CHG 2314
Heat Transfer Operations
Winter 2011

Assignment 3

Professor: **Dr. Boguslaw Kruczek** **January 24, 2011**

DATE DUE: January 31, 2011 at 4:00 p.m. in the Assignment Box

SOLUTIONS

Problem 1

Answer the following questions and draw a diagram whenever necessary:

a) Write the expressions for thermal resistance to conduction in plane wall, cylindrical shell, and spherical shell.

Ans: - Plane Wall: $R = \dfrac{L}{kA}$

- Cylindrical Shell: $R = \dfrac{\ln\left(r_2/r_1\right)}{2\pi Lk}$

- Spherical Shell: $R = \dfrac{\left(1/r_1\right) - \left(1/r_2\right)}{4\pi k}$

b) What is a contact resistance and what modes of heat transfer are associated with the contact resistance?

Ans: Materials are not perfectly smooth. Because of their roughness, contact spots are interspaced with gaps that are air filled. The mode of heat transfer between the contact area is conduction, while conduction and/or radiation may occur across the gaps. The contact resistance may be viewed as two parallel resistances: that due to the contact spots and that due to the gaps. The contact area is typically small, and especially for rough surfaces, the major contribution to the resistance is made by gaps. Hence the mode of heat transfer is conduction at the contact spots, and radiation/conduction between the gaps.

c) Consider cylindrical and spherical shells made from the same material, having the same inner and outer radii, the same outer surface areas, and the same inner (T_i) and outer (T_o) surface temperatures. Compare heat flow through these two shells.

Ans: Cylindrical Shell: $q_{cyl} = \frac{2\pi k \Delta T}{\ln\left(r_2/r_1\right)}$, Spherical Shell: $q_{sph} = \frac{4\pi k \Delta T}{\left[\left(1/r_1\right)-\left(1/r_2\right)\right]}$

Since outer surface areas are the same: $4\pi r_2^2 = 2\pi r_2 L \Rightarrow L = 2r_2$

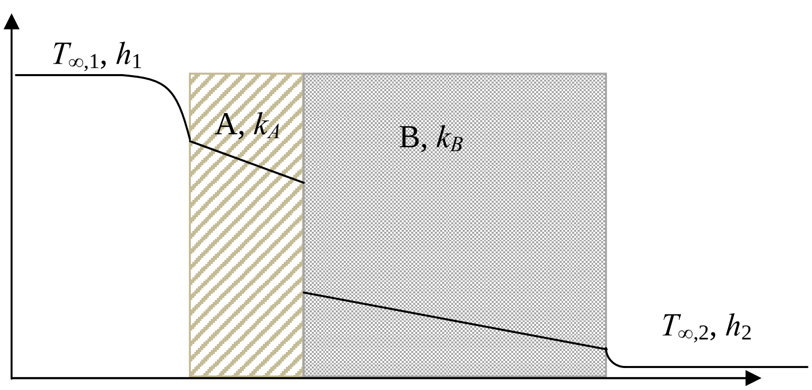
Substituting expression for L into the rate equation for cylinder, and dividing the two equations leads to:

$$\frac{q_{cyl}}{q_{sph}} = \frac{r_2\left[\left(1/r_1\right)-\left(1/r_2\right)\right]}{\ln\left(r_2/r_1\right)} = \frac{r_2\left[\frac{r_2-r_1}{r_1 r_2}\right]}{\ln\left(r_2/r_1\right)} = \frac{\frac{r_2}{r_1}-1}{\ln\left(r_2/r_1\right)} > 1 \text{ for } \frac{r_2}{r_1} > 1$$

Therefore, the heat flow through cylindrical shell for such specified problem is greater that heat flow through the cylindrical shell.

Alternatively, for the same outer surface areas, the inner surface area of cylinder is greater than the inner surface area for sphere, thus the resistance to conduction through cylindrical shell will be less than the resistance to conduction through the spherical shell.

d) Consider a composite plane wall below and answer the following questions (you will need a ruler to answer some parts of this question):

