CVG2140 – Solutions to Assignment No. 5 (Flexural Stress)

Q1. A beam having a tee-shaped cross section is subjected to equal 12 kN-m bending moments, as shown in Figure 1a. The cross-sectional dimensions of the beam are shown in Figure 1b. Determine:

(a) The centroid location, the moment of inertia about the z axis, and the controlling section modulus about the z axis.
(b) The bending stress at point H. State whether the normal stress at H is tension or compression.
(c) The maximum bending stress produced in the cross section. State whether the stress is tension or compression

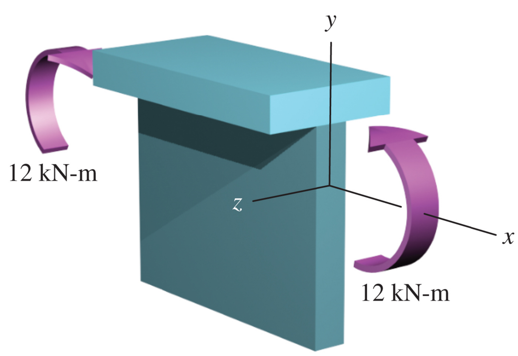


Figure 1a

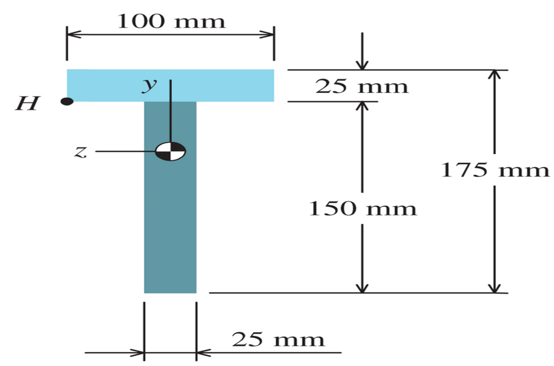


Figure 1b

Solution

(a) Centroid location in *y* direction: (reference axis at bottom of tee shape)

Shape	Area Ai	*yi* (from bottom)	*yi Ai*
	(mm2)	(mm)	(mm3)
top flange	2,500.0	162.5	406,250.0
stem	3,750.0	75.0	281,250.0
	6,250.0 mm2		687,500.0 mm3

$$\bar{y} = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{687{,}500.0 \text{ mm}^3}{6{,}250.0 \text{ mm}^2} = 110 \text{ mm}$$ (measured upward from bottom edge of stem)

Moment of inertia about the z axis:

Shape	*IC*	*d = yi – y*	*d²A*	*IC + d²A*
	(mm4)	(mm)	(mm4)	(mm4)
top flange	130,208.33	52.50	6,890,625.00	7,020,833.33
stem	7,031,250.00	−35.00	4,593,750.00	11,625,000.00

Moment of inertia about the *z* axis (mm^4) = 18,645,833.33

I_z = 18,646,000 mm^4

Section moduli:

$$S_{top} = \frac{I_z}{c_{top}} = \frac{18{,}645{,}833.33 \text{ mm}^4}{(175 \text{ mm} - 110 \text{ mm})} = 286{,}858.974 \text{ mm}^3$$

$$S_{bot} = \frac{I_z}{c_{bot}} = \frac{18{,}645{,}833.33 \text{ mm}^4}{110 \text{ mm}} = 169{,}507.576 \text{ mm}^3$$

$S = 169{,}500 \text{ mm}^3$

(b) Bending stress at point *H*: (y = 175 mm − 25 mm − 110 mm = 40 mm)

$$\sigma_x = -\frac{M_y}{I_z} = -\frac{(12 \text{ kN–m})(40 \text{ mm})\left(1{,}000\frac{\text{N}}{\text{kN}}\right)\left(1{,}000\frac{\text{mm}}{\text{m}}\right)}{18{,}654{,}833.33 \text{ mm}} = -25.743 \text{ MPa} = 25.743 \text{ MPa (C)}$$

(c) Maximum bending stress:

The maximum bending stress occurs at either the top or the bottom surface of the beam. The top of the cross section is at y = +65 mm, and the bottom of the cross section is at y = −110 mm. The larger bending stress magnitude occurs at the larger magnitude of these two values; in this case, at the bottom of the cross section.

$$\sigma_x = -\frac{M_y}{I_z} = -\frac{(12 \text{ kN–m})(\ 110 \text{ mm})(1{,}000 \text{ N/kN})(1{,}000 \text{ mm/m})}{18{,}654{,}833.33 \text{ mm}} = 70.793 \text{ MPa (T)}$$

