{-6}\right)\text{ m}^4$$

$$Q_A = 0$$



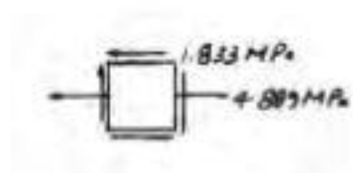
$$\sigma_A = \frac{M_x r}{I} = \frac{60(0.025)}{0.306796\left(10^{-6}\right)} = 4.889\text{ MPa}$$

$$\tau_A = \frac{T_y r}{J} = \frac{45(0.025)}{0.613592\left(10^{-6}\right)} = 1.833\text{ MPa}$$

$\sigma_x = 4.889$ MPa $\qquad \sigma_y = 0 \qquad \tau_{xy} = -1.833$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{4.889 + 0}{2} \pm \sqrt{\left(\frac{4.889 - 0}{2}\right)^2 + (-1.833)^2}$$



$$\sigma_1 = \boxed{5.50\text{ MPa}}$$

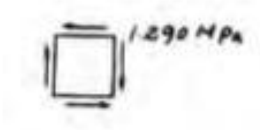
$$\sigma_2 = \boxed{-0.611\text{ MPa}}$$

For Point *B*

$$I_x = I_y = \frac{\pi}{4}(0.025)^4 = 0.306796\left(10^{-6}\right)\text{ m}^4$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.613592\left(10^{-6}\right)\text{ m}^4$$

$$Q_B = \bar{y}A' = \frac{4(0.025)}{3\pi}\left(\frac{1}{2}\right)\pi(0.025)^2 = 10.4167\left(10^{-6}\right)\text{ m}^3$$



$$\sigma_B = 0$$

$$\tau_B = \frac{V_z Q_B}{It} - \frac{T_y r}{J} = \frac{800(10.4167)\left(10^{-6}\right)}{0.306796\left(10^{-6}\right)(0.05)} \frac{45(0.025)}{0.613592\left(10^{-6}\right)} = -1.290\text{ MPa}$$

$$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = -1.290\text{ MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 0 \pm \sqrt{(0)^2 + (-1.290)^2}$$

$$\sigma_1 = \boxed{1.29\text{ MPa}}$$

$$\sigma_2 = \boxed{-1.29\text{ MPa}}$$

Q3. The state of stress at a point is shown in Figure 3. By using Mohr's circle, determine:

a. The principal stresses and their orientations

b. The maximum in-plane shear stress and average normal stress, specifying the corresponding orientation

c. The state of stress if the coordinate system is oriented 30° clockwise from the element shown in Figure 3.

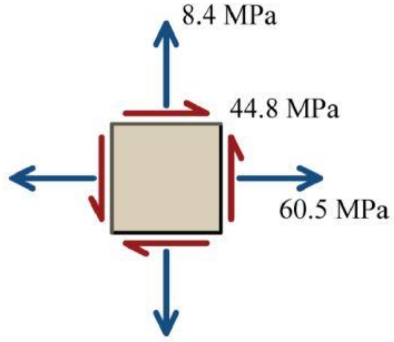


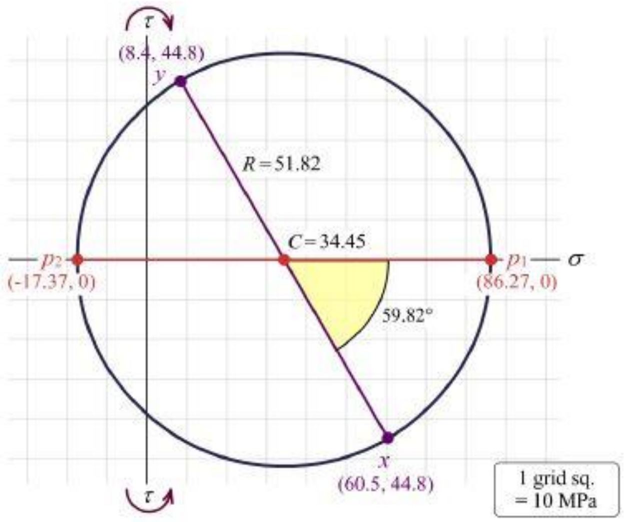
Figure 3

<u>Solution</u>:

