CHG 2314
Heat Transfer Operations
Winter 2011

Assignment 6 Solution

Professor: Dr. Boguslaw Kruczek **February 17, 2011**

DATE DUE: March 9, 2011 at 4:00 p.m. in the Assignment Box

Problem 1

Answer the following questions and draw a diagram whenever necessary:

a) *What is the definition of the Fourier number? What is the physical interpretation of the Fourier number?*

$Fo = \frac{\alpha t}{L_c^2}$,

where:

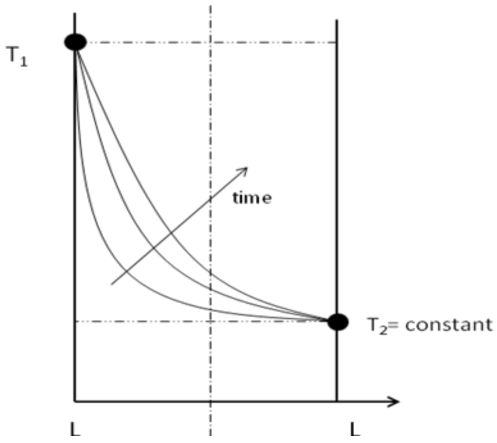
α - thermal conductivity, m^2/s
t - characteristic time, s
L_c - characteristic length, m

Fourier number is a dimensionless number that characterizes thermal response in heat conduction. In other words, it is the ratio of the heat conduction rate to the rate of thermal energy stored.

b) *What is a difference between plane wall and semi-infinite solid? Can the plane wall be treated as a semi-infinite solid? In what circumstances?*

Plane wall has a defined thickness (L), whereas the semi-infinite solid extends to infinity. Thus, for plane wall two exact boundary conditions exist and for semi-infinite solid only one. The second boundary is set at “an infinite distance” from the surface.

Plane wall can be treated as a semi-infinite solid for as long as the conditions on the other side of the wall are not affected by whatever changes occurred at the opposite side. Consider example below, where temperature on the right side changes from T_2 to T_1:



c) *Consider spheres A, B, and C, which are initially at uniform temperature T_1 and are suddenly immersed in the new environment at temperature T_2.*

i. *If the spheres had the same diameter but were made from different materials (A = Aluminum Alloy 2024-T6, B = Constantan (Copper Alloy), C = Carbon Steel AISI 1010), which sphere would first reach the new thermal equilibrium?*

From the Appendix A:
$\alpha_A = 73 \times 10^{-6}\ m^2/s$
$\alpha_B = 6.7 \times 10^{-6}\ m^2/s$
$\alpha_C = 18.8 \times 10^{-6}\ m^2/s$

Recall that: $Fo = \dfrac{\alpha}{R^2} t$. Therefore, since the spheres have the same diameter, at any time: $Fo_c = 2.8\ Fo_B = 0.26\ Fo_A$. The spheres will reach the new thermal equilibrium according the numerical values of the Fourier number, i.e., the first sphere to reach the new thermal equilibrium will be A, then C and then B.

ii. *Would your answer from part i) change if the temperature of the new environment for each sphere were different, i.e., $(T_1 - T_{2A}) = 2(T_1 - T_{2B}) = 4(T_1 - T_{2C})$, while the corresponding convective heat transfer coefficients were the same?*

No, the answer would be the same provided that we have constant properties. The new thermal equilibrium is reached when:

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} \to 0$$

Regardless of the value of the denominator, which are different for different spheres, the nominator changes accordingly.

iii. *If the spheres had the same diameter but were made from different materials (A = Aluminum Alloy 2024-T6, B = Constantan (Copper Alloy), C = Carbon Steel AISI 1010), and the convective coefficient in the new thermal environment were also different ($h_A = 10\ W/m^2\ K$, $h_B = 100\ W/m^2\ K$, $h_C = 1000\ W/m^2\ K$), which sphere would first reach the new thermal equilibrium?*

From the Appendix A:
$k_A = 177\ W/mK$
$k_B = 23\ W/mK$
$k_C = 63.9\ W/mK$

Then

$$Bi_A = \frac{hr}{k} = \frac{10}{177} r = 0.056r;\ Bi_B = \frac{100}{23} r = 4.3r;\ and\quad Bi_C = \frac{1000}{63.9} r = 15.6r$$

Consequently: $Bi_C \approx 3.6\ Bi_B \approx 279\ Bi_A$

At the same time, as already noted, at any time: $Fo_c = 2.8\ Fo_B = 0.26\ Fo_A$.

