mech 371/4

1.12

FUNDAMNETALS OF CONTROL SYSTEMS Section X

Mid-Term Thursday March 7, 2002

Answer All 4 Questions (total 45 marks)

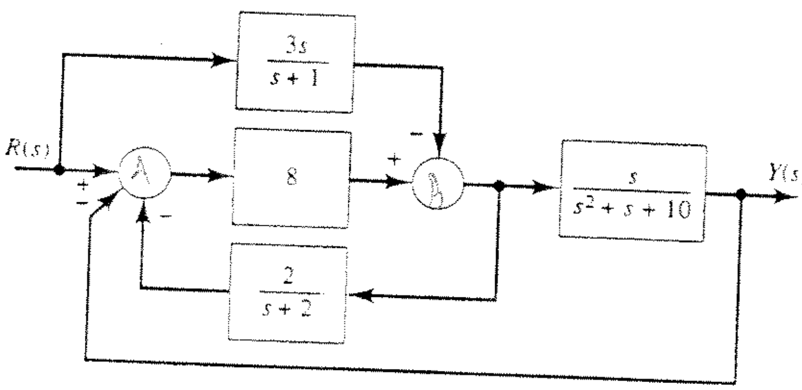
Question #1 (10 marks)

For the system shown in the block diagram:

a) Draw the signal flow graph.

b) Using Mason's loop rule, determine the overall system transfer function $\frac{Y(s)}{R(s)}$.

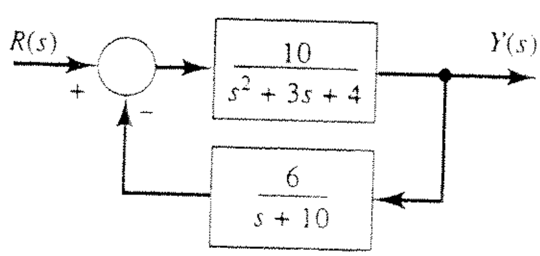
Show all mathematical steps.



Question #2.A (10 marks)

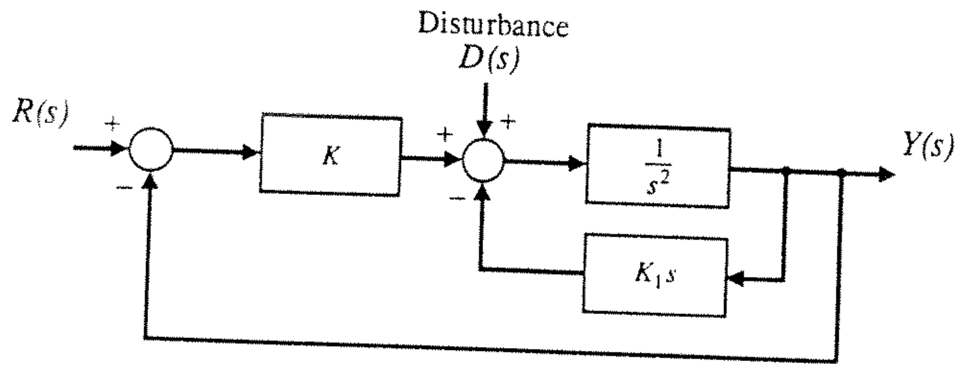
a) Determine the type number of the system.
b) Using the concept of system type number, determine the steady state error to a unit step input.
c) Using the concept of system type number, determine the steady state error to a unit ramp input.
d) Using the concept of system type number, determine the steady state error to a unit parabola input.

Show all equations and mathematical steps.
Answers without mathematical proofs will be given a mark of zero.



Question #2.B (5 marks)

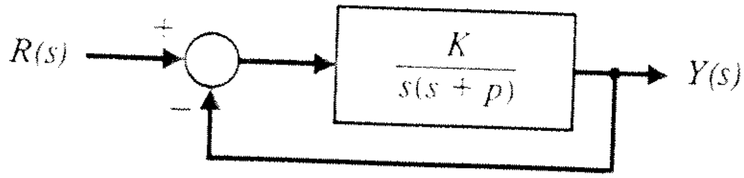
For the system shown, the command signal $R(s)$ and disturbance signal $D(s)$ are both unit step inputs. Determine the gain K so that the steady state output $y(t)$ is 1.01 .



