CHG 2314
Heat Transfer Operations
Winter 2011

Assignment 5 Solutions

Professor: **Dr. Boguslaw Kruczek** **February 7, 2011**

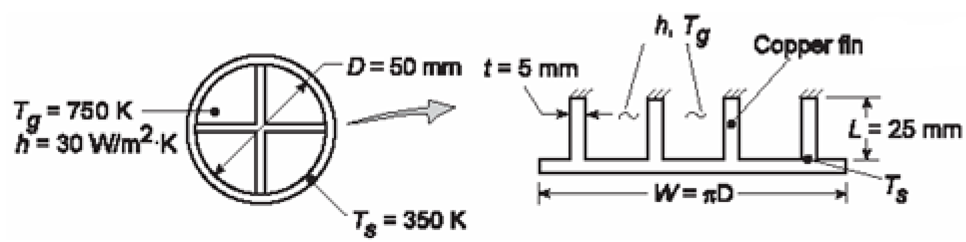
<u>DATE DUE: February 16, 2010 at 4:00 p.m. in the Assignment Box</u>

Problem 1

Known: Diameter and internal fin configuration of copper tubes emerged in water. Tube wall temperature and temperature and convection coefficient of gasses flow through the tube.

Find: i) System thermal circuit, ii) Individual fin efficiency, iii) efficiency of finned surface, iv) rate of heat transfer per unit length, v) repeat part (iv) without the presence of the fin.

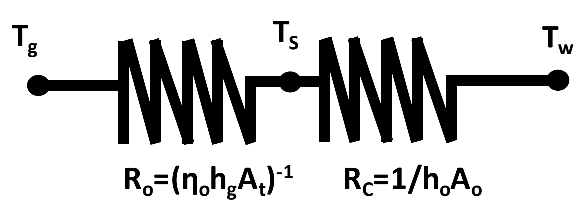
Schematic:



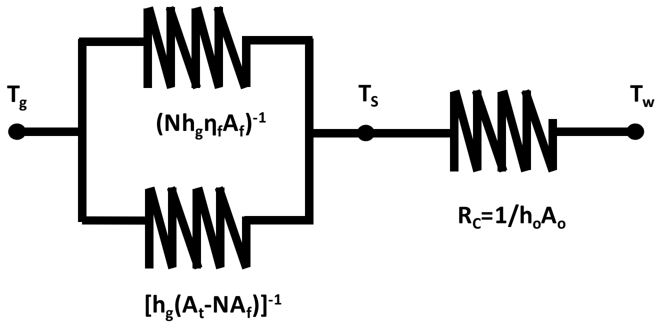
Assumptions:
- Steady State
- 1D Heat conduction
- Constant properties
- Negligible radiation
- Tube walls me be unfolded as illustrated on the diagram above.

Analysis:

i)



Or



ii)

$$mL_c = \left(\frac{h_g P}{kA_C}\right)^{\frac{1}{2}} L_c$$

(Fin perimeter can be estimated as 2w since w >> t)

$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

$$mL_c = \left(\frac{h_g(2w)}{k(wt)}\right)^{\frac{1}{2}} L_c$$

$$mL_c = \left(\frac{2h_g}{kt}\right)^{\frac{1}{2}} L_c$$

$L_c = L$ (Since it's an adiabatic tip)

$\eta_f = 0.990$ ← $mL_c = 0.137$

iii)

$$\eta_o = 1 - \frac{NA'_f}{A'_t}(1-n_f)$$

$$\eta_o = 1 - \frac{4(2L)}{0.337m}(1-0.9938)$$

$$\eta_o = 0.991$$

$$A'_t = NA'_f + A'_b$$

$$A'_t = NA'_f + (\pi D - 4t)$$

$$A'_t = 4(2L) + (\pi D - 4t)$$

$$A'_t = 0.337m$$

iv)

Based on thermal circuit from part (i):

$$A'_o = \pi D$$

$$q_f = \frac{T_g - T_o}{\frac{1}{\eta_o h_g A_t} + \frac{1}{h_o A_o}} = \frac{750K - 300K}{\frac{1}{(0.991)(30)(0.337)} + \frac{1}{(3000)(0.05\pi)}}$$

$$q_f = 4414.69W$$

v)