The fastest way to answer this question is to use Heisler charts for sphere. We can use either the chart for the center temperature (Fig. D.7) or the chart for internal energy change (Fig. D.9). In the former case the sphere having the minimum θ_o^* will reach first the new thermal equilibrium, in the latter the sphere having the maximum Q/Q_o will reach first the new thermal equilibrium. Consider the latter case, that is, let us compare Q/Q_o values for the three spheres.

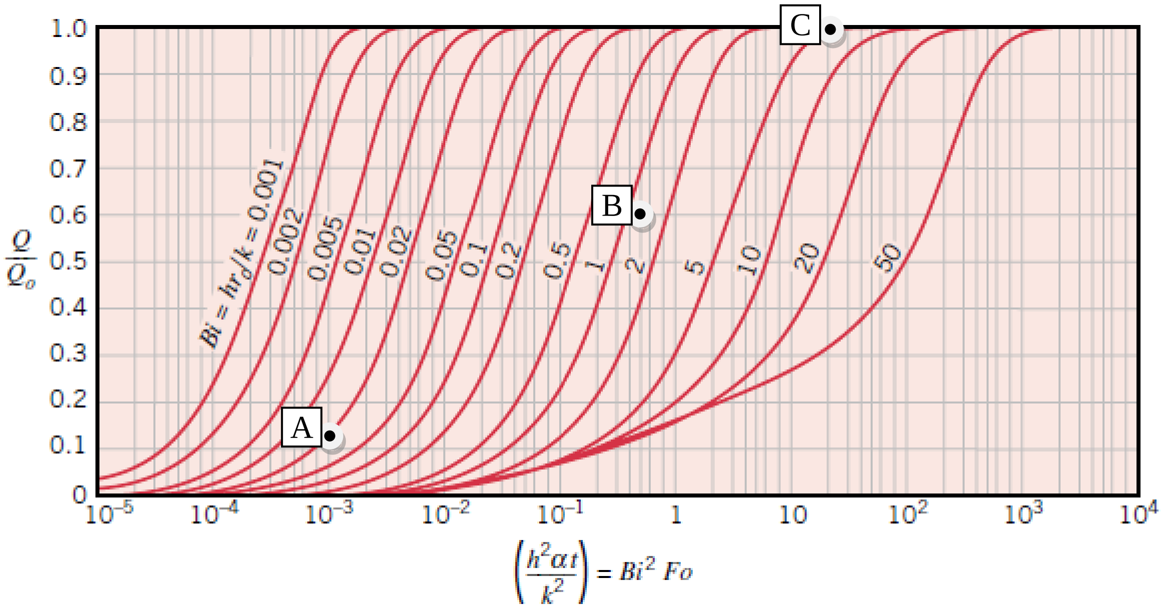


FIGURE 5S.9 Internal energy change as a function of time for a sphere of radius r_o [2]. Adapted with permission.

Now, the criterion of reaching the new thermal equilibrium is no longer Fourier number but the product of square of Biot number and Fourier number. For the spheres in different thermal environments, at any given time we have:

$(Bi^2Fo)_C \approx 37(Bi^2Fo)_B \approx 20{,}000(Bi^2Fo)_A$
At the same time: $Bi_C \approx 3.6\ Bi_B \approx 279\ Bi_A$

In the above Figure let set $(Bi^2Fo)_A = 10^{-3}$ and $Bi_A = 0.02$ (these are arbitrary values), the corresponding $(Q/Q_o)_A \approx 0.12$ (Point A). Consequently, the corresponding $(Bi^2Fo)_B \approx 5.4\ \text{x}10^{-1}$ and $Bi_B \approx 1.5$ from which $(Q/Q_o)_B \approx 0.6$ (Point B), and the corresponding $(Bi^2Fo)_C \approx 20$ and $Bi_C \approx 5.6$ from which $(Q/Q_o)_C \approx 1.0$ (Point C)

Therefore, the new thermal equilibrium will be reach in the following order: C followed by sphere B and sphere A

d) *Consider a plane wall of thickness 2L, which is initially at temperature T_1. Suddenly, the temperature of one surface of the wall is decreased from T_1 to T_2 while the other surface is perfectly insulated.*