Question #3 (10 marks)

For the system shown, it is required that the settling time $T_S \leq 4$ seconds and the percent overshoot $P.O. \leq 4.3\%$. Determine:

a) The gain K and parameter p necessary to satisfy the above performance requirements.
b) On the s-plane, plot the region of valid characteristic root locations.
c) Determine the equation for the sensitivity of the overall system transfer function with respect to parameter p. ie: determine S_p^T



Question #4 (10 marks)

The characteristic equation of a system is given by: $s^6 + s^5 + 5s^4 + s^3 + 2s^2 - 2s - 8$

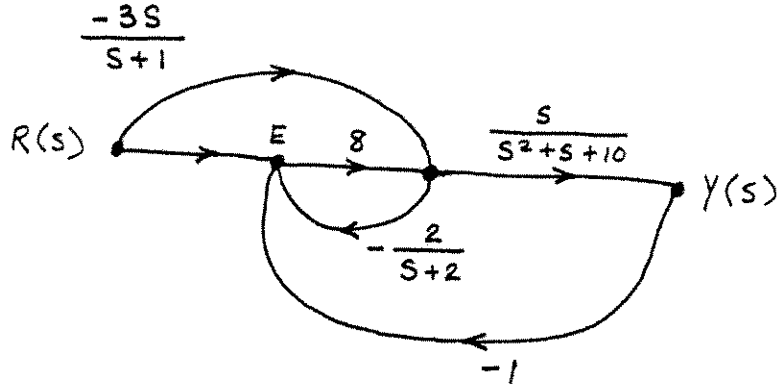
Using the Routh-Hurwitz test, determine:

a) If any, the marginally stable roots, and their numerical values.
b) If any, the total number of unstable roots.

HINT: The negative number -4 is a vector written as $-4+0j$ or as $4e^{+180j}$.

1

a)



b) $P_1 = -\frac{3S}{S+1}\left(\frac{S}{S^2+S+10}\right)$

$P_2 = \frac{8S}{S^2+S+10}$

$L_1 = -\frac{8\times 2}{S+2}$ $\qquad L_2 = -\frac{8S}{S^2+S+10}$

$$T(s) = \frac{-\frac{3S}{S+1}\left(\frac{S}{S^2+S+10}\right) + \frac{8S}{S^2+S+10}}{1 + \frac{16}{S+2} + \frac{8S}{S^2+S+10}}$$

$$= \frac{-3S^2\frac{S+2}{S+1} + 8S(S+2)}{(S+2)(S^2+S+10) + 16(S^2+S+10) + 8S(S+2)}$$

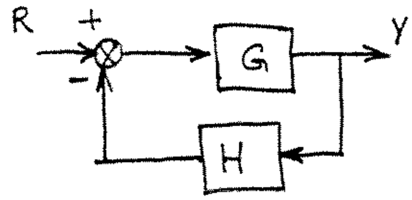
$$\begin{array}{r} S^3+S^2+10S \\ +2S^2+2S+20 \\ +16S^2+16S+160 \\ +8S^2+16S \\ \hline S^3+27S^2+44S+180 \end{array}$$

$$= \frac{-3S^2(S+2) + 8S(S+2)(S+1)}{(S+1)(S^3+27S^2+44S+180)}$$

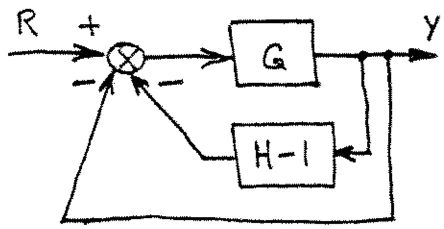
$$= \frac{5S^3+18S^2+16S}{S^4+28S^3+71S^2+224S+180}$$

$$\begin{array}{r} S^4+27S^3+44S^2+180S \\ +\quad S^3+27S^2+44S+180 \\ \hline S^4+28S^3+71S^2+224S+180 \end{array}$$

