CHG 2314
Heat Transfer Operations
Winter 2011

Professor: **Dr. Boguslaw Kruczek** **January 10, 2011**

DATE DUE: January 17, 2011 at 4:00 p.m. in the Assignment Box

ASSIGNMENT 1 SOLUTIONS

Problem 1

Provide short answers to the following questions:

a) Is energy transferring always a heat transfer process? Explain.

Ans: No. Energy transfer also includes changes in the kinetic energy, potential energy, and work done by the system.

b) Is a system at steady state conditions also in thermal equilibrium?

Ans: Not necessarily. Steady state implies that temperature profile within a medium does not change with time. In the special case of steady state, when the temperature within the system is constant, the system is also in thermal equilibrium.

c) Is a system in thermal equilibrium also at steady state?

Ans: Yes. See the answer for b).

d) Why do you feel colder when you put your hand in water at 10^{o}C than when you exposed it to air at 0^{o}C?

Ans: In both cases heat is being transferred (lost) by convection, and Newton's cooling law is applicable: $q''_{loss} \approx q''_{conv} = h\left(T_s - T_\infty\right)$. Since $h_{conv,water}$ is at least one order of magnitude greater than $h_{conv,air}$ (see Problem 2 from Tutorial 1 and/or Table 1.1), the heat loss in water is greater that that is air despite the fact that $\left(T_s - T_\infty\right)_{air} > \left(T_s - T_\infty\right)_{water}$.

e) What is a temperature gradient within an object sitting in a room in which the temperature is maintained constant after a sufficiently long time?

Ans: Assuming that an object does not generate heat, there will be no temperature gradient within the object, since the object will eventually be at thermal equilibrium.

f) Is it possible that the temperature of the object from part e) be different than the room air temperature? If yes, please explain why?

Ans: Yes. One obvious example is the case when the object generates heat internally (e.g. human body). Also, even if the object does not generate energy internally, its temperature will be different than that of air when the surroundings are at different temperature than the air.

g) Would the answer to part f) be the same if the object were immersed in water for a long time?

Ans: If the object generates energy, then yes. If the object does not generate energy, then no, because there is no radiation through liquids.

h) What is a difference between an opaque and semitransparent object?

Ans: In opaque surfaces, a portion of the irradiated heat is reflected. While in semi-transparent objects, portions of it is reflected *and* transmitted.

i) Two grey surfaces (for a grey surface: $\varepsilon = \alpha$) having different ε are exposed to solar radiation. Which of these two surfaces will reach a higher temperature, the one having lowε, or the one having highε? Explain.

Ans: The surface with the higher emissivity (ε). From our experience we know that a black surface (high ε) is hotter than a silver surface (high ε). The energy balance on a grey surface ($\varepsilon = \alpha$) exposed to a solar radiation will be: $q^{''}_{solar}\varepsilon = \varepsilon\sigma T_s^4 + h\left(T_s - T_\infty\right)$. Putting a reasonable values for the solar radiation, heat transfer coefficient and air temperature, it can be shown that T_s increases with ε.

j) What is the difference between the open and closed systems?

Ans: Open systems allow the transfer of mass across the boundary of the system while the closed system does not.

k) What is a difference between energy storage at steady and unsteady state conditions?

Ans: Energy storage (accumulation with time) is 0 at steady state conditions.

l) Why the form of the energy balance for a surface of an object at steady and unsteady state conditions is the same?

Ans: Because the surface has no volume so it cannot accumulate and store the energy.

Problem 2

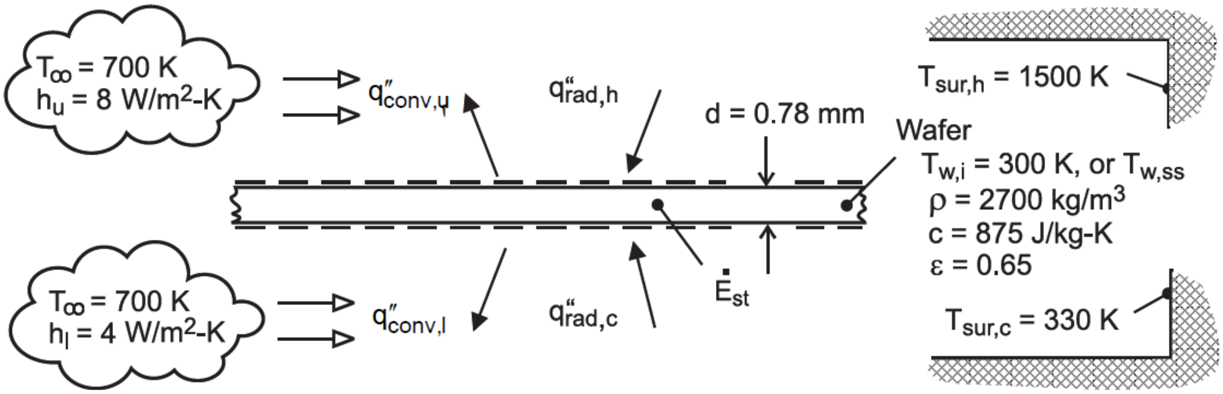
Known:

Silicon wafer positioned in furnace with top and bottom surfaces exposed to hot and cool ones, respectively.