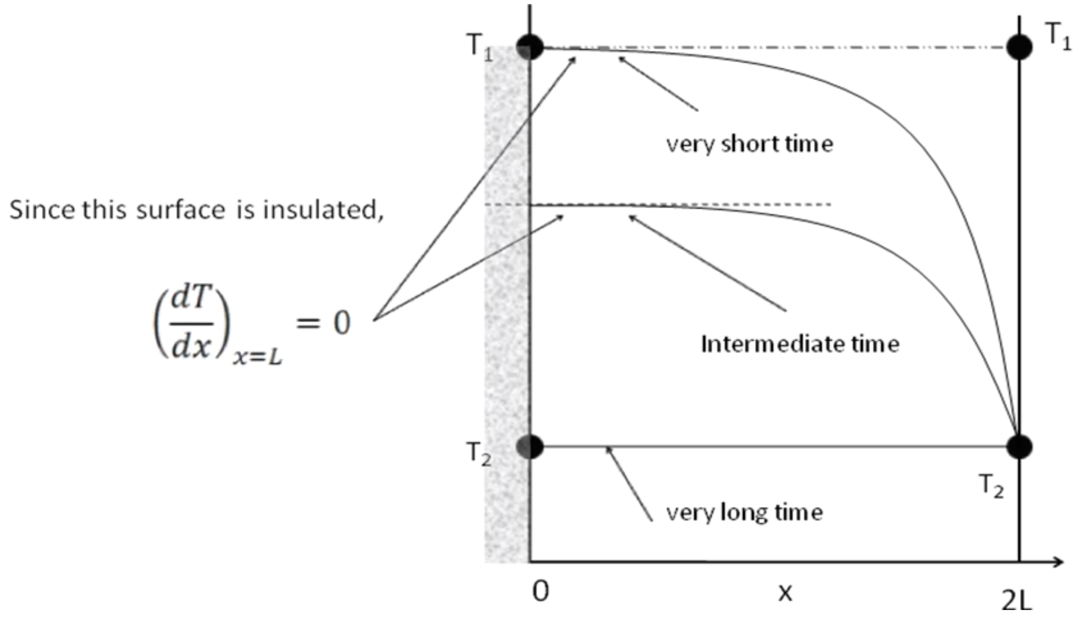
i. *What is the characteristic length for heat conduction in this plane wall?*

Because the temperature profile is not symmetrical in the wall, $L_c = 2L$;

ii. *How is it possible to increase the surface temperature instantaneously to a new constant value? What is the corresponding Bi number?*

By exposing the surface to a fluid with high convective coefficients, $Bi \rightarrow \infty$

iii. *On a single diagram, draw the temperature profiles within the wall after short, intermediate and long time.*



iv. *If T_1 = 80°C, T_2 = 20°C, what is the average wall temperature at the time when the fractional energy loss by the wall is 0.6? Please note that the fractional energy gain is defined as: $\Phi = Q/Q_o$, where Q is the total energy transfer from the plane wall at time t and Q_o is the maximum energy transfer from the plane wall when it reaches a new equilibrium state.*

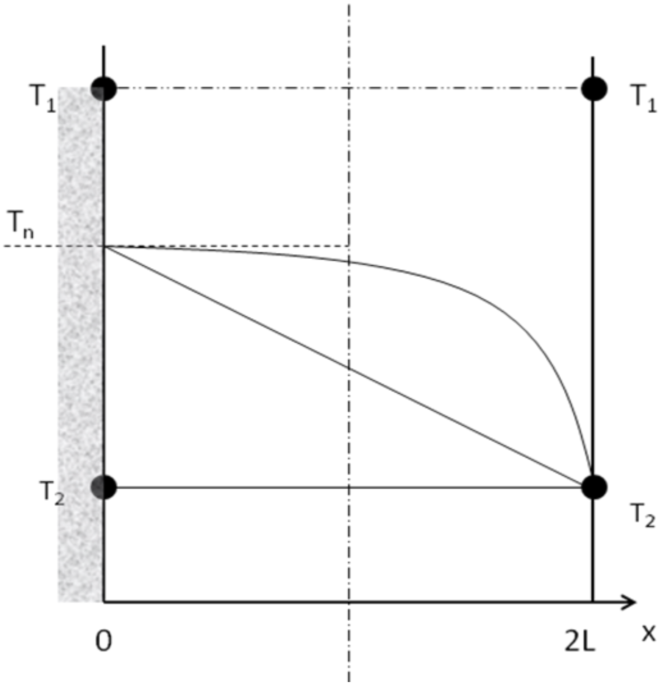
$Q = mc_p(\bar{T} - T_i)$ *and* $Q_0 = mc_p(T_\infty - T_i)$

Where $\bar{T}$ - average temperature at time t, T_i– initial temperature, 80°C and T_∞ - temperature of environment, 20 °C. Then:

$$\frac{Q}{Q_0} = \frac{mc_p(\bar{T} - T_i)}{mc_p(T_\infty - T_i)} = \frac{\bar{T} - T_i}{T_\infty - T_i} = 0.6 = \frac{\bar{T} - 80}{20 - 80}$$

$$\bar{T} = 80 + 0.6(20 - 80) = \mathbf{44°C}$$

v. *Referring to part iv), underline the temperature that represents the closest approximation of the corresponding centre plane temperature: 67°C, 70°C, 73°C. Justify your answer.*



