SYSC-3200 Fall 2013. Assignment 1 Solutions.

Marking notes: Formulations include marks for labelling constraints to explain what they mean, giving units for all variables, constraints, and the objective function, and explicitly listing the variable bounds (even when they are all nonnegativity bounds). Simplex solutions also include a mark for a concluding statement.

Question 1 [5 marks]

Variables: WW: tons of wheat in WakeUp HW: tons of wheat in Healthy
WS: tons of sugar in WakeUp HS: tons of sugar in Healthy
WB: tons of bran in WakeUp HB: tons of bran in Healthy

Objective function:
maximize total revenue from sales ($)
Maximize Z = 830(WW+WS+WB) + 770(HW+HS+HB)

Constraints:
Wheat availability (tons): WW+HW ≤ 100
Sugar availability (tons): WS+HS ≤ 20
Bran availability (tons): WB+HB ≤ 30
WakeUp sugar fraction: WS/(WW+WS+WB) ≥ 0.15
⇒ 0.85WS – 0.15WW – 0.15WB ≥ 0
Healthy wheat fraction: HW/(HW+HS+HB) ≥ 0.5
⇒ 0.5HW – 0.5HS – 0.5HB ≥ 0
Healthy bran fraction: HB/(HW+HS+HB) ≥ 0.2
⇒ 0.8HB – 0.2HS – 0.2HW ≥ 0

Variable bounds:
WW, WS, WB, HW, HS, HB ≥ 0

Question 2 [5 marks]

Variables: MJ, MF, MM: machinists hired at the beginning of January, February, and March
TJ, TF, TM: trainees hired at the beginning of January, February, and March
EJ, EF, EM: excess machinists at the beginning of January, February, and March

Objective: minimize staffing costs ($)
Min Z = (trainee staffing costs) + (needed machinist staffing costs)
+ (excess machinist staffing costs)
Min Z = 400(TJ+TF+TM) + 700(100+0.1TJ + 150 + 0.1TF + 200 + 0.1TM) +
500(EJ + EF + EM)
⇒ 470(TJ+TF+TM) + 500(EJ+EF+EM) + 315,000
Note: Constants are not a problem in the objective function.

Constraints: These take the general form of *needs + excess = available*.
All constraints are in units of machinists.
January 1: $100 + 0.1TJ + EJ = 130 + MJ$
$\Rightarrow 0.1TJ + EJ - MJ = 30$
February 1: $150 + 0.1TF + EF = 130 + MJ + 0.7TJ + MF$
$\Rightarrow MJ + 0.7TJ + MF - 0.1TF - EF = 20$
March 1: $200 + 0.1TM + EM = 130 + MJ + MF + MM + 0.7TJ + 0.7TF$
$\Rightarrow MJ + MF + MM + 0.7TJ + 0.7TF - 0.1TM - EM = 70$
April 1: $250 \le 130 + MJ + MF + MM + 0.7TJ + 0.7TF + 0.7TM$
$\Rightarrow MJ + MF + MM + 0.7TJ + 0.7TF + 0.7TM \ge 120$
Variable bounds: $MJ, MF, MM, TJ, TF, TM, EJ, EF, EM \ge 0$

Question 3 [10 marks]

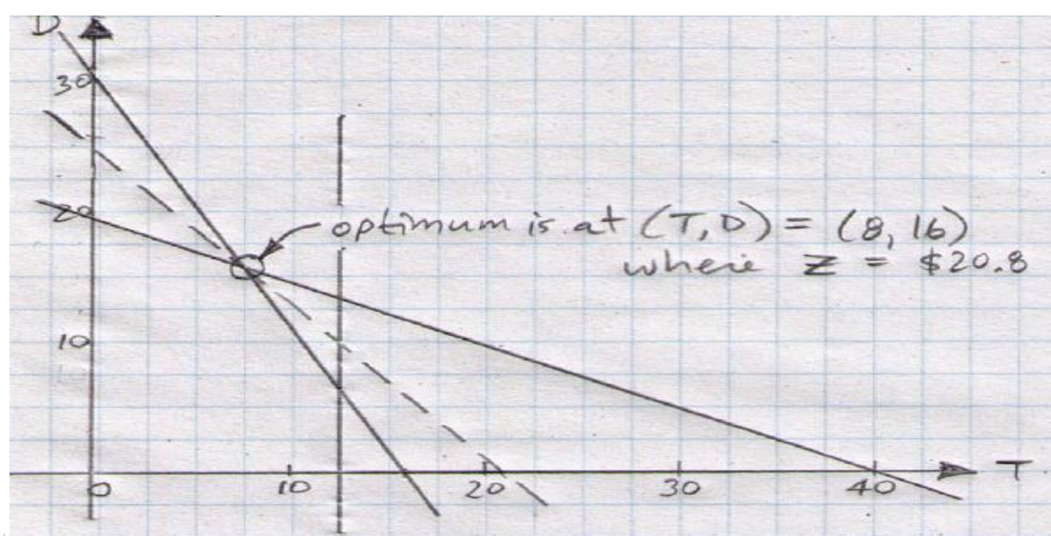
(i) Formulation [3 marks]