Find:

(a) Initial rate of change of the wafer temperature corresponding to the wafer temperature $T_{W,i}$ = 300 K, (b) Steady-state temperature reached if the wafer remains in this position, and (c) hot zone temperature if the wafer temperature was 100 K lower than that of part (b). Provide other reasons that could bring the wafer to a lower temperature.

Schematic:



Assumptions:

- Uniform wafer temperatures
- Transient conditions when wafer is initially positioned
- Hot and cool zones have uniform temperatures
- Radiation exchange occurs between wafer and enclosed chamber surfaces
- Negligible heat losses from wafer to mounting pin holder

Analysis:

The energy balance on the wafer illustrated in the schematic above includes convection from the upper (u) and lower (l) surfaces with the ambient gas, radiation exchange with the hot- and cool-zone (chamber) surroundings, and the rate of energy storage term for the transient condition.

$$\dot{E}_{in}''+\dot{E}_{gen}''-\dot{E}_{out}''=\dot{E}_{st}''$$

$$\dot{E}_{gen}''=0$$

$$\dot{E}_{in}''-\dot{E}_{out}''=\dot{E}_{st}''$$

$$\dot{q}_{rad,h}+\dot{q}_{rad,c}-\dot{q}_{conv,u}-\dot{q}_{conv,l}=\rho cV\frac{dT_W}{dt}$$

The rate equation above is re-expressed in terms of heat flux by factoring out the surface area. The volume term on the right hand side is expressed in terms of thickness (d) and surface area (V).

$$\dot{q}_{rad,h}''A+\dot{q}_{rad,c}''A-\dot{q}_{conv,u}''A-\dot{q}_{conv,l}''A=\rho cdA\frac{dT_W}{dt}$$

$$\dot{q}_{rad,h}''+\dot{q}_{rad,c}''-\dot{q}_{conv,u}''-\dot{q}_{conv,l}''=\rho cd\frac{dT_W}{dt}$$

$$\varepsilon\sigma\left(T_{sur,h}^4-T_W^4\right)+\varepsilon\sigma\left(T_{sur,c}^4-T_W^4\right)-h_u\left(T_W-T_\infty\right)-h_l\left(T_W-T_\infty\right)=\rho cd\frac{dT_W}{dt}$$

(a) For the initial condition, the time rate of temperature change of the wafer is determined using the energy balance above with $T_W = T_{W,i} = 300$ K.

$$\begin{aligned}&(0.65)\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)\left(1500^4\,\text{K}^4-300^4\,\text{K}^4\right)+(0.65)\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)\left(330^4\,\text{K}^4-300^4\,\text{K}^4\right)\\&-\left(8\text{W/m}^2\cdot\text{K}\right)(300\text{K}-700\text{K})-\left(4\text{W/m}^2\cdot\text{K}\right)(300\text{K}-700\text{K})=\left(2700\text{kg/m}^3\right)(875\text{J/kg}\cdot\text{K})(0.00078\text{m})\left(\frac{dT_W}{dt}\right)_i\end{aligned}$$

$$\left(\frac{dT_W}{dt}\right)_i=104\text{K/s}$$

(b) For the steady-state condition, the energy storage term is zero, and the energy balance can be solved for the steady-state wafer temperature, $T_W = T_{W,SS}$.

$$\rho cd\left(\frac{dT_W}{dt}\right)_i=0$$

$$\begin{aligned}&(0.65)\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)\left(1500^4\,\text{K}^4-T_{w,ss}^4\right)+(0.65)\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)\left(330^4\,\text{K}^4-T_{w,ss}^4\right)\\&-\left(8\text{W/m}^2\cdot\text{K}\right)\left(T_{W,SS}-700\text{K}\right)-\left(4\text{W/m}^2\cdot\text{K}\right)\left(T_{W,SS}-700\text{K}\right)=0\end{aligned}$$

Solve for $T_{W,SS}$

$$T_{W,SS}=1251\text{K}$$

(c) Same expression as part (b), only $T_{sur,h}$ (which was 1500 K) becomes an unknown, while $T_{W,SS}$ is set to 1151K.

$$(0.65)\left(5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\right)\left(\mathrm{T_{sur,h}^4}-1151^4\,\mathrm{K^4}\right)+(0.65)\left(5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\right)\left(330^4\,\mathrm{K^4}-1151^4\,\mathrm{K^4}\right)$$
$$-\left(8\,\mathrm{W/m^2\cdot K}\right)(1151\mathrm{K}-700\mathrm{K})-\left(4\,\mathrm{W/m^2\cdot K}\right)(1151\mathrm{K}-700\mathrm{K})=0$$

Solve for $\mathrm{T_{sur,h}}$

$$\mathrm{T_{sur,h}}=1382\mathrm{K}$$

(d)

- The elevator not reaching the desired position, in case of which the upper surface will exchange radiation not only with the surfaces in the hot zone, but also in the cold zone.
- Lower emissivity of the surfaces in hot zone. To increase the emissivity, the surface may be coated with a coat having high ε and the coat may deteriorate over a time in high temperature.
- Too much ventilation causing increase in h. The disk is losing heat by convection.
- Improper insulation of the chamber, which will lead to decrease in surface temperature for a given power input to the hot zone.