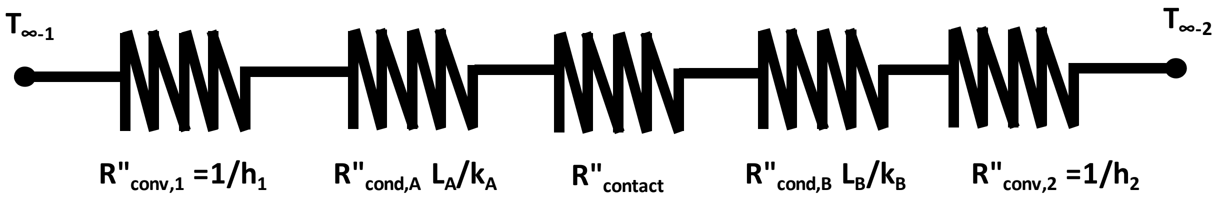


i) Identify resistances for the heat transfer from $T_{\infty,1}$ to $T_{\infty,2}$.
Ans: Along the x-axis (left to right): convection-conduction-contact-conduction-convection.

ii) Which of the two heat transfer coefficients, h_1 and h_2, is greater?
Ans: h_2 (temperature of the surface on the right closer to the environment temperature)

iii) Which of the two walls, A and B, has greater thermal conductivity?
Ans: Wall B (smaller slope)

iv) Draw the thermal circuit corresponding to the above diagram
Ans:

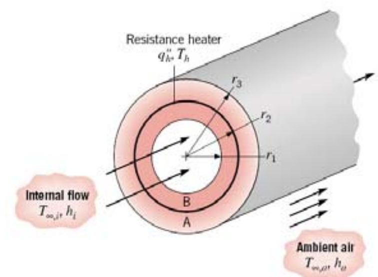


v) List all resistances in the above figure in the order of increasing values.

At steady state with constant area for heat flow, the greater the resistance the greater the temperature drop. Consequently, by measuring the temperature drop due to each resistance, we can assess their relative values, which are as follow:

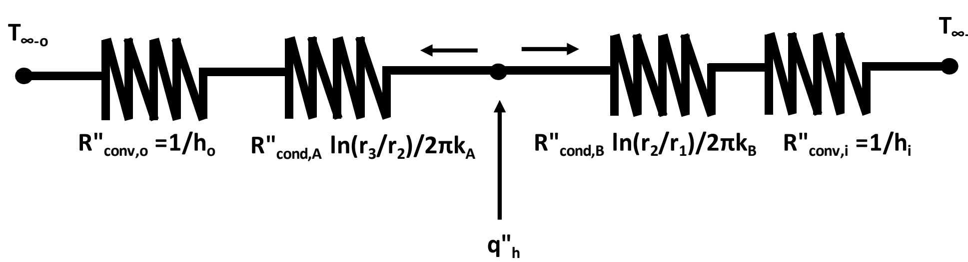
$R''_{conv,2} < R''_{cond,A} < R''_{cond,B} < R''_{conv,1} < R''_{contact}$

e) A composite cylindrical wall is composed of two materials of thermal conductivity k_A and k_B, which are separated by a very thin, elctric resistance heater for which interfacial contact resistance is negligible, and which under steady state conditions dissipates uniformly heat flux q_h''. The inner and outer surfaces are exposed to two different fluids at different temperatures resulting in two different heat transfer coefficients, as shown on the diagram.



i) Sketch the equivalent thermal circuit of the system and express all resistances in terms of relevant variables

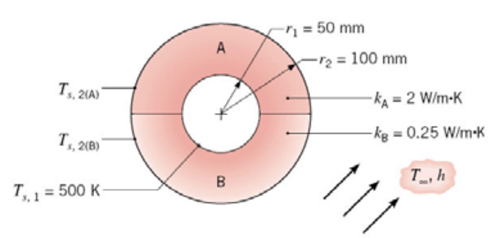
Ans: Heat flow is split in two directions; therefore we end up in two branches of resistances that are parallel to each other, with each branch consisting of two resistances in series.



ii) Why despite heat generation within the system you can draw thermal circuit?

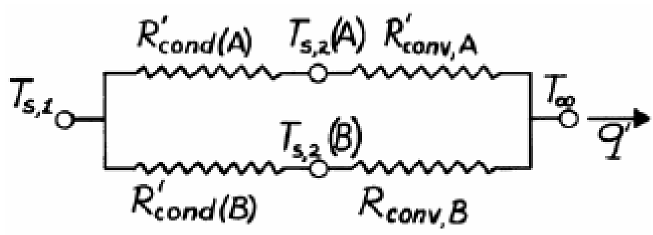
That is because the heat generation for this case occurs on a surface (interface), which has no volume. Besides, even if the generating part of the system had a finite volume, the thermal circuit can be applied from the boundaries of the generating element.

f) Steam flowing through a long, thin-walled pipe maintains the pipe wall at a uniform temperature. The pipe is covered with an insulation comprised of two different materials, A and B. The



interface between the materials may be assumed to have an infinite contact resistance, and the outer surface is convectively cooled by air. Sketch the thermal circuit of the system and label using the symbols on the diagram all pertinent resistances.