From the Figure above, T_n represents temperature at x = L at certain time when $Q/Q_o = 0.6$. Arithmetic temperature average ($T_{ar.average}$) in the wall can be represented in the following way: it is the area below the straight line that connects T_2 and T_n:

$$T_{ar.average} = \frac{T_2 + T_n}{2}$$

And it has to be less than the actual average temperature which is the area below the curve line that connects T_2 and T_n:

$$\frac{T_2 + T_n}{2} < \bar{T}$$

From above, $T_n < 2\bar{T} - T_2 < 68°\text{C}$

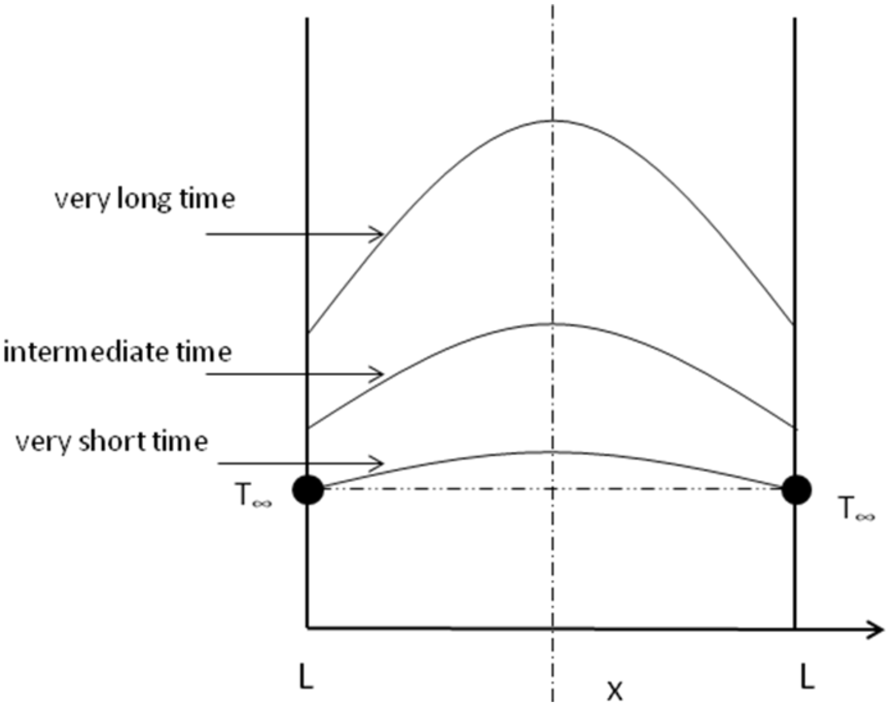
Since $T_n < 68°\text{C}$, the center plane temperature must be lower than 68°C and is closer to 67°C than to any of the other proposed temperatures.

e) *Consider a plane wall of thickness 2L, which is initially in equilibrium with ambient fluid at T_∞. Suddenly electrical current starts to flow through the wall resulting in uniform heat generation within the wall. The heat generated by the flow of the electrical current is* *dissipated from the wall surfaces by convection characterized by a constant convective heat transfer coefficient.*

i. *What is the characteristic length for heat conduction in this plane wall?*

The system is symmetrical, therefore $L_c = L$;

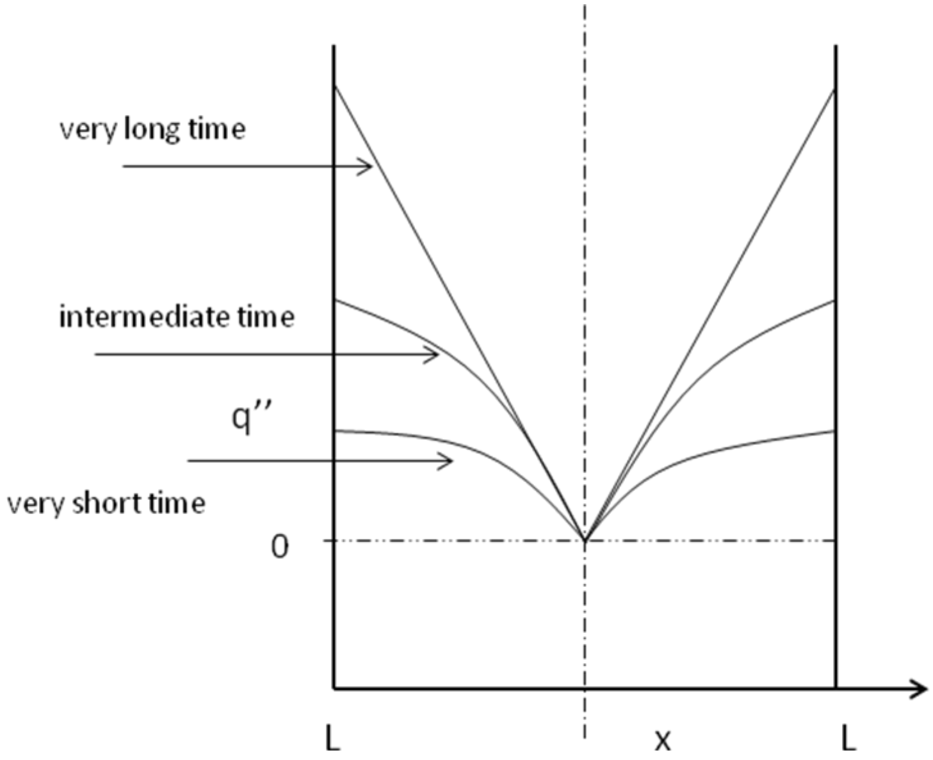
ii. *On a single diagram, draw the temperature profiles within the wall after short, intermediate and long times.*



