

Concordia University
Comp 232 Sample Review Questions

1. State truth value of: If $1 + 1 = 2$ or $1 + 1 = 3$ then $2 + 2 = 3$ and $2 + 2 = 4$.
 True False Don't know
2. Are the following propositions logically equivalent? $p \rightarrow (\neg q \wedge r)$, $\neg p \vee \neg(r \rightarrow q)$.
 Yes No Don't know
3. Determine whether the following proposition is a tautology: $((p \rightarrow \neg q) \wedge q) \rightarrow \neg p$
 Tautology Not Tautology Don't know
4. $P(x)$ represents $x + 2y = xy$. What is the truth value of each of the following?
 - a) $\exists y \forall x P(x, y)$
 True False Don't know
 - b) $\neg \forall x \exists y \neg P(x, y)$
 True False Don't know
5. $P(m, n)$ means $m \leq n$, where the domain of discourse for m and n is the set of non negative integers. What is the truth value of the following statements?
 - a) $\exists n \forall m P(m, n)$
 True False Don't know
 - b) $\forall m \exists n P(m, n)$
 True False Don't know
6. Are the following statements valid?
 - a) $\forall x \forall y [P(x) \wedge \neg Q(y)] \equiv \forall x P(x) \wedge \neg \exists y Q(y)$
 Valid Not Valid Don't know
 - b) $\forall x \forall y [P(x) \vee Q(y)] \equiv \forall x P(x) \vee \forall y Q(y)$
 Valid Not Valid Don't know
7. Variable x represents students, y represents courses. $M(y)$ rep. y is a math course, $F(x)$ rep. x is a freshman, $B(x)$ rep. x is a full-time student, $T(x, y)$ rep. x is taking y . Write the statement in good English without using variables in your answers.
 - a) $\forall x \exists y T(x, y)$
 - b) $\exists x \forall y T(x, y)$
 - c) $\forall x \exists y [(B(x) \wedge F(x)) \rightarrow (M(y) \wedge T(x, y))]$

8. Suppose the variables x and y represent real numbers, and $L(x, y) : x < y$, $Q(x, y) : x = y$, $E(x) : x$ is even, $I(x) : x$ is an integer. Write the statement using these predicates and any needed quantifiers.

a) Every integer is even.

b) If $x < y$, then x is not equal to y .

c) There is no largest real number.

9. Determine whether the following argument is valid:

She is a Math Major or a Computer Science Major.

If she does not know discrete math, she is not a Math Major.

If she knows discrete math, she is smart.

She is not a Computer Science Major.

Therefore, she is smart.

Valid

Not Valid

Don't know

10. Place the correct symbol from the list \subseteq , $=$, \supseteq between each pair of sets below

a) $A \cup B$, $A \cup (B - A)$

b) $A \cup (B \cap C)$, $(A \cup B) \cap C$

c) $(A - B) \cup (A - C)$, $A - (B \cap C)$

d) $(A - C) - (B - C)$, $A - B$

11. Suppose $f : R \rightarrow Z$ where $f(x) = \lceil 2x - 1 \rceil$.

a) Is f one to one ?

Yes

No

Don't know

b) Is f onto Z ?

Yes

No

Don't know

12. Suppose $g : R \rightarrow R$ where $g(x) = \lfloor \frac{x-1}{2} \rfloor$. List the answer for each.

a) If $S = \{x | 1 \leq x \leq 6\}$, find $g(S)$

b) If $T = \{2\}$, find $g^{-1}(T)$

13. For each of the following statements below state whether it is True or False:

a) For all integers a, b, c , if $a|c$ and $b|c$, then $(a + b)|c$.

True

False

Don't know

b) For all integers a, b, c, d , if $a|b$ and $c|d$ then $(ac)|(b + d)$.

True

False

Don't know

c) If a and b are rational numbers (not equal to zero), then a^b is rational.

True

False

Don't know

d) If $f(n) = n^2 - n + 17$, then $f(n)$ is prime for all positive integers n .

True

False

Don't know

e) If $a \equiv b \pmod{m^2}$ then $a \equiv b \pmod{m}$.

True

False

Don't know

18. If Relation R is on the set of all integers and $(x, y) \in R$ iff $x \geq y^2$ determine if R is:

a) Reflexive

True False Don't know

b) Symmetric

True False Don't know

c) Antisymmetric

True False Don't know

d) Transitive

True False Don't know

19. Consider R and S are relations on $\{a, b, c, d\}$, where $R = \{(a, b), (a, d), (b, c), (c, c), (d, a)\}$ and $S = \{(a, c), (b, d), (d, a)\}$ Find value of each:

a) R^2

b) R^3

c) $S \circ R$

d) The transitive closure of R

20. Suppose A is the set composed of all ordered pairs of positive integers. Let R be the relation defined on A where $(a, b)R(c, d)$ means that $a + d = b + c$. Is R an equivalence relation ?

True False Don't know

21. Find the value of each:

a) The smallest equivalence relation on $\{1, 2, 3\}$ that contains $(1, 2)$ and $(2, 3)$.

b) The smallest partial order relation on $\{1, 2, 3\}$ that contains $(1, 1), (3, 2), (1, 3)$

22. Let R be the relation defined on the set of integers defined by $(a, b) \in R$ if and only if $a \geq b$. Is R a partial order ?