Ans: Because of perfect insulation (infinite contact resistance), the heat transfer through A and B occurs in parallel. Moreover, since inner surfaces of A and B are exposed to the same fluid inside and the outer surfaces of A and B are also exposed to the same fluid, we have a closed thermal circuit rather than open one is in the case of question e).



g) In a solid cylinder experiencing uniform volumetric heating and convection heat transfer from its surface, how does the heat flux and heat flow vary with radius?

For a cylinder with uniform heat generation, which is in symmetrical convection environment, the temperature distribution is given by Eq.(3.53):

$$T(r) = \frac{\dot{q}r_o^2}{4k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s \quad \text{where: } r_o \text{ is the outer rarius of the cylinder}$$

To obtain the expression for heat flux we apply Fourier's law:

$$q_r'' = -k\frac{dT}{dr} \text{ Differetiating Eq.(3.53) and subsituting to Fourier's law gives: } q_r'' = \frac{\dot{q}}{2}r$$

Therefore, the flux increases linearly from 0 in the middle to the largest value (not the mathematical maximum though) at the outer surface.

Heat flow is related to heat flux through:

$$q_r = q_r'' A_r \text{ where } A_r = 2\pi r L \text{ in which } L = \text{is the length of cylinder}$$

Therefore: $q_r = q_r'' A_r = \pi L \dot{q} r^2$

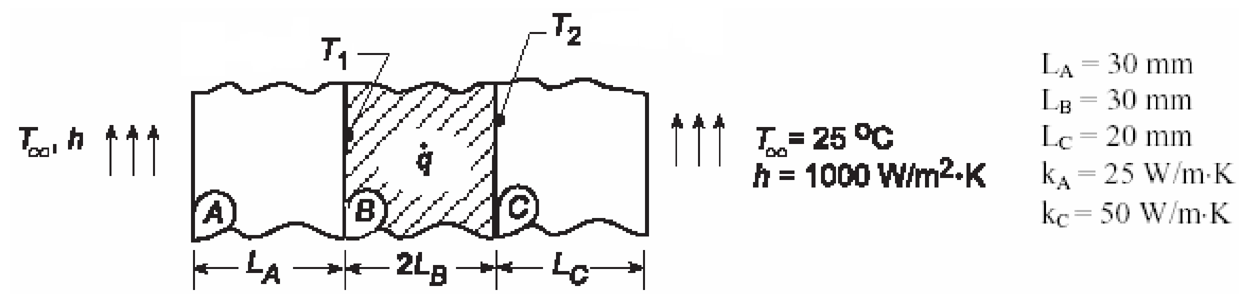
Therefore, heat flow increases also increases in radial direction, but it is proportional to r^2.

Problem 2

<u>Known</u>: Composite wall with outer surfaces exposed to convection process.

<u>Find:</u> (a) Volumetric heat generation within material B and temperature B-C interface (T_2), (b) Plot of temperature distribution, (c) T_1 and T_2 where h = 0 on surface of C.

<u>Schematic:</u>



<u>Assumptions:</u>

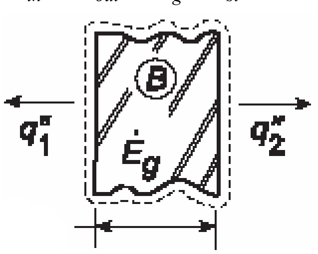
- Steady-state 1D heat transfer
- Negligible contact resistance at interfaces
- Uniform heat generation present in B only

<u>Analysis:</u>

(a)

Perform energy balance on wall B:

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

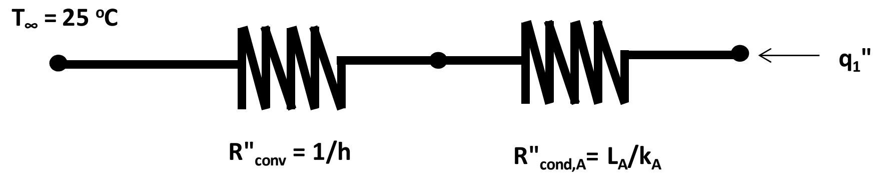


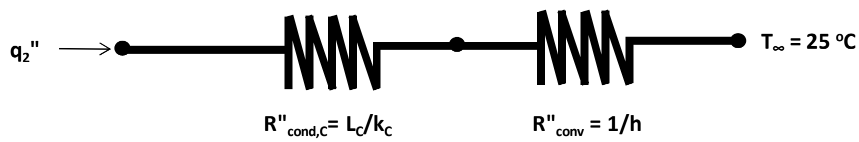
$$\dot{E}_{in} = \dot{E}_{st} = 0$$

Therefore:

$$q_1'' + q_2'' = 2\dot{q}L_B \quad (1)$$

Proceed to constructing a thermal circuit across the system.