The temperature increases with time, because not all heat generated is removed from the solid. This implies, as time increases the heat flux at the surface increases (the slope at each surface increases). As a result, we start with flatten parabolic curve, which eventually becomes the true parabola at steady state.

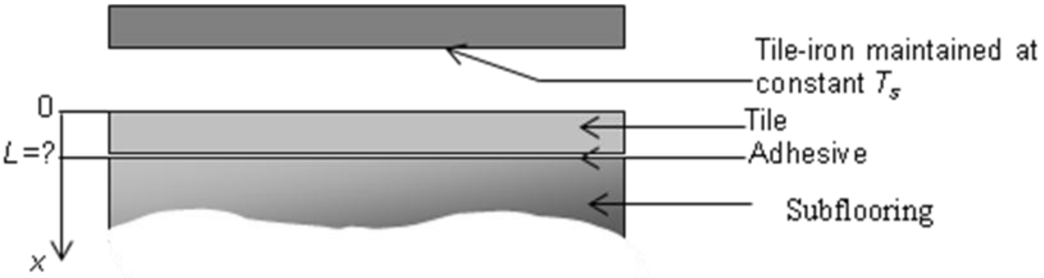
iii. *On a single diagram, draw the heat flux as a function of the position within the wall after short, intermediate and long times.*

At steady state heat flux increases linearly from zero at the mid-plane to the maximum value at the surface. At short and intermediate times, heat flux at the mid-plane is also zero and the heat flux increases as we approach the surface. This increase in flux as we approach the surface is not linear at the transient state. The rate of flux increase decreases as we approach the surface. The heat flux curves result from the shape of temperature profiles in part (ii) We just need to remember that: $q'' = -k\frac{dT}{dx}$


very long time
intermediate time
q''
very short time
0
L
x
L

Problem 2

A tile-iron consists of a massive plate maintained at 150^{o}C by an imbedded electrical heater. The iron is placed in contact with a tile to soften the adhesive, allowing the tile to be easily lifted from the subflooring. The adhesive will soften sufficiently if heated above 50^{o}C for at least 2 min, but its temperature should not exceed 90^{o}C to avoid deterioration of the adhesive. Assume the tile and subflooring to have an initial temperature of 25^{o}C and to have equivalent thermophysical properties of k = 0.15 W/m K and $\rho c = 1.5 \times 10^6$ J/m^3 K. If there is no contact resistance between the plate of the tile-iron and the tile and $(k\rho c)_{iron\ plate} >> (k\rho c)_{tile}$, what should be the minimum thickness of the tile for which this tile-iron could be utilized without damaging the adhesive? For the calculated minimum thickness, how long would it take to lift the tile using the tile-iron? What is the corresponding energy to lift the tile?



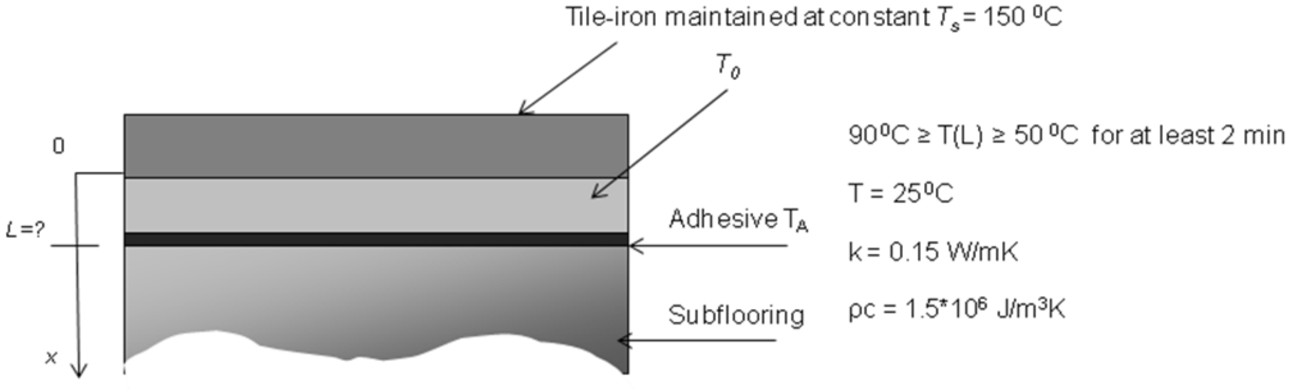
Known:

1. Surface temperature of a tile used to soften adhesive underneath tiles in order to lift tiles.
2. Thickness and properties of tiles and subflooring.

Unknown:

1. Minimum thickness of tiles to meet the requirements for softing of the adhesive while without damaging it.
2. The required time to lift the tile of calculated minimum thickness.
3. Energy required to lift the tile.

Schematic:



Assumptions:

1. 1-D conduction along x-tile, adhesive and subflooring represents a system which can be treated as semi-infinite solid;
2. No contact resistance between tile-iron and tile so that surface temperature of tile becomes 150^{o}C instantaneously upon the contact with the tile-iron.
3. Adhesive does not impose any resistance to heat transfer or the properties of adhesive are the same as those of the tile and subflooring.
4. Constant properties of tile and subflooring.

Analysis:

(1 and 2) Due to assumptions 1 and 2, the tile can be treated as semi-infinite solid subjected to sudden change in surface temperature. In addition, because of assumptions 3 and 4 and because

$(k\rho c)_{\text{iron plate}} >> (k\rho c)_{\text{tile}}$, the temperature response in the system consisting of the tile, adhesive and subflooring is given by Eq (5.57)

$$\frac{T - T_s}{T_i - T_s} = erf\left(\frac{x}{(4\alpha t)^{1/2}}\right)$$

Where T is T(t,x) , $\alpha = \frac{k}{\rho c} = \frac{0.15}{1.5*10^6} = 1*10^{-7}\ m^2/s$, T_i = 25 ^{0}C, T_s= 150 ^{0}C

Using the specifications for softening of adhesive without damaging it, we can write the following equations:

$$\frac{T(t, L) - T_s}{T_i - T_s} = erf\left(\frac{L}{(4\alpha t)^{1/2}}\right)$$

$$\frac{T(t+2, L) - T_s}{T_i - T_s} = erf\left(\frac{L}{(4\alpha (t+2))^{1/2}}\right)$$

Where $T(t, L)$ = 50 ^{0}C and $T(t+2, L)$ = 90 ^{0}C

We have a system of two equations and two unknowns L and t, solving the equations:

$$\frac{50 - 150}{25 - 150} = 0.8 = erf\left(\frac{L}{(4\alpha t)^{1/2}}\right)$$

$$\frac{90 - 150}{25 - 150} = 0.48 = erf\left(\frac{L}{(4\alpha (t+2))^{1/2}}\right)$$

From the table B2,

$\mathrm{erf}(w) = 0.8$, using interpolation: slope = (0.80677 - 0.78669)/(0.92 - 0.88) = 0.502
Slope = 0.502 = (0.8 - 0.78669)/(w - 0.88)
w_1 = 0.88 + (0.8-0.78669)/0.502 = **0.9065**
$\mathrm{erf}(w) = 0.48$, again using interpolation: **w_2 = 0.4550**

Then:

$$w_1 = \left(\frac{L}{(4\alpha t)^{1/2}}\right) \qquad (1)$$
$$w_2 = \left(\frac{L}{(4\alpha(t+2))^{1/2}}\right) \qquad (2)$$

Combining and rearranging (1) and (2):

$$L = w_1(4\alpha t)^{1/2} = w_2(4\alpha(t+2))^{1/2}$$

$$w_1 t^{1/2} = w_2(t+2)^{1/2}$$

$$\left(\frac{t}{t+2}\right)^{1/2} = \frac{w_2}{w_1}$$

$$\frac{t}{t+2} = \left(\frac{w_2}{w_1}\right)^2$$

$$t = \frac{2\left(\frac{w_2}{w_1}\right)^2}{1-\left(\frac{w_2}{w_1}\right)^2} = \frac{2\left(\frac{0.4550}{0.9065}\right)^2}{1-\left(\frac{0.4550}{0.9065}\right)^2} = 0.674\,\mathrm{min} = 60 * 0.674 = \mathbf{40.4\ s};$$

$$L = w_1(4\alpha t)^{1/2} = 0.9065 * \left(4 * 1 * 10^{-7}\frac{m^2}{s} * 40.4\ s\right)^{1/2} = 0.0036\ \mathrm{m} = \mathbf{3.6\ mm}$$