#2.A



$$\Downarrow$$



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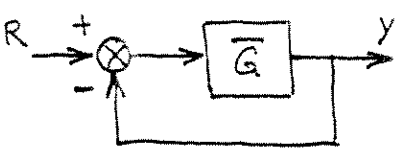


where

$$\overline{G} = \frac{G}{1+G(H-1)}$$

$$\overline{G} = \frac{\frac{10}{s^2+3s+4}}{1+\frac{10}{s^2+3s+4}\cdot\left(\frac{6}{s+10}-1\right)}$$

$$= \frac{\frac{10}{s^2+3s+4}}{1+\frac{10}{s^2+3s+4}\cdot\left(\frac{-s-4}{s+10}\right)}$$

$$= \frac{10(s+10)}{(s^2+3s+4)(s+10)-10(s+4)}$$

$$\begin{array}{r} s^3+3s^2+4s \\ +10s^2+30s+40 \\ -10s-40 \\ \hline s^3+13s^2+24s \end{array} = s(s^2+13s+24)$$

$$\overline{G} = \frac{10(s+10)}{s(s^2+13s+24)} \Rightarrow \text{Type 1}$$

b) $$k_p = \lim_{s\to 0}\overline{G} = \frac{100}{0} = \infty$$

$$e_{ss} = \frac{1}{1+k_p} = \frac{1}{\infty} = 0$$

c) $$k_v = \lim_{s\to 0} s\overline{G} = \lim_{s\to 0}\frac{10(s+10)}{s^2+13s+24} = \frac{100}{24}$$

$$e_{ss} = \frac{1}{k_v} = \frac{24}{100} = \frac{6}{25}$$

d) $$k_a = \lim_{s\to 0} s^2\overline{G} = \lim_{s\to 0}\frac{s(10)(s+10)}{s^2+13s+24} = 0$$

$$e_{ss} = \frac{1}{k_a} = \frac{1}{0} = \infty$$ cannot track acceleration input

#2.B

$$\frac{Y_R}{R} = \frac{K/S^2}{1 + (K/S^2 + K_1/S)} = \frac{K}{S^2 + K_1 S + K}$$

$$\frac{Y_D}{D} = \frac{1/S^2}{1 + (K/S^2 + K_1/S)} = \frac{1}{S^2 + K_1 S + K}$$

$$Y_{SSR} = \lim_{S \to 0} S \left(\frac{K}{S^2 + K_1 S + K} \right) \cdot \frac{1}{S} = 1$$

$$Y_{SSD} = \lim_{S \to 0} S \left(\frac{1}{S^2 + K_1 S + K} \right) \frac{1}{S} = \frac{1}{K}$$

$$Y_{SS} = 1.01 = Y_{SSR} + Y_{SSD} = 1 + \frac{1}{K}$$

$$\therefore K = 100$$

#3

P.O. $\Rightarrow \quad 100 \exp\left[\dfrac{-\xi\pi}{\sqrt{1-\xi^2}}\right] \leq 4.3$

$$-\frac{\xi\pi}{\sqrt{1-\xi^2}} \leq \ln\frac{4.3}{100} = -\pi$$

$$\xi \geq \sqrt{1-\xi^2} \quad \text{or} \quad \xi^2 \geq 1-\xi^2$$

$$\therefore \quad \xi \geq \frac{1}{\sqrt{2}} \quad (0.707)$$

$T_s \Rightarrow \quad \dfrac{4}{\xi\omega_n} \leq T_s = 4$

$$\therefore \quad \omega_n \geq \frac{1}{\xi} = \sqrt{2}$$

(1.414)