Q2. A W360 × 72 standard steel shape is used to support the loads shown on the beam in Figure 2a. The shape is oriented so that bending occurs about the weak axis as shown in Figure 2b. Consider the entire 6-m length of the beam and determine:

(a) the maximum tension bending stress at any location along the beam
(b) the maximum compression bending stress at any location along the beam

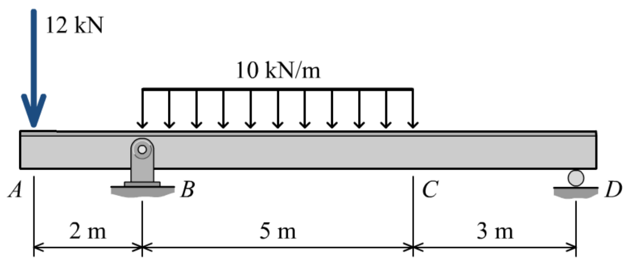


Figure 2a

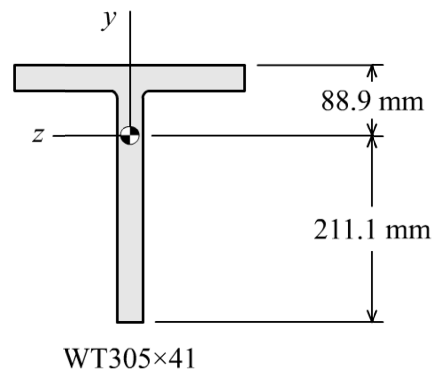
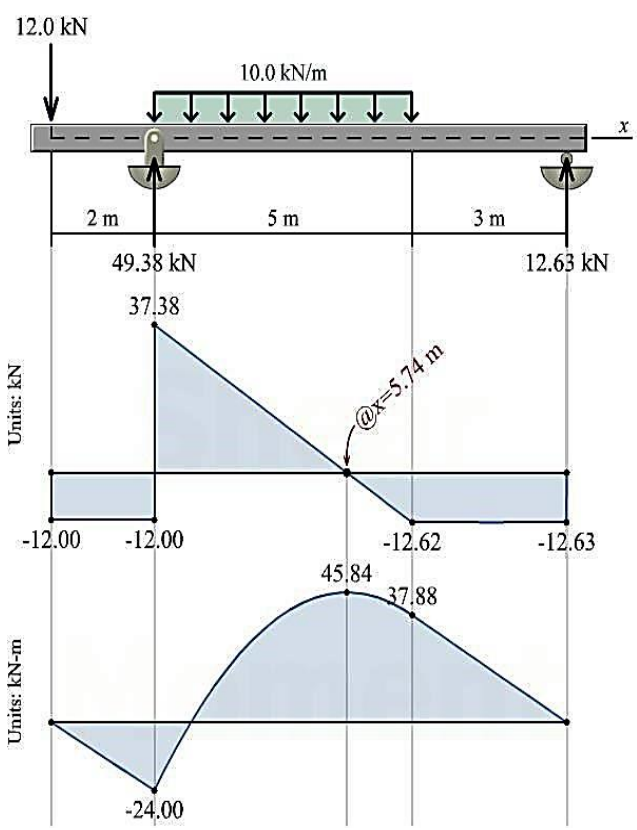


Figure 2b

Solution (I_z =48.7 ×10^6 mm^4)

Shear-force and bending-moment diagrams



Maximum bending moments

positive $M = 45.84$ kN-m
negative $M = -24.00$ kN-m

Bending stresses at max positive moment

$$\sigma_x = -\frac{(45.84 \text{ kN-m})(88.9 \text{ mm})(1{,}000)^2}{48.7\times10^6 \text{ mm}^4}$$
$$= 83.7 \text{ MPa (C)}$$

$$\sigma_x = -\frac{(45.84 \text{ kN-m})(-211.1 \text{ mm})(1{,}000)^2}{48.7\times10^6 \text{ mm}^4}$$
$$= 198.7 \text{ MPa (T)}$$

Bending stresses at max negative moment

$$\sigma_x = -\frac{(-24 \text{ kN-m})(88.9 \text{ mm})(1{,}000)^2}{48.7\times10^6 \text{ mm}^4}$$
$$= 43.8 \text{ MPa (T)}$$

$$\sigma_x = -\frac{(-24 \text{ kN-m})(-211.1 \text{ mm})(1{,}000)^2}{48.7\times10^6 \text{ mm}^4}$$
$$= 104.0 \text{ MPa (C)}$$

(a) Maximum tension bending stress 198.7 MPa (T)
(b) Maximum compression bending stress 104.0 MPa (C)

Q3. A simply supported wood beam (Figure 3a) with a span of L = 15 ft supports a uniformly distributed load of w0 = 320 lb/ft. The allowable bending stress of the wood is 1,200 psi. If the aspect ratio of the solid rectangular wood beam is specified as h/b = 2.0 (Figure 3b), calculate the minimum width b that can be used for the beam.

