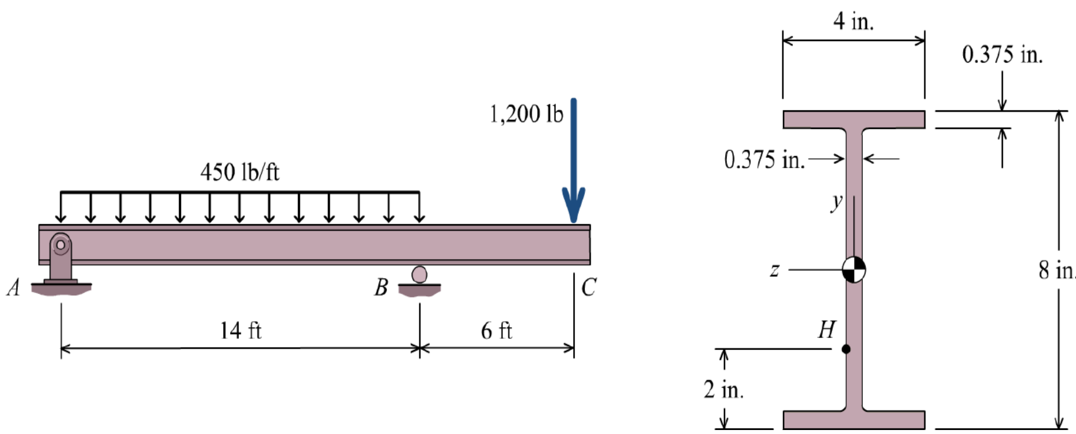
CVG2140 Winter – 2014
Mechanics of Materials I
ASSIGNMENT #7
Solutions

Shear and Torsion

Q1. A simply supported beam is subjected to the loads shown in Figure 1a. The cross-sectional dimensions of the plastic wide-flange shape are shown in Figure 1b.

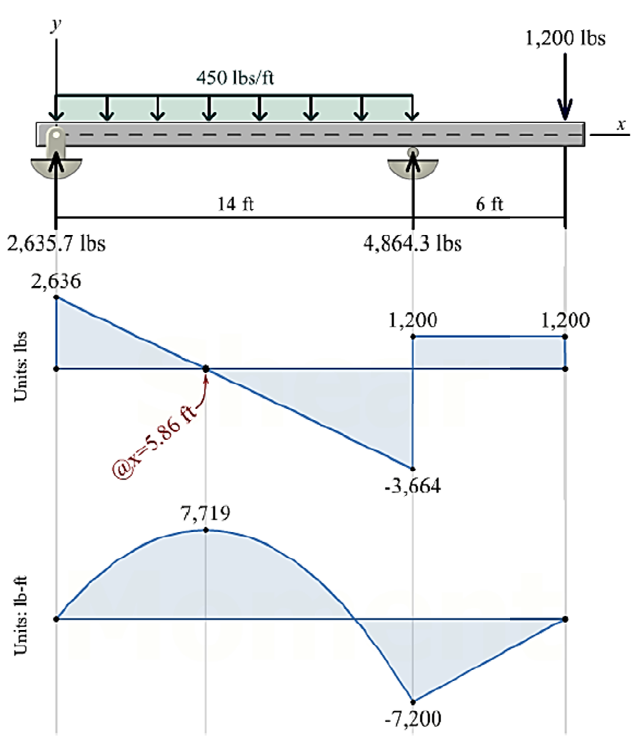
a. Determine the magnitude of the maximum shear force in the beam.
b. At the section of maximum shear force, determine the shear stress magnitude in the cross section at point H, which is located 2 in. above the bottom surface of the wide-flange shape.



(a) (b)

Figure 1

Solution:



Section properties:

$$I_z = \frac{(4\text{ in.})(8\text{ in.})^3}{12} - \frac{(3.625\text{ in.})(7.25\text{ in.})^3}{12}$$

$$= 55.5493\text{ in.}^4$$

(a) Maximum shear force magnitude:

$V = 3{,}664$ lb

(b) Shear stress magnitude at *H*:

$$Q_H = (4\text{ in.})(0.375\text{ in.})(3.8125\text{ in.}) + (0.375\text{ in.})(1.625\text{ in.})(2.8125\text{ in.})$$

$$= 7.4326\text{ in.}^3$$

$$\tau_H = \frac{(3{,}664\text{ lb})(7.4326\text{ in.}^3)}{(55.5493\text{ in.}^4)(0.375\text{ in.})}$$

$= 1{,}307$ psi

Q2. For the beam shown in Figure 2, determine the maximum shear stress acting in the fiber glass beam at the section where the internal shear force is maximum.