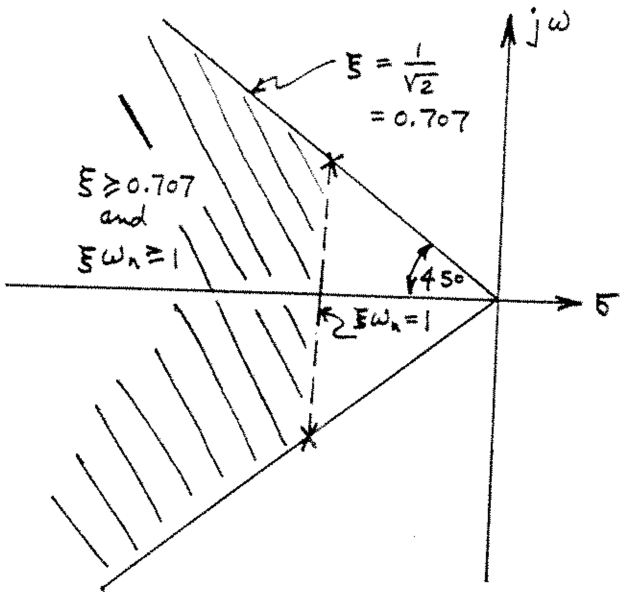
a) $$T(s) = \frac{\dfrac{K}{s(s+p)}}{1+\dfrac{K}{s(s+p)}} = \frac{K}{s^2+ps+K} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\therefore \quad K = \omega_n^2 = 2 \quad \text{and} \quad p = 2\xi\omega_n = 2\cdot\frac{1}{\sqrt{2}}\cdot\sqrt{2} = 2$$

$$s^2+ps+K = s^2+2s+2$$

b) $\xi\omega_n \geq 1$

$\theta = \cos^{-1}\xi = 45°$



c) $$T(s) = \frac{K}{s^2+ps+K}$$

$$\boxed{S_p^T = \frac{\partial T}{\partial p}\cdot\frac{p}{T}}$$

$$= \frac{-sK}{(s^2+ps+K)^2}\cdot\frac{s^2+ps+K}{K}\cdot p$$

$$\therefore \quad S_p^T = \frac{-ps}{s^2+ps+K}$$

$$T(s) = \frac{K}{s^2+ps+K}$$

$$S_p^T = \frac{\partial T}{\partial p}\cdot\frac{p}{T} = \frac{-sK}{(s^2+ps+K)^2}\cdot\frac{s^2+ps+K}{K}\cdot p$$

#4

s^6	1	5	2	-8
s^5	1	1	-2	
s^4	4	4	-8	
s^3	0	0		

Because row s^3 are all zero, there are roots on the imaginary axis.
From the auxiliary equation (row s^4):

$$4s^4 + 4s^2 - 8 = 0 \implies s^4 + s^2 - 2 = 0$$

$$\therefore \quad s_{1,2}^2 = \frac{-1 \pm \sqrt{1-4(-2)}}{2} = \frac{-1 \pm 3}{2} \implies s_1^2 = +1 \text{ and } s_2^2 = -2$$

$\therefore \; s_1 = \pm 1$

Since $-2 = 2e^{180j}$

$\therefore \; s_2 = \sqrt{2}\, e^{\pm 90j} \implies$ marginally stable roots

To fill the Routh-Hurwitz table, replace row s^3 by:

$$\frac{d}{ds}(4s^4 + 4s^2 - 8) = 16s^3 + 8s$$

s^6	1	5	2	-8	
s^5	1	1	-2		
s^4	4	4	-8		
s^3	16	8			
s^2	2	-8			$\leftarrow \frac{16\times4 - 4\times8}{16} = 2$
s^1	72				$\leftarrow \frac{2\times8 - 16\times(-8)}{2} = 72$
s^0	-8	change in sign indicates 1 root in RHP			

Imaginary Roots = 2   Roots in the RHP (unstable) = 1

CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
ENGR 372/4 sec. Y FUNDAMENTALS OF CONTROL SYSTEMS

Mid-Term **Instructor: Dr. H. Hong** **Date: March 8, 2001**

Answer all three (3) questions. **Closed book exam.** **Total time: 1 hour 15 minutes**

Problem 1.→23; Problem 2.→7; Problem 3.→5; = 35 marks total.

Problem 1. [a→1, b→7, c→3, d→2, e→1, f→3, g→3, h→3 = 23 marks]

A negative feedback control system has a block diagram as shown below.

(1) Determine the closed-loop transfer function T(s) = Y(s)/R(s).

(2) For a unit step input, find the value for the gain K so that the system has 20% overshoot.
What is the natural frequency ω_n of the system?
What is the settling time Ts of the system?

(3) Sketch the closed-loop poles on the s-plane. Clearly indicate ω_n (natural frequency) and θ (angular representation of the damping ratio ζ), and their numerical values, on the figure.