(3) The total energy per unit area when T_s is 150 ^{0}C is

$$Q'' = Q/A = \int_0^t q_s dt = \int_0^t \frac{k(T_s - T_i)}{(\pi\alpha t)^{1/2}} dt = \frac{k(T_s - T_i)}{(\pi\alpha)^{1/2}} \cdot 2t^{1/2}$$

For T_s = 150 ^{0}C and t = 160.4 s

$$Q'' = \frac{k(T_s - T_i)}{(\pi\alpha)^{1/2}} \cdot 2t^{1/2} = \frac{0.15(150-25)}{(\pi \times 1.0 \times 10^{-7})^{1/2}} \cdot 2 \cdot 160.4^{1/2} = \mathbf{847\ kJ}$$

Problem 3

Recalling that your mother once said that meat should be cooked until every portion has attained a temperature of 80°C, how long will it take to cook a 2.25-kg roast? Assume that the meat is initially at 6°C and that the oven temperature is 175°C with a convection heat transfer coefficient of 15 W/m^2 K. Treat the roast as a cylinder with properties of liquid water, having a diameter equal to its length.

What is the corresponding maximum temperature of the roast?

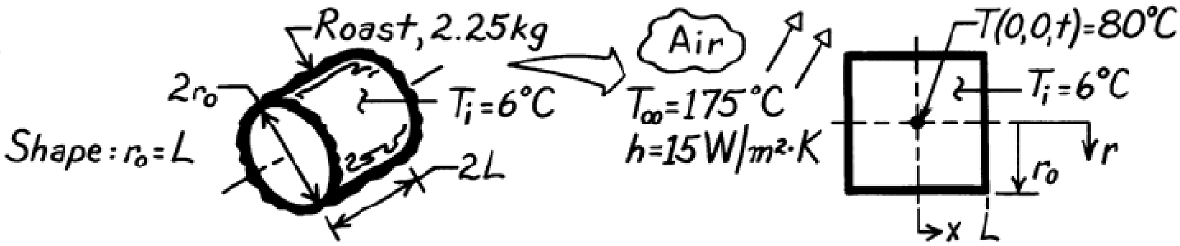
Known: Cylindrical-shaped meat roast weighing 2.25 kg, initially at 6 ^{0}C, is placed in an oven and subjected to convection heating o with prescribed (T_∞, h).

Find: 1. Time required for the center to reach a done temperature of 80 ^{0}C.
2. Maximum temperature of the roast?

Assumptions:

1. Two-dimensional conduction in x and r directions,
2. Uniform and constant properties,
3. Properties approximated as those of water.

Schematic:



Properties: From Table A-6, Water, liquid ($\bar{T}$ = (80 + 6) ^{0}C /2 ≈ 315K):
$\rho = 1/v_f = 1/1.009 \times 10^{-3}$ m^3/kg = 991.1 kg/m^3,
$c_{p,f}$ = 4179 J/kg K,
k = 0.634 W/m K,
$\alpha = k/\rho c = 1.531 \times 10^{-7}$m^2/s.

Analysis: The dimensions of the roast are determined from the requirement $r_o = L$ and knowledge of its weight and density,

$$M = \rho V = \rho 2L \cdot \pi r_0^2\ ,\ \ then\ r_0 = L = \left[\frac{M}{2\pi\rho}\right]^{1/3} = \left[\frac{2.25\ kg}{2\pi \cdot 991.1kg/m^3}\right]^{1/3} = 0.0712m$$

1. From the Supplemental Material (Figure 5S.11, Case (i))

$$\frac{T(x,r,t)-T_\infty}{T_i-T_\infty}=P(x.t)\times C(r,t)$$

Where $P(x,t)=[C_1\exp(-\zeta_1^2Fo)]_{wall}\ \ and\ \ C(r,t)=[C_1\exp(-\zeta_1^2Fo)]_{cylinder}$

For both shapes,

$$Bi=\frac{hr_0}{k}=\frac{hL}{k}=\frac{15W/m^2\times 0.0712m}{0.634\ W/m\cdot K}=1.68$$

$$Fo=\alpha\ t/r_0^2=\alpha\ t/L^2=1.53\times10^{-7}m^2/s\times t/(0.0712m)^2=3.020\times10^{-5}t$$