Variables: T: rate at which to manufacture T-shirts (shirts/day)
D: rate at which to manufacture dress shirts (shirts/day)

Objective function: maximize profit rate ($/day)
Max $Z = T + 0.8D$

Constraints:
- Cotton availability (m2/day): $T + 2D \le 40$
- Labour availability (hours/day): $T + 0.5D \le 16$
- Market limit (shirts/day): $T \le 12$

Variable bounds: $T \ge 0$, $D \ge 0$
(ii) Graphical Solution [3 marks]



So T-shirts should be made at the rate of 8 per day and dress shirts at the rate of 16 per day to achieve a maximum profit rate of $20.80 per day.

(iii) Solution via simplex tableau [4 marks]
Convert to equality format:
Max Z – T – 0.8D = 0
T + 2D + s_1 = 40
T + 0.5D + s_2 = 16
T+s_3=12

basic	Z	T	D	s_1	s_2	s_3	RHS	MRT
Z	1	-1	-0.8	0	0	0	0	-
s_1	0	1	2	1	0	0	40	40/1=40
s_2	0	1	0.5	0	1	0	16	16/1=16
s_3	0	1	0	0	0	1	12	12/1=12
Z	1	0	-0.8	0	0	1	12	-
s_1	0	0	2	1	0	-1	28	28/2=14
s_2	0	0	0.5	0	1	-1	4	4/0.5=8
T	0	1	0	0	0	1	12	No limit
Z	1	0	0	0	1.6	-0.6	18.4	-
s_1	0	0	0	1	-4	3	12	12/3=4
D	0	0	1	0	2	-2	8	No limit
T	0	1	0	0	0	1	12	12/1=12

basic	Z	T	D	s_1	s_2	s_3	RHS	MRT
Z	1	0	0	0.2	0.8	0	20.8	
s_3	0	0	0	0.33	-1.33	1	4	
D	0	0	1	0.67	-0.67	0	16	
T	0	1	0	-0.33	1.33	0	8	

The simplex method produces the same solution as the graphical method: T-shirts should be made at the rate of 8 per day and dress shirts at the rate of 16 per day to achieve a maximum profit rate of $20.80 per day.

Question 4 [10 marks]

[4 marks formulation, 4 marks simplex solution, 2 marks recognizing and explaining unboundedness]

Variables: R: production of red paint (litres/min)
B: production of blue paint (litres/min)

Objective function: maximize profit rate ($/min)

Max Z = 3R + 6B

Constraints:

- Red paint production (litres/min): $R \leq B + 4 \rightarrow R - B \leq 4$
- Blue paint production (litres/min): $B \leq R+1 \rightarrow -R + B \leq 1$

Variable bounds: $R \geq 0$, $B \geq 0$

Convert to equality format:
Max Z – 3R – 6B = 0
R – B + s1 = 4
-R + B + s2 = 1

Basic	Z	R	B	s_1	s_2	RHS	MRT
Z	1	-3	-6	0	0	0	-
s_1	0	1	-1	1	0	4	No limit
s_2	0	-1	1	0	1	1	1/1=1
Z	1	-9	0	0	6	6	-
s_1	0	0	0	1	1	5	No limit
B	0	-1	1	0	1	1	No limit

The possible leaving basic variables are all tied at "no limit". This shows that the model is unbounded, which usually indicates that a constraint has been omitted, in this case likely a constraint on the amount of paint that can actually be sold, or on the capacity limits at the production facility.