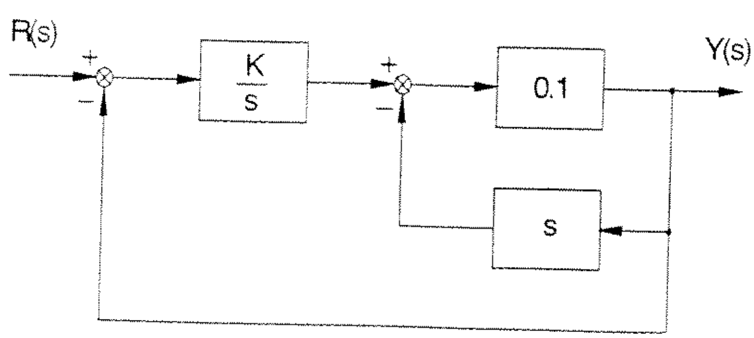
(4) Using the Routh-Hurwitz stability criterion determine the range of gain K for the system to be stable?

n (5) What is the "type number" of the system?

(6) For a unit step input, determine the position error constant Kp, and the steady-state error. Show all mathematical steps leading to your answer. lim G.

(7) For a unit ramp input, determine the velocity error constant Kv, and the steady-state error. Show all mathematical steps leading to your answer. lim sG

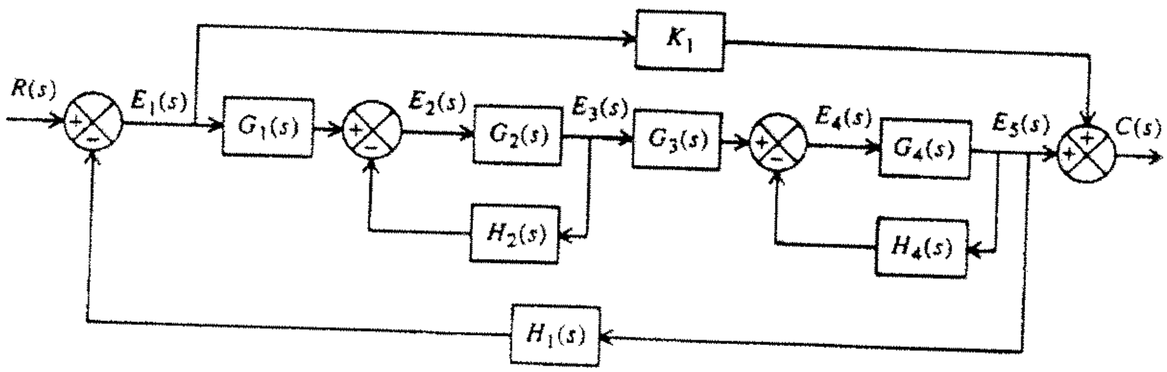
(8) For a unit acceleration input, determine the acceleration error constant Ka, and the steady-state error. Show all mathematical steps leading to your answer.
s^2G



Problem 2. [a→1½, b→5½ = 7 marks]

For the system represented in the block diagram below, find:

(1) the signal flow graph representation.
(2) the transfer function T(s) = C(s)/R(s), by using (only) Mason's signal-flow gain formula.



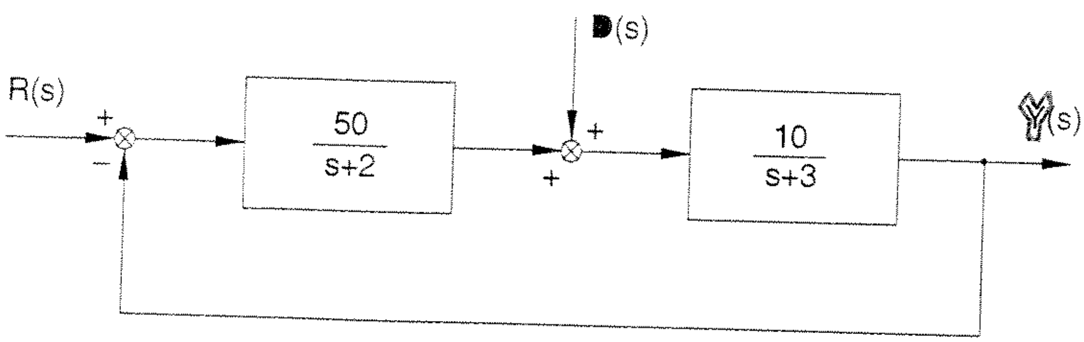
$$T = \frac{\sum_k P_k \Delta_k}{\Delta}$$