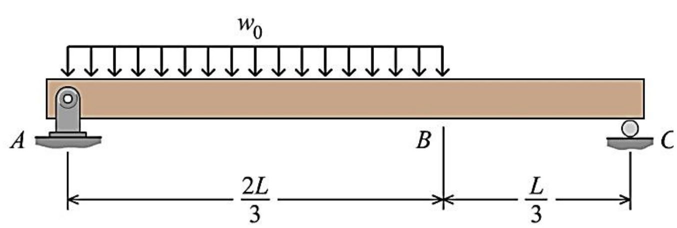


Figure 3a

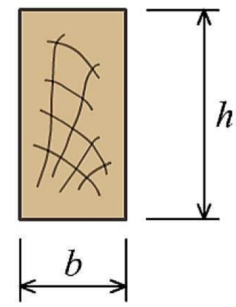
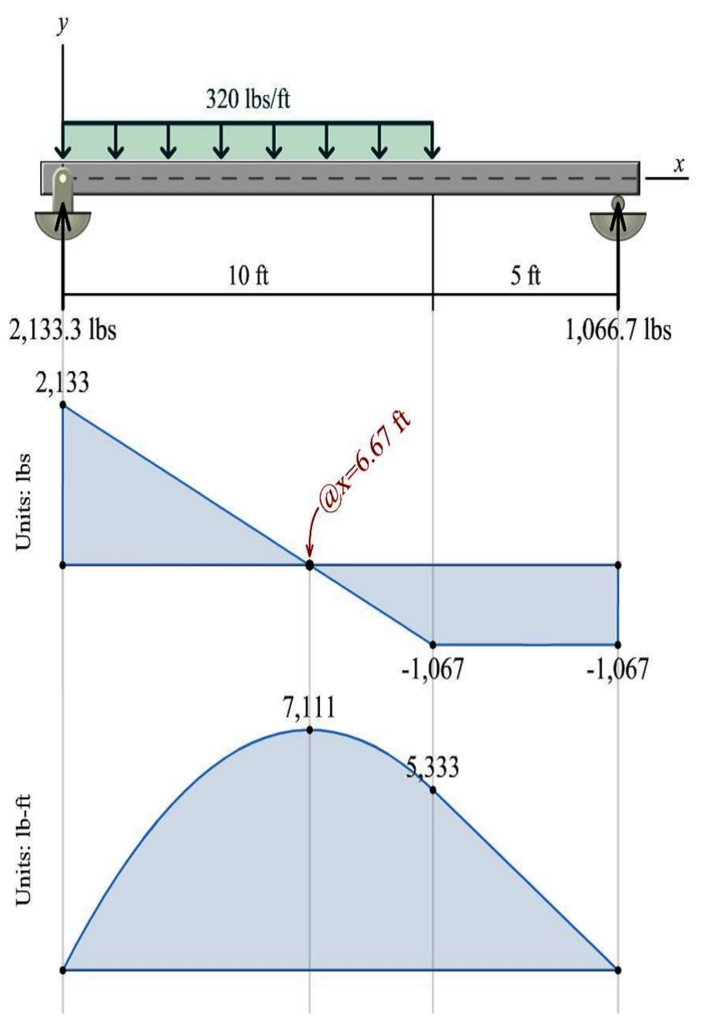


Figure 3b

Solution

Shear-force and bending-moment diagrams



Maximum bending moment magnitude

$$M = 7{,}111 \text{ lb-ft} = 85{,}332 \text{ lb-in.}$$

Minimum required section modulus

$$\sigma_x \geq \frac{M}{S}$$

$$\therefore S \geq \frac{M}{\sigma_x}$$

$$\geq \frac{85{,}332 \text{ lb-in.}}{1{,}200 \text{ psi}} = 71.110 \text{ in.}^3$$

Section modulus for solid rectangular section

$$S = \frac{I}{c} = \frac{bh^3/12}{h/2} = \frac{bh^2}{6}$$

The aspect ratio of the solid rectangular wood beam is specified as h/b = 2.0; therefore, the section modulus can be expressed as:

$$s = \frac{bh^2}{6} = \frac{b(2b)^2}{6} = 0.6667b^3$$

Minimum allowable beam width :

$0.6667b^3 > 71.110 \text{ in}^3$

b= 4.74 in

h=2b=2*(4.74 in) = 9.48 in