From the Table 5.1, based on the Bi = 1.68,

$Wall$: using interpolation, $\zeta_1=0.8603+\frac{(1.68-1)*(1.0769-0.8603)}{(2-1)}=1.0076$

Similarly, $C_1=1.1594$

$Cylinder$: $\zeta_1=1.4894\ \ and\ C_1=1.2964$

Then:

$$P(x,t)=1.1594\times\exp(-1.0076^2\times3.020\times10^{-5}t)=1.1594\times\exp\left(-3.066\times10^{-5}t\right)$$

$$C(r,t)=1.2964\times\exp(-1.4894^{\ 2}\times3.020\times10^{-5}t)=1.2964\times\exp\left(-6.6997\times10^{-5}t\right)$$

For the center of the cylinder,

$$\frac{T(x,r,t)-T_\infty}{T_i-T_\infty}=\frac{T(0,0,t)-T_\infty}{T_i-T_\infty}=\frac{(80-175)°\mathrm{C}}{(6-175)°\mathrm{C}}=0.56$$

The final equation to solve is:

$$0.56=P(x,t)C(r,t)=1.1594\times\exp\left(-3.066\times10^{-5}t\right)\times1.2964\times\exp\left(-6.6997\times10^{-5}t\right)$$

$$\begin{aligned}0.56&=1.503\times\exp(-3.066\times10^{-5}t)\times\exp(-6.6997\times10^{-5}t)\\&=1.503\times\exp(-3.066\times10^{-5}t-6.6997\times10^{-5}t)\end{aligned}$$

$$0.3726=\exp\left(-9.7657\times10^{-5}t\right)$$

Taking natural logarithm of both sides:

$$\begin{aligned}&\ln\left(0.3726\right)=ln(\exp(-9.7657\times10^{-5}t))\\&-0.98725=-9.7657\times10^{-5}t\\&t=10109\ s=168.5\ min=\mathbf{2.8\ hours}\end{aligned}$$

2. The maximum temperature will be at the edge of the roast:

T(L,r_0, 2.8h), or:

$$\frac{T(L,r_0,t)-T_\infty}{T_i-T_\infty}=\left.\frac{T(L,t)-T_\infty}{T_i-T_\infty}\right|_{Plane\ Wall}\cdot\left.\frac{T(r_0,t)-T_\infty}{T_i-T_\infty}\right|_{Cylinder}$$

Plane Wall (eq.5.40b):

$$\left.\frac{T(L,t)-T_\infty}{T_i-T_\infty}\right|_{Plane\ Wall}=\theta^*=\theta_0^*\cos\left(\zeta_1 x^*\right)$$

Where $x^*=\frac{x}{L}=1$ and

$$\theta_0^*=C_1\exp(-\zeta_1^2 Fo)=1.1594\times\exp(-3.066\times10^{-5}t)=1.1594\times\exp(-3.066\times10^{-5}\times10109\ s)=0.8504$$

Then

$$\left.\frac{T(L,t)-T_\infty}{T_i-T_\infty}\right|_{Plane\ Wall}=0.8504\times\cos(1.0076rad\times1)=0.4540$$

Cylinder (eq.5.49b):

$$\left.\frac{T(r_0,t)-T_\infty}{T_i-T_\infty}\right|_{Cylinder}=\theta_0^* J_0\left(\zeta_1 r^*\right)$$

Where $r^*=\frac{r}{r_0}=1$

From the Table B.4, the Bessel function is determined:

$$J_0\left(\zeta_1 r^*\right)=J_0(1.4894\times1)$$

Using interpolation,

$$J_0(1.4894)=0.5118+(0.5669-0.5118)*\frac{1.4894-1.4}{1.5-1.4}=0.5611$$

$$\theta_0^* = C_1 \exp(-\zeta_1^2 Fo) = 1.2964 \times \exp(-6.6997 \times 10^{-5} t)$$
$$= 1.2964 \times \exp\left(-6.6997 \times 10^{-5} \times 10109\ s\right) = 0.6586$$

$$\left.\frac{T(r_0,t) - T_\infty}{T_i - T_\infty}\right|_{Cylinder} = 0.6586 \times 0.5611 = 0.370$$

Overall:

$$\frac{T(L,r_0,t) - T_\infty}{T_i - T_\infty} = \left.\frac{T(L,t) - T_\infty}{T_i - T_\infty}\right|_{Plane\ Wall} \cdot \left.\frac{T(r_0,t) - T_\infty}{T_i - T_\infty}\right|_{Cylinder} = 0.4540 \times 0.370$$
$$= 0.131$$

Then:

$$T(L,r_0,2.8h) = T_\infty + 0.131(T_i - T_\infty) = 175 + 0.131 * (6 - 175) = \mathbf{153}\ ℃$$