$\Delta_1 = 1 - ($ not touching path ... $)$, $\Delta_2 = 1 -)$ not touching in path 2

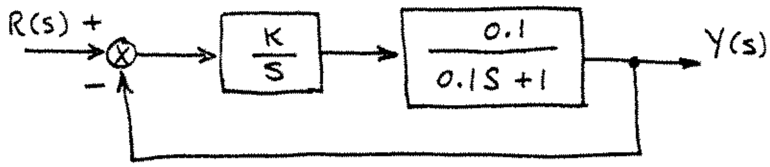
Problem 3. [5 marks]

review

For the system shown below find the total steady-state error due to a unit step input R(s) and a unit step disturbance D(s).



Problem # 1



inner loop: $\dfrac{0.1}{1+0.1S}$

(1) $$\frac{Y(s)}{R(s)} = \frac{G}{1+G} = \frac{\dfrac{0.1K}{S(0.1S+1)}}{1+\dfrac{0.1K}{S(0.1S+1)}} = \frac{0.1K}{S(0.1S+1)+0.1K}$$

$$= \frac{0.1K}{0.1S^2+S+0.1K} = \frac{K}{S^2+10S+K}$$

(2) From the characteristic equation

$$S^2+10S+K = S^2+2\xi\omega_n S+\omega_n^2$$

$\omega_n = \sqrt{K}$ ⟶ equation (a)

$2\xi\omega_n = 10$ ⟶ equation (b)

given 20% overshoot

$$P.O. = 100\exp\left[\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right]$$

$$-\frac{\xi\pi}{\sqrt{1-\xi^2}} = \ln\left[\frac{20}{100}\right] = -1.6094$$

$$1-\xi^2 = 3.8102\,\xi^2$$

$$\therefore\ \xi = 0.4559$$

From equation (b)

$$2\xi\omega_n = 10$$

$$\therefore \omega_n = \frac{10}{2\times 0.4559} = 10.9661 \ \frac{\text{rad}}{\text{sec}}$$

From equation (a)

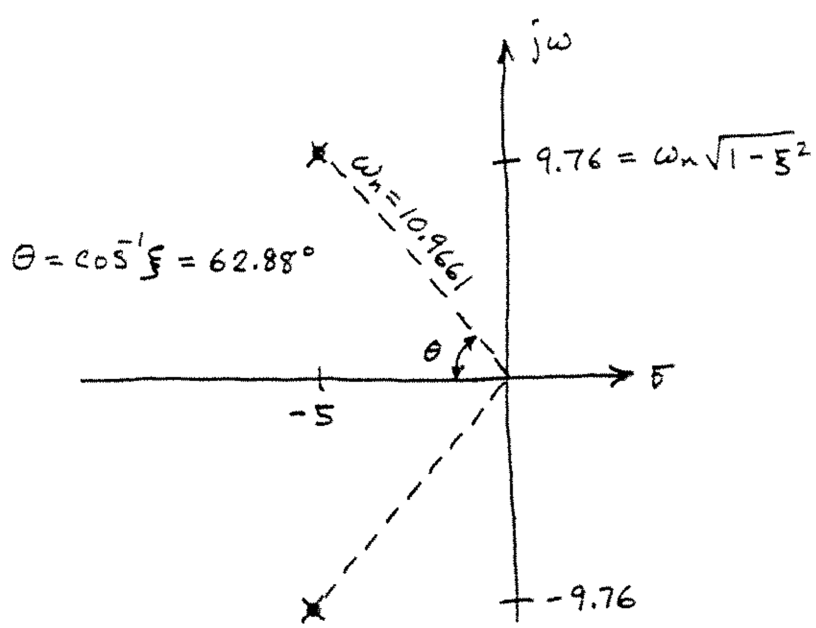
$$K = \omega_n^2 = (10.9661)^2 = 120.2558$$

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{0.4559\times 10.9661} = 0.8 \ \text{sec}$$

(3) $s^2 + 10s + 120.2558 = 0$

$$s_1, s_2 = \frac{-10 \pm \sqrt{(10)^2 - 4\times 120.2558}}{2}$$

$$= \frac{-10 \pm j\,19.52}{2} = -5 \pm j\,9.76$$



(1) (4) $s^2 + 10s + K = 0$

$$
\begin{array}{c|cc}
s^2 & 1 & K \\
s^1 & 10 & 0 \\
s^0 & K &
\end{array}
$$

(1) From row s^0, $K > 0$ for the system to be stable.