True False Don't know

Answers

1. False 2. Yes 3. Tautology 4 a) False 4 b) False 5 a) False 5 b) True 6 a) Valid 6 b) Valid 7 a) Every student is taking a course 7b) Some student is taking every course 7 c) Every full-time freshman is taking a math course 8 a) $\forall x(I(x) \rightarrow E(x))$ 8 b) $\forall x\forall y(L(x, y) \rightarrow \neg Q(x, y))$ 8 c) $\forall x\exists yL(x, y)$ 9. Valid 10 a) = 10 b) \supseteq 10 c) = 10 d) \subseteq 11 a) No 11 b) Yes 12 a) $\{0, 1, 2\}$ 12 b) $5 \leq x < 7$ 13 a) False: $a = b = c = 1$ 13 b) False: $a = b = 2, c = d = 1$ 13 c) False $(\frac{1}{2})^{\frac{1}{2}} = \frac{\sqrt{2}}{2}$ which is not a Rational 13 d) False, $f(17)$ is divisible by 17 13 e) True 14 a) 12 14 b) $b = 0, \pm 5, \pm 10, \pm 15, \dots$; a any integer 14 c) 8 15 a) 15 b) proofs see below 16 a) True 16 b) False 16 c) False 16 d) False 17 a) True 17 b) True 17 c) False 17 d) True 18 a) False 18 b) False 18 c) True 18 d) True 19 a) $\{(a, a), (a, c), (b, c), (c, c), (d, b), (d, d)\}$ 19 b) $\{(a, b), (a, c), (a, d), (b, c), (c, c), (d, a), (d, c)\}$ 19 c) $\{(a, a), (a, d), (d, c)\}$ 19 d) $\{(a, a), (a, b), (a, c), (a, d), (b, c), (c, c), (d, a), (d, b), (d, c), (d, d)\}$ 20 Yes : Reflexive: $a + b = b + a$; Symmetric: if $a + d = b + c$, then $c + b = d + a$; Transitive: if $a + d = b + c$ and $c + f = d + e$, then $a + d - (d + e) = (b + c) - (c + f)$, therefore $a - e = b - f$, or $a + f = b + e$. 21 a) $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$ 21 b) $\{(1, 1), (2, 2), (3, 3), (3, 2), (1, 3), (1, 2)\}$ 22 True

15 a) Prove $P(n) : 5 \mid (7^n - 2^n) \forall n \geq 0$

Step1 (Base case) Prove $P(1) : 5 \mid (7^1 - 2^1)$

Proof $5 \mid 5 \rightarrow 5 \mid (7 - 2) \rightarrow 5 \mid (7^1 - 2^1) \rightarrow P(1)$

Step2 (Inductive hypothesis) Assume $P(k) : 5 \mid (7^k - 2^k)$

Step3 (What must be proved in the inductive Step4)

Prove $P(k) \rightarrow P(k+1) : 5 \mid (7^k - 2^k) \rightarrow 5 \mid (7^{k+1} - 2^{k+1})$

Step4 (Proof of the inductive step) Prove $P(k+1) : 5 \mid (7^{k+1} - 2^{k+1})$

Proof

$$P(k) \rightarrow 5 \mid (7^k - 2^k) \rightarrow 5 \mid 7(7^k - 2^k)$$

Assumption, then Def. of Division

$$P(1) \rightarrow 5 \mid (7 - 2) \rightarrow 5 \mid 2^k(7 - 2)$$

$P(1)$, then Def. of Division

$$\rightarrow 5 \mid [7(7^k - 2^k) + 2^k(7 - 2)]$$

Addition, Def. of Division

$$\rightarrow 5 \mid [7^{k+1} - 7 \times 2^k + 7 \times 2^k - 2^{k+1}]$$

Multiplication

$$\rightarrow 5 \mid [7^{k+1} - 2^{k+1}]$$

Cancel

$$\rightarrow P(k+1)$$

$$\rightarrow P(n) : 5 \mid (7^n - 2^n) \forall n \geq 0$$

By Mathematical Induction

15 b) Prove $P(n) : a_n \leq 3^n \forall n \in \mathbb{Z}^+$ Use Strong Induction

Step1 (Base cases) Prove $P(1) : a_1 \leq 3^1$ and $P(2) : a_2 \leq 3^2$

Proof LHS = $a_1 = 2$, RHS = $3^1 \rightarrow a_1 \leq 3^1 \rightarrow P(1)$

LHS = $a_2 = 9$, RHS = $3^2 \rightarrow a_2 \leq 3^2 \rightarrow P(2)$

Step2 (Inductive hypothesis) Assume $P(k) : a_k \leq 3^k$ for $1 \leq k < n$ where $n \geq 3, n \in \mathbb{Z}^+$

Step3 (What must be proved in the inductive Step4)

Prove $P(k) \rightarrow P(k+1) : a_k \leq 3^k$ for $1 \leq k < n \rightarrow a_{k+1} \leq 3^{k+1}$

Step4 (Proof of the inductive step) Prove $P(k+1) : a_{k+1} \leq 3^{k+1}$

Proof

$$a_{k+1} = 2a_k + 3a_{k-1}$$

Since $k+1 \geq 3$ use recursive definition of a_n

$$\leq 2 \times 3^k + 3 \times 3^{k-1}$$

By Assumption replace a_k and a_{k-1}

$$= 2 \times 3^k + 3^k$$

Multiplication

$$= 3 \times 3^k$$

Addition

$$= 3^{k+1}$$

Multiplication

$$\rightarrow a_{k+1} \leq 3^{k+1}$$

$$\rightarrow P(k+1)$$

$$\rightarrow P(n) : a_n \leq 3^n \forall n \in \mathbb{Z}^+$$

Using Strong Induction