STAT 251

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mean $= \bar{x}$

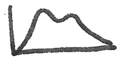
Variance $= s^2 = \sigma^2 = \frac{1}{n-1} \Sigma (x_i - \bar{x})^2$

IQR $= Q_3 - Q_1$ $\quad Q_1 = 25\%$ below, $Q_3 = 75\%$ below

Unimodal:

right skewed
mean > median

Left skewed
median > mean



Bimodal

Outlier if $< Q_1 - (1.5 \times IQR)$
or $> Q_3 + (1.5 \times IQR)$

$P(A \cap B) = P(A \text{ and } B)$

$P(A \cup B) = P(A \text{ or } B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A^c) = 1 - P(A)$

If A and B independant $P(A \cap B) = P(A) \times P(B)$

Conditional Probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (*)$$

when ind

$$P(A|B) = (*) = \frac{P(A) \cdot P(B)}{P(B)} = P(A)$$

$$P(A_j|B) = \frac{P(A_j \cap B)}{\sum_1^k P(B|A_i) \times P(A_i)}$$

cdf $= F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$

$f(t) =$ pdf $= F'(x)$

$P(a \le X \le b) = F(b) - F(a)$

$P(x > a) = 1 - F(a)$

$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

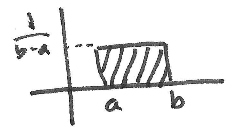
$E(g(x)) = \int_{-\infty}^{\infty} g(x) f(x)\,dx$

$Var(x) = E(x^2) - E(x)^2 = \sigma^2$

$\sigma =$ std dev.

Uniform dist:

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$



$E(x) = \frac{a+b}{2}$

$Var(x) = \frac{(b-a)^2}{12}$

$$F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x > b \end{cases}$$

Exp Dist:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

$E(x) = \frac{1}{\lambda}$

$Var(x) = \frac{1}{\lambda^2}$

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-\lambda x} & x \ge 0 \end{cases}$$

memoryless.

For Discrete Ran V:

$E(h(x)) = \Sigma h(x) \cdot P(x)$

$Var(x) = E(x^2) - E(x)^2$

For Rand V:

$E(aX+b) = a\,E(x) + b$

$E(X+Y) = E(X) + E(Y)$

$E(XY) = E(X)\,E(Y)$

$Var(aX+b) = a^2 Var(X)$

$Var(X+Y) = Var(X) + Var(Y)$

$Var(X-Y) = Var(X) + Var(Y)$

$E(\bar{X}) = \frac{1}{n}(E(X_1) + E(X_2) \ldots)$

$Var(\bar{X}) = \frac{1}{n^2}(Var(X_1) + Var(X_2) \ldots)$

Finding max of n independant R Variables.

Let $Y = \max(X_1, X_2 \dots X_n)$
get pdf of Y.

$F_Y(y) = P(Y \le y)$

$= P(X_1 \le y, X_2 \le y, \dots X_n \le y)$

$= F_{X_1}(y) \cdot F_{X_2}(y) \cdot F_{(X_3)}(y) \dots$

If X_1 and X_2 have same PdF:

$F_Y(y) = [F_X(y)]^n$

$f_Y(y) = F_Y'(y)$

$= n[F_X(y)]^{n-1} \times f_X(y)$

Finding min.

$Y = \min(X_1, X_2 \dots)$

$F_Y(y) = P(Y \le y)$

$= 1 - P(Y > y)$

$= 1 - P(X_1 > y) \cdot P(X_2 > y) \dots$

$= 1 - [1 - F_{X_1}(y)] \cdot [1 - F_{X_2}(y)]$

$\Rightarrow$ If same $\Rightarrow$ $F_Y(y) = 1 - [1 - F_X(y)]^n$

$f_Y(y) = F'_Y(y)$

$= -n[1 - F_X(y)]^{n-1} \times (-f_X(y))$

$F_U(u) = P(U \le u)$

$= P(Y^2 \le u)$

$= P(-\sqrt{u} \le Y \le \sqrt{u})$

$= F(\sqrt{u}) - \cancel{F(\sqrt{u})}\ F(-\sqrt{u})$

or

$\int_{-\sqrt{u}}^{\sqrt{u}} f(y)\, dy$

$\int_0^{\sqrt[3]{u}} f(y)\, dy$

* watch bounds

ie For $y = x^2$

$-1 < x < 1 \Rightarrow 0 < y < 1$

$a < x < b$

$a^2 < x^2 < b^2$