R_o from parts (i) and (iv) changes to:

$$R_o = \frac{1}{h_g A_o}$$

$$q_f = \frac{T_g - T_o}{\dfrac{1}{h_g A_o} + \dfrac{1}{h_o A_o}} = \frac{750\text{K} - 300\text{K}}{\dfrac{1}{(30)(0.05\pi)} + \dfrac{1}{(3000)(0.05\pi)}}$$

$$q_f = 2099.57\text{W}$$

Problem 2

a) What is an isotherm? What is an adiabat? How are the two lines related geometrically?
Ans:

- Isotherm: Line of constant temperature
- Adiabat: Line of heat flow
- Geometrically, the isotherm is perpendicular to the adiabat

b) What is a difference between the heat flow line and the adiabat? What is the difference between heat flow line and lane?
Ans:

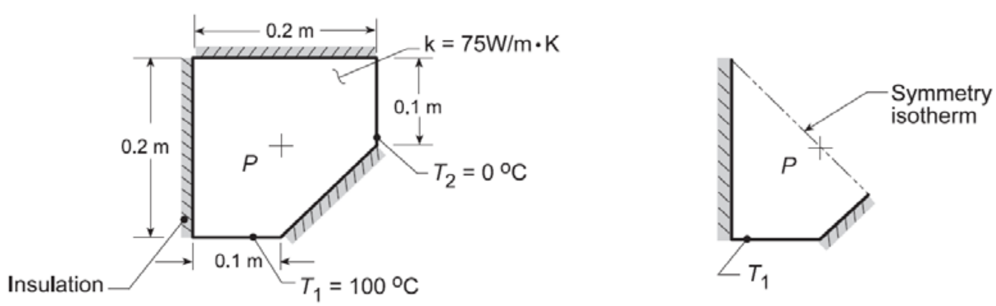
- Adiabat is a heat flow line
- Lane is a region between two adjoining adiabats

c) A supporting strut fabricated from a material with a thermal conductivity of 75 W/m K has the cross section shown. The end faces are at different temperatures T_1 = 100°C and T_2 = 0°C, while the remaining sides are insulated.
Ans:

Known: Structural member with known thermal conductivity subjected to a temperature difference.

Find: (i) Temperature at a prescribed point P, (ii) Estimate shape factor using heat flux method, (iii) Rate of heat transfer per unit length

Schematic:



Assumptions:

- Two dimension conduction
- Steady state conditions
- Constant properties

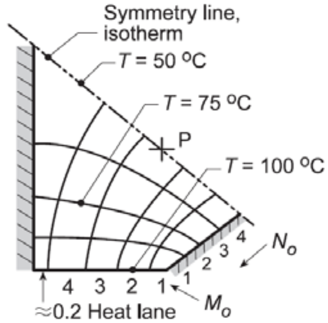
Analysis:

i) Symmetry isotherm indicated in the schematic above. T at P is obtained as follows:

$$T(P) = \frac{T_1 + T_2}{2} = \frac{100^o C + 0^o C}{2} = 50^o C$$

ii) The flux plot on the symmetrical section is now constructed to obtain the shape factor:

$$S = \frac{Ml}{N}$$



M = 4.2
N=4

Now take entire section:

M = 4
N = 2(4) = 8 (since it was halved when taking isolating a single symmetrical side)

$$S = \frac{4.2}{2(4)} l = 0.53 l$$

iii) The heat rate can easily be determined using the equation below:

$$q = kS(T_1 - T_2)$$
$$q = (75 W/mK)(0.53)(100^o C + 0^o C)$$
$$q = 3975 W/m$$

Problem 3

a) Under what conditions, may the lumped capacitance method be used to predict the transient response of a solid to a change in its thermal environment?
Ans:

It's when it's reasonable to assume that there is a uniform temperature distribution within the solid immersed in a fluid. When R_{cond} (with solid) << R_{conv}.