Continuing from Equation (1):

$$\frac{T_1 - T_\infty}{1/h + L_A/k_A} + \frac{T_2 - T_\infty}{1/h + L_C/k_C} = 2\dot{q}L_B \qquad (2)$$

Two unknowns present in Equation (2): $\dot{q}$ and T_2

Next, perform surface energy balance using B/C where unknown T_2 is present:

$$-k_B \frac{dT}{dx}\bigg|_{x=L_B} = q_2'' = \frac{T_2 - T_\infty}{1/h + L_C/k_C} \qquad (3)$$

Before proceeding, must develop temperature profile within B in order to be able to calculate derivative (dT/dx)…

General Differential Equation:

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k_B} = 0$$

Integrate once...

$$\frac{dT}{dx} = -\frac{\dot{q}}{k_B}x + C_1$$

Integrate again...

$$T = -\frac{\dot{q}}{2k_B}x^2 + C_1 x + C_2$$

Apply BC1: At $x = 0, T = T_1$ (Note: Origin is A/B interface)

Apply BC2: At $x = 2L_B, T = T_2$

$$T(x) = T_1 + \left(\frac{\dot{q}L_B}{k_B} + \frac{T_2 - T_1}{2L_B}\right)x - \frac{\dot{q}}{2k_B}x^2 \qquad (4)$$

Now back to Equation (3), apply profile in Equation (4) to differential equation:

(Note: B/C interface redefined as $2L_B$, feel free to define it whichever way suits you)

$$-k_B \left.\frac{dT}{dx}\right|_{x=L_B} = \frac{T_2 - T_\infty}{1/h + L_C/k_C}$$

$$-k_B \frac{d}{dx}\left[T_1 + \left(\frac{\dot{q}L_B}{k_B} + \frac{T_2 - T_1}{2L_B}\right)x - \frac{\dot{q}}{2k_B}x^2\right]\Bigg|_{x=2L_B} = \frac{T_2 - T_\infty}{1/h + L_C/k_C}$$

$$-k_B\left[\frac{\dot{q}L_B}{k_B} + \frac{T_2 - T_1}{2L_B} - \frac{2L_B\dot{q}}{k_B}\right] = \frac{T_2 - T_\infty}{1/h + L_C/k_C} - k_B\left[\frac{\dot{q}L_B}{k_B} + \frac{T_2 - T_1}{2L_B} - \frac{2L_B\dot{q}}{k_B}\right] = \frac{T_2 - T_\infty}{1/h + L_C/k_C}$$

$$\dot{q}L_B - \frac{k_B(T_2 - T_1)}{2L_B} = \frac{T_2 - T_\infty}{1/h + L_C/k_C} \qquad (5)$$

Now we have two equations and two unknowns:

$$\frac{T_1 - T_\infty}{1/h + L_A/k_A} + \frac{T_2 - T_\infty}{1/h + L_C/k_C} = 2\dot{q}L_B \qquad (2)$$

$$\dot{q}L_B - \frac{k_B(T_2 - T_1)}{2L_B} = \frac{T_2 - T_\infty}{1/h + L_C/k_C} \qquad (5)$$

Multiply Equation (5) by 2 then substitute generation term into Equation (2):

$$\frac{T_1 - T_\infty}{\frac{1}{h} + \frac{L_A}{k_A}} + \frac{T_2 - T_\infty}{\frac{1}{h} + \frac{L_C}{k_C}} = \frac{2(T_2 - T_\infty)}{\frac{1}{h} + \frac{L_C}{k_C}} + \frac{k_B(T_2 - T_1)}{L_B}$$

$$\frac{T_1 - T_\infty}{\frac{1}{h} + \frac{L_A}{k_A}} - \frac{T_2 - T_\infty}{\frac{1}{h} + \frac{L_C}{k_C}} = \frac{k_B(T_2 - T_1)}{L_B}$$