(a)and(b)

$$C = \frac{60.5 + 8.4}{2} = 34.45 \text{ MPa}$$

$$R = \sqrt{26.05^2 + 44.8^2} = 51.8232 \text{ MPa}$$



$$\sigma_{p1} = C + R = 34.45 + 51.8232 = \boxed{86.3 \text{ MPa}}$$

$$\sigma_{p2} = C - R = 34.45 - 51.8232 = \boxed{-17.37 \text{ MPa}}$$

$$\tau_{\max} = R = \boxed{51.8 \text{ MPa}}$$

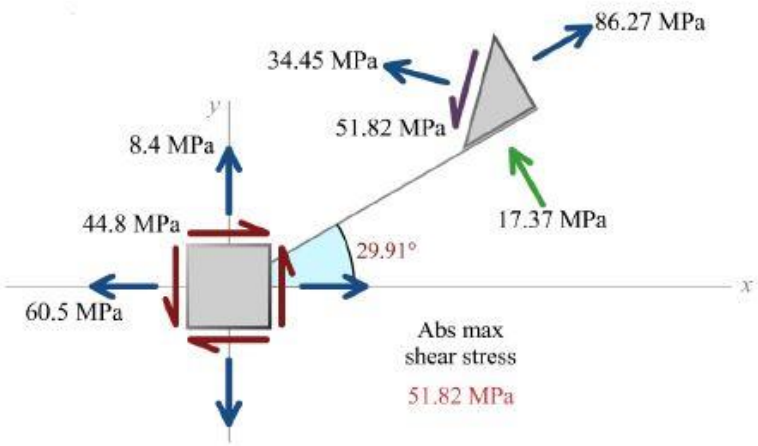
$$\sigma_{avg} = C = \boxed{34.5 \text{ MPa (T)}}$$

The magnitude of the angle $2\theta_p$ between point *x* (i.e., the *x* face of the stress element) and point 2 (i.e., the principal plane subjected to σ_{p2}) is found from:

$$\tan 2\theta_p = \frac{44.8}{|60.5 - 34.45|} = \frac{44.8}{26.05} = 1.7198 \;\therefore\; 2\theta_p = 59.8231°, \quad \theta_p = \boxed{29.9°}$$

By inspection, the angle □*p* from point *x* to point 1 is turned **counterclockwise.**

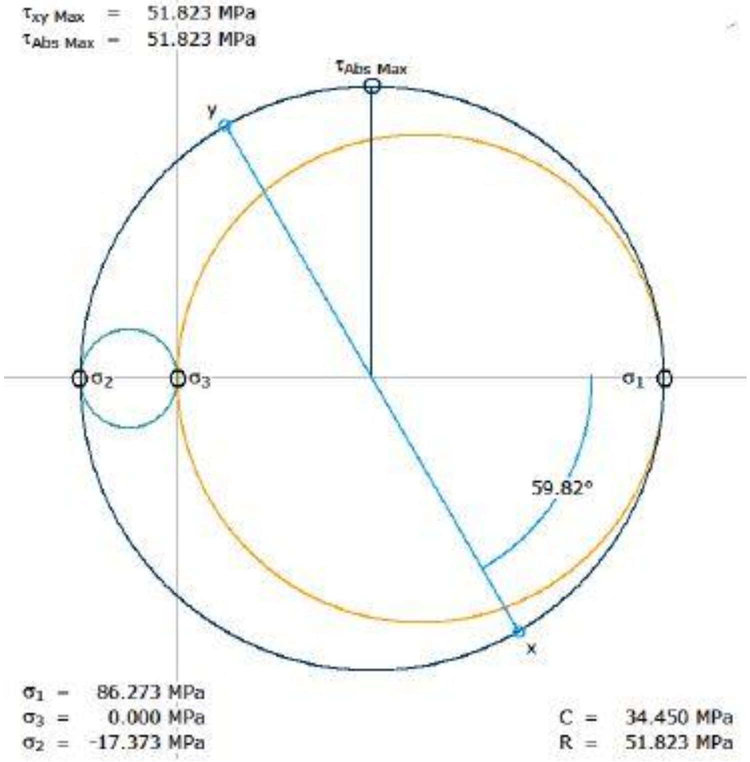
(c) The orientation of the principal stresses and the maximum in-plane shear stress is shown in the sketch below



(d) Since the point in a structural member is subjected to plane stress

$\sigma_z = \sigma_{p3} = 0$

Three Mohr's circles can be constructed to show stress combinations in the $\sigma_{p1} - \sigma_{p2}$ plane, the $\sigma_{p1} - \sigma_{p3}$ plane, and the $\sigma_{p2} - \sigma_{p3}$ plane. These three circles are shown below.



In this case, the absolute maximum shear stress occurs in the $\sigma_{p1} - \sigma_{p2}$ plane (which is also the *x*-*y* plane). Therefore

$\tau_{abs,\max} = \tau_{\max} = \boxed{51.8\text{ MPa}}$