This is when Bi << 0.1 (where $Bi = \frac{R_{cond}}{R_{conv}} = \frac{hL}{k}$)

b) Is the lumped thermal capacitance method of analysis likely to be more applicable for cooling of a hot solid made of copper or aluminum? Please justify.
Ans:

The solid made of copper. Since copper has a higher thermal conductivity (k_{Al} = 237 W/mK vs. k_{Cu} = 401 W/mK), it would yield the lower resistance due to conduction, and hence a lower Biot number.

c) What is the physical interpretation of the Biot number?
Ans:

It provides a measure of the temperature drop in a solid relative to the temperature difference between the solid surface and the fluid.

d) What is the characteristic length for the purpose of evaluation of the Biot number of:

i. A plane wall of thickness 2*L* with both surfaces exposed to convection heating/cooling
Ans: $L_C = L$

ii. A cylindrical shell with inner and outer radii r_i and r_o whose inner surface is perfectly insulated and the outer surface is exposed to convective heating/cooling.
Ans: $L_C = r_o - r_i$

iii. A cube of edge *L*, which is perfectly insulated except for a single edge.
Ans: $L_C = \sqrt{2}L$

iv. A cube of edge *L*, which is perfectly insulated except for a single corner.
Ans: $L_C = \sqrt{3}L$

e) A hot coffee is poured into a cylindrical cup having inner and outer radii of r_i = 3 cm and r_o = 3.5 cm, respectively. The thermal conductivity of the cup k_c = 0.12 W/m K. The convective heat transfer coefficient between the coffee and the inner surface of the cup is 2,500 W/m^2 K, while the convective heat transfer coefficient between air and the outside surface of the cup is 30 W/m^2 K. Assuming that heat transfer from hot coffee occurs only through the side of the cylindrical cup, is it reasonable to assume a negligible temperature gradient within the liquid coffee during the cooling process?

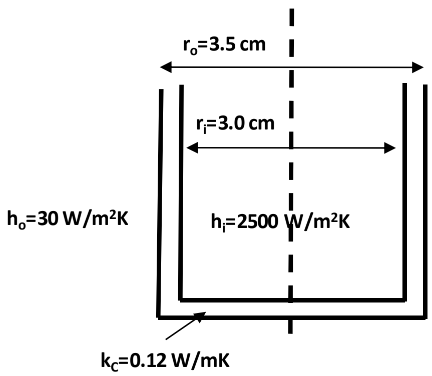
Hint: Define what constitutes the internal and external resistances in this problem.

Ans:

Known: Radial dimensions of cup, subject to known convections in the interior and exterior.

Find: Whether it is reasonable to assume negligible temperature gradient within the liquid coffee as it is being cooled from the outside.

Schematic:



Assumptions:

- One dimension conduction
- Steady state conditions
- Constant properties
- Lumped capacitance applies on coffee medium only.
- Internal Resistance from coffee only, while external resistance includes both conduction within cup and outer convection

Analysis:

Calculate Biot number as follows:

$$\mathrm{Bi} = \frac{R_{\text{internal}}}{R_{\text{external}}}$$

For negligible temperature gradient within the coffee, check for applicability of lumped capacitance model.

$$\mathrm{Bi} = \frac{R_{\text{internal}}}{R_{\text{external}}} = \frac{R_{\text{conv,i}}}{R_{\text{conv,o}} + R_{\text{cond}}} = \frac{\dfrac{1}{h_i \pi r_i L}}{\dfrac{1}{h_o \pi r_o L} + \dfrac{\ln\left(r_o / r_i \right)}{2\pi L k}}$$

$$\mathrm{Bi} = \frac{\dfrac{1}{h_i r_i}}{\dfrac{1}{h_o r_o} + \dfrac{\ln\left(r_o / r_i \right)}{2k}} = 0.00836$$

Since the calculated Biot number Bi = 0.00836 << 0.1, assuming the lumped capacitance model on the coffee is applicable. Therefore it is reasonable to assume a negligible temperature drop within the coffee.