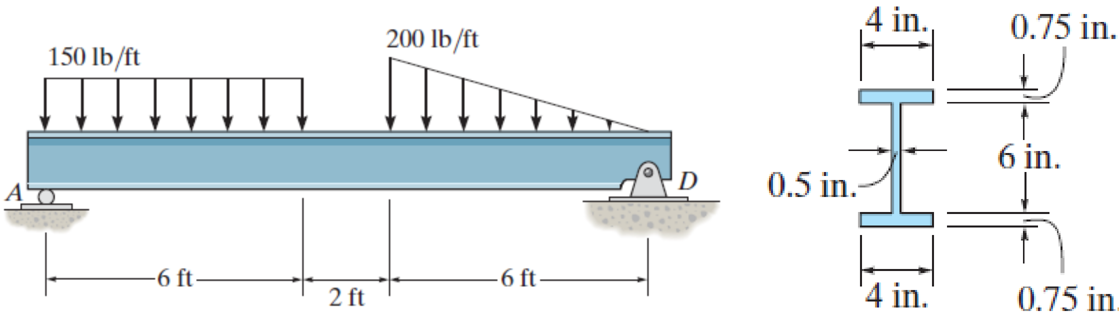
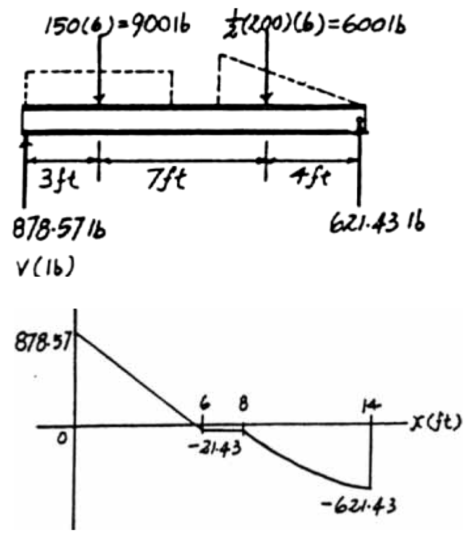


Figure 2



Support Reactions: As shown on FBD.

Internal Shear Force: As shown on shear diagram,

V_{max} = 878.57 Ib

Section Properties:

$$I_{NA} = \frac{1}{12}(4)\left(7.5^3\right) - \frac{1}{12}(3.5)\left(6^3\right) = 77.625 \text{ in}^4$$

$$Q_{max} = \Sigma \bar{y}' A'$$

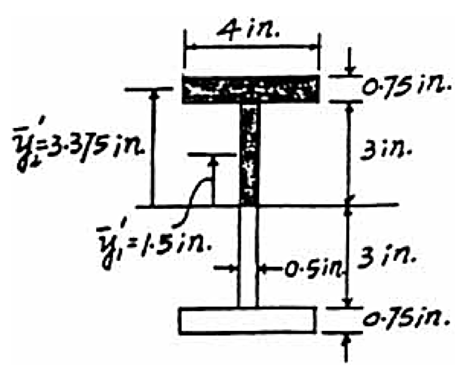
$$= 3.375(4)(0.75) + 1.5(3)(0.5) = 12.375 \text{ in}^3$$

Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section.

Applying the shear formula:

$$\tau_{max} = \frac{VQ_{max}}{It}$$

$$= \frac{878.57(12.375)}{77.625(0.5)} = 280 \text{ psi}$$



Q3. A solid steel [G = 80 GPa] shaft of variable diameter is subjected to the torques shown in Figure 3. The diameter of the shaft in segments (1) and (3) is 50 mm, and the diameter of the shaft in segment (2) is 80 mm. The bearings shown allow the shaft to turn freely.

a. Determine the maximum shear stress in the compound shaft.
b. Determine the rotation angle of pulley D with respect to pulley A. Plot a torque diagram showing the internal torque in segments (1), (2), and (3) of the shaft.