$$\frac{T_1 - T_\infty}{\frac{1}{h} + \frac{L_A}{k_A}} - \frac{T_2}{\frac{1}{h} + \frac{L_C}{k_C}} + \frac{T_\infty}{\frac{1}{h} + \frac{L_C}{k_C}} = \frac{k_B T_2}{L_B} - \frac{k_B T_1}{L_B}$$

$$\frac{T_2}{\frac{1}{h} + \frac{L_C}{k_C}} + \frac{k_B T_2}{L_B} = \frac{T_1 - T_\infty}{\frac{1}{h} + \frac{L_A}{k_A}} + \frac{T_\infty}{\frac{1}{h} + \frac{L_C}{k_C}} + \frac{k_B T_1}{L_B}$$

$$T_2\left(\frac{1}{\frac{1}{h} + \frac{L_C}{k_C}} + \frac{k_B}{L_B}\right) = \frac{T_1 - T_\infty}{\frac{1}{h} + \frac{L_A}{k_A}} + \frac{T_\infty}{\frac{1}{h} + \frac{L_C}{k_C}} + \frac{k_B T_1}{L_B}$$

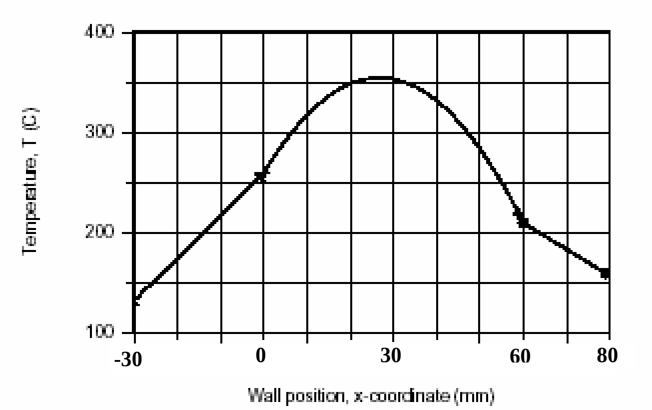
Now that T_2 is isolated, it can be solved for...

$T_2 = 209.3^{o}C$

Then calculate $\dot{q}$ using either (2) or (5):

$\dot{q} = 4.05 \times 10^6 \text{ W/m}^3$

b)



With the origin (x = 0) at A/B interface.

Important features:

- Quadratic distribution in B due to heat generation, however it's non-symmetrical since the conductivities of A and C or not the same.

- The magnitude of the gradient at wall A < wall C. This is indicative that q_2" (across wall C) > q_1" (across wall A)

(c)

The energy balance changes in this case:

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

$$q_1'' = 2\dot{q}L_B$$

Now the exclusion of q_2" since there is no heat loss towards C.

Calculate new T_1 using thermal circuit of heat loss towards direction of A (refer to Equation (2) above):

$$\frac{T_1 - T_\infty}{1/h + L_A/k_A} = 2\dot{q}L_B$$

$$T_1 = T_\infty + 2\dot{q}L_B\left(\frac{1}{h} + \frac{L_A}{k_A}\right)$$

$$T_1 = 559.6^{o}\,C$$

Since the right boundary is adiabatic, material C will be isothermal to T_2 (uniform T_2 temperature throughout C):

Develop temperature profile at B, in a similar manner as done at part (a) in Equation (4) above. Only this time, the second boundary condition BC2 changes to an adiabatic boundary (dT/dx=0).

General Differential Equation:

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k_B} = 0$$

Integrate once...

$$\frac{dT}{dx} = -\frac{\dot{q}}{k_B}x + C_1$$

$$\text{Apply BC2: At } x = 2L_B, \frac{dT}{dx} = 0$$

$$C_1 = \frac{\dot{q}}{k_B} 2L_B$$

So for now, we have:

$$\frac{dT}{dx} = \frac{2\dot{q}L_B}{k_B} - \frac{\dot{q}}{k_B} x$$

Then integrate again...

$$T = \frac{2\dot{q}L_B}{k_B} x - \frac{\dot{q}}{2k_B} x^2 + C_2$$

$$\text{Apply BC1: At } x = 0, T = T_1 \qquad \text{(Note: Origin is A/B interface again)}$$

$$T(x) = \frac{2\dot{q}L_B}{k_B} x - \frac{\dot{q}}{2k_B} x^2 + T_1 \qquad (6)$$

Finally, use Equation (6) to solve for T_2 at $x = 2L_B$

$T_2 = 1288.6\ ^oC$