(1) (5) From the reduced block diagram, in unity feedback form, the system is "Type 1".

(3) (6) $e_{ss} = \dfrac{1}{1+K_p}$ for $R(s) = \dfrac{1}{s}$

$$K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0}\left[\frac{0.1K}{s(0.1s+1)}\right] = \infty$$

$$e_{ss} = \frac{1}{1+\infty} = 0$$

(3) (7) $e_{ss} = \dfrac{1}{K_v}$ for $R(s) = \dfrac{1}{s^2}$

$$K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} s\left[\frac{0.1K}{s(0.1s+1)}\right] = 0.1K$$

For $K = 120.2558$ $K_v = 12.026$

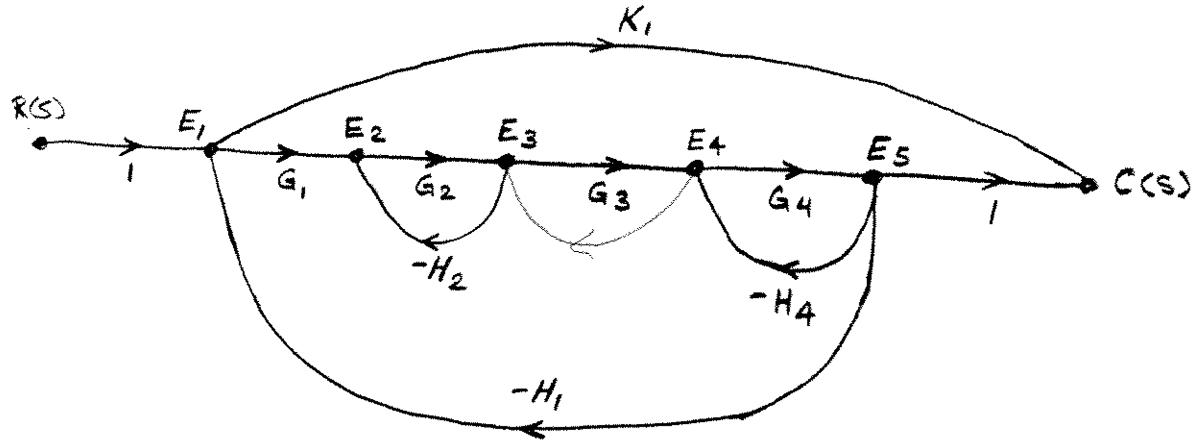
$$e_{ss} = \frac{1}{0.1K} = 0.0832$$

(3) (8) $e_{ss} = \dfrac{1}{K_a}$ for $R(s) = \dfrac{1}{s^3}$

$$K_a = \lim_{s\to 0} s^2G(s) = \lim_{s\to 0} s^2\left[\frac{0.1K}{s(0.1s+1)}\right] = 0$$

$e_{ss} = \dfrac{1}{0} = \infty$ system cannot track a parabola input

Problem # 2



$\frac{1}{4}$ $L_1 = -G_2 H_2$

$\frac{1}{4}$ $L_2 = -G_4 H_4$

$\frac{1}{4}$ $L_3 = -G_1 G_2 G_3 G_4 H_1$

$P_1 = G_1 G_2 G_3 G_4$ $\frac{1}{4}$

$P_2 = K_1$ $\frac{1}{4}$

$\frac{1}{2}$ $\frac{1}{2}$

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2)$$

$$= 1 + G_2 H_2 + G_4 H_4 + G_1 G_2 G_3 G_4 H_1 + G_2 H_2 G_4 H_4$$

$\frac{1}{2}$ $\frac{1}{2}$

$\Delta_1 = 1$ $\quad P_1 \Delta_1 = G_1 G_2 G_3 G_4$

$3/4$ $\frac{1}{2}$

$$\begin{cases} \Delta_2 = 1 - (L_1 + L_2) + L_1 L_2 = 1 + G_2 H_2 + G_4 H_4 + G_2 H_2 G_4 H_4 \\ P_2 \Delta_2 = K_1 (1 + G_2 H_2 + G_4 H_4 + G_2 H_2 G_4 H_4) \end{cases}$$