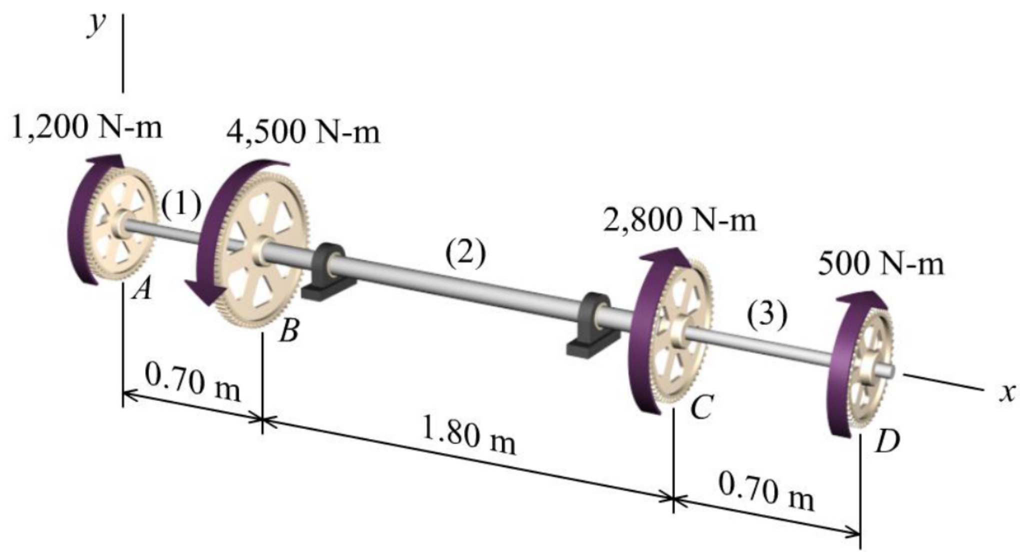


Figure 3

Solution

Section properties: The polar moments of inertia for the shaft segments will be needed for this calculation.

$$J_1 = \frac{\pi}{32}(50\text{ mm})^4 = 613{,}592.32\text{ mm}^4 = J_3$$

$$J_2 = \frac{\pi}{32}(80\text{ mm})^4 = 4{,}021{,}238.60\text{ mm}^4$$

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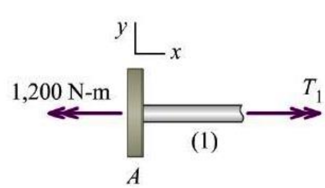
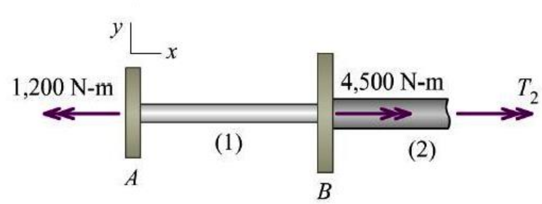
(a) Equilibrium

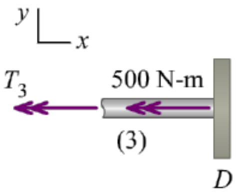
$$\Sigma M_x = -1{,}200\text{ N-m} + T_1 = 0$$
$$\therefore T_1 = 1{,}200\text{ N-m}$$



$$\Sigma M_x = -1{,}200\text{ N-m} + 4{,}500\text{ N-m} + T_2 = 0$$
$$\therefore T_2 = -3{,}300\text{ N-m}$$



$$\Sigma M_x = -T_3 - 500 \text{ N-m} = 0$$
$$\therefore T_3 = -500 \text{ N-m}$$



Shear stress:

$$\tau_1 = \frac{T_1 c_1}{J_1} = \frac{(1{,}200 \text{ N-m})(50 \text{ mm}/2)(1{,}000 \text{ mm/m})}{613{,}592.32 \text{ mm}^4} = 48.892 \text{ MPa} = 48.9 \text{ MPa}$$

$$\tau_2 = \frac{T_2 c_2}{J_2} = \frac{(-3{,}300 \text{ N-m})(80 \text{ mm}/2)(1{,}000 \text{ mm/m})}{4{,}021{,}238.60 \text{ mm}^4} = -32.826 \text{ MPa} = -32.8 \text{ MPa}$$

$$\tau_3 = \frac{T_3 c_3}{J_3} = \frac{(-500 \text{ N-m})(50 \text{ mm}/2)(1{,}000 \text{ mm/m})}{613{,}592.32 \text{ mm}^4} = -20.372 \text{ MPa} = -20.4 \text{ MPa}$$

The maximum shear stress in the compound shaft is $\tau_{max} = \tau_1 = 48.9 \text{ MPa}$

Angles of twist:

The angles of twist in the three shaft segments are:

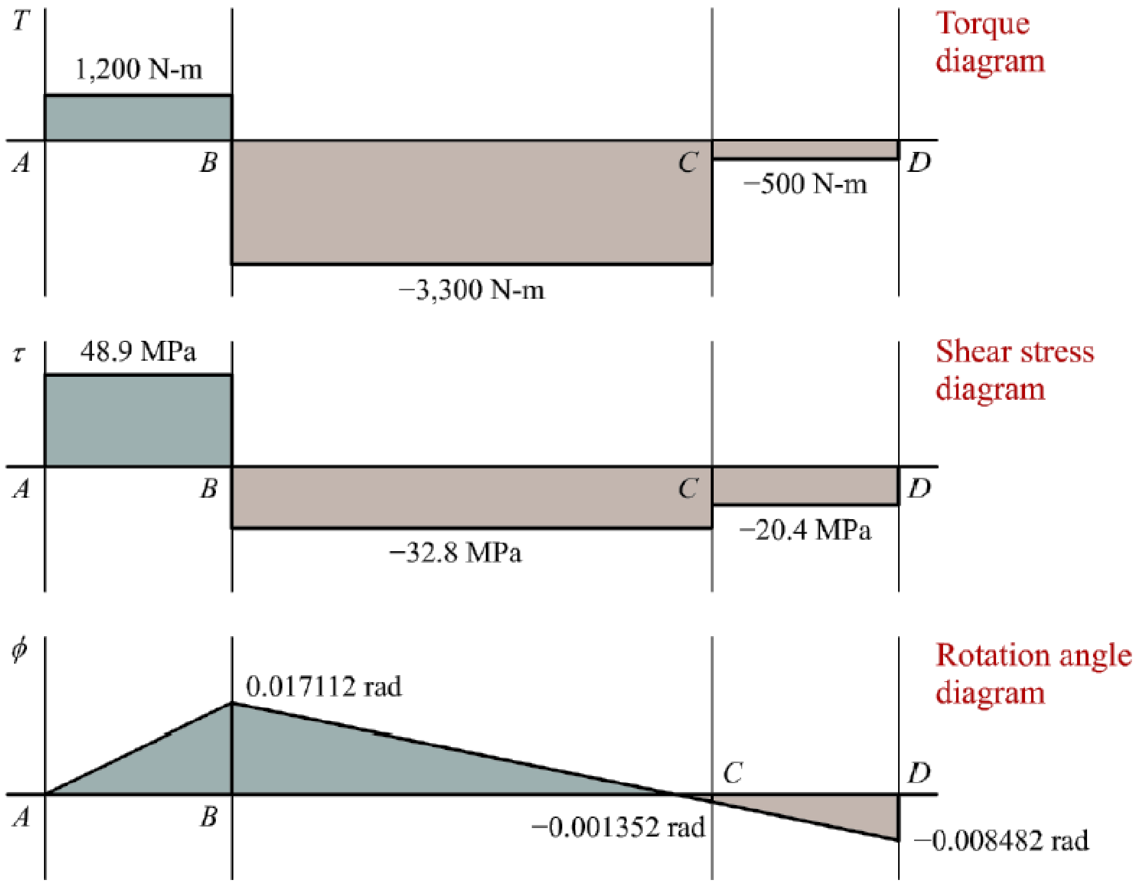
$$\phi_1 = \frac{T_1 L_1}{J_1 G_1} = \frac{(1{,}200 \text{ N-m})(0.7 \text{ m})(1{,}000 \text{ mm/m})^2}{(613{,}592.32 \text{ mm}^4)(80{,}000 \text{ N/mm}^2)} = 0.017112 \text{ rad}$$

$$\phi_2 = \frac{T_2 L_2}{J_2 G_2} = \frac{(-3{,}300 \text{ N-m})(1.8 \text{ m})(1{,}000 \text{ mm/m})^2}{(4{,}021{,}238.60 \text{ mm}^4)(80{,}000 \text{ N/mm}^2)} = -0.018464 \text{ rad}$$

$$\phi_3 = \frac{T_3 L_3}{J_3 G_3} = \frac{(-500 \text{ N-m})(0.7 \text{ m})(1{,}000 \text{ mm/m})^2}{(613{,}592.32 \text{ mm}^4)(80{,}000 \text{ N/mm}^2)} = -0.007130 \text{ rad}$$

(b) Rotation angles:

$$\phi_A = 0 \text{ rad}$$
$$\phi_B = \phi_A + \phi_1 = 0 \text{ rad} + 0.017112 \text{ rad} = 0.017112 \text{ rad}$$
$$\phi_C = \phi_B + \phi_2 = 0.017112 \text{ rad} + (-0.018464 \text{ rad}) = -0.001352 \text{ rad}$$
$$\phi_D = \phi_C + \phi_3 = -0.001352 \text{ rad} + (-0.007130 \text{ rad}) = -0.008482 \text{ rad}$$


T
1,200 N-m
A
B
C
D
−500 N-m
−3,300 N-m
Torque diagram
τ
48.9 MPa
−32.8 MPa
−20.4 MPa
Shear stress diagram
ϕ
0.017112 rad
−0.001352 rad
−0.008482 rad
Rotation angle diagram