$\frac{1}{2}$

$\frac{1}{2}$ $$\frac{C(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 G_3 G_4 + K_1 (1 + G_2 H_2 + G_4 H_4 + G_2 H_2 G_4 H_4)}{1 + G_2 H_2 + G_4 H_4 + G_1 G_2 G_3 G_4 H_1 + G_2 H_2 G_4 H_4}$$

Problem #3

$$E(s) = E_R(s) + E_D(s)$$

$\left(1\tfrac{1}{2}\right)$ $$E_R(s) = \frac{1}{1+\left(\frac{50}{s+2}\right)\left(\frac{10}{s+3}\right)} \cdot R(s) = \frac{(s+2)(s+3)}{(s+2)(s+3)+500} \cdot \frac{1}{s}$$

$\left(1\tfrac{1}{2}\right)$ $$E_D(s) = -\frac{\frac{10}{s+3}}{1+\left(\frac{50}{s+2}\right)\left(\frac{10}{s+3}\right)} \cdot D(s) = \frac{-10(s+2)}{(s+2)(s+3)+500} \cdot \frac{1}{s}$$

$\left(\tfrac{1}{2}\right)$ $$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\left[E_R(s) + E_D(s)\right]$$

$\left(1\tfrac{1}{2}\right)$ $$= \lim_{s\to 0} s\left[\frac{(s+2)(s+3)-10(s+2)}{(s+2)(s+3)+500}\right]\cdot\frac{1}{s}$$

$$= \frac{6-20}{6+500} = 0.0119 - 0.0395 = -0.0276$$

OR

$\left(1\tfrac{1}{2}\right)$ $$\frac{Y_R(s)}{R(s)} = \frac{\left(\frac{50}{s+2}\right)\left(\frac{10}{s+3}\right)}{1+\left(\frac{50}{s+2}\right)\left(\frac{10}{s+3}\right)} = \frac{500}{(s+2)(s+3)+500}$$

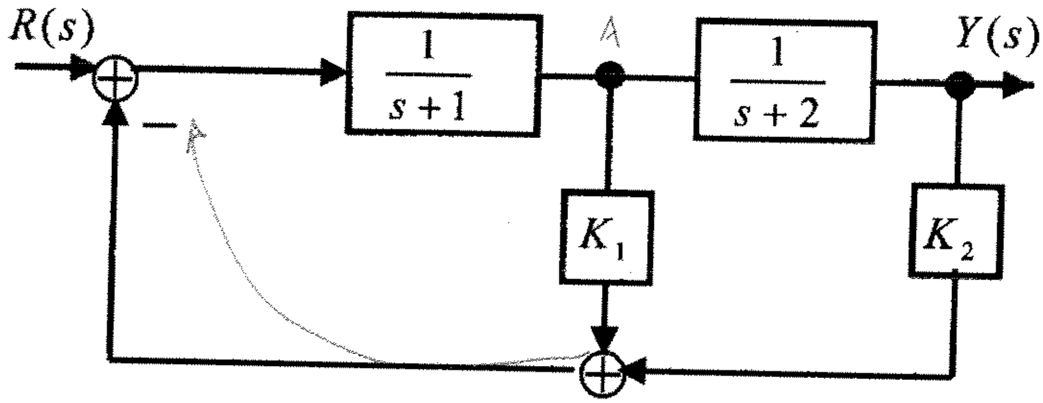
$\left(\tfrac{1}{2}\right)$ $$Y_R\left(\begin{smallmatrix}\text{steady}\\ \text{state}\end{smallmatrix}\right) = \lim_{s\to 0} s\left[\frac{500}{(s+2)(s+3)+500}\right]\cdot\frac{1}{s} = \frac{500}{506} = 0.9881$$

$\left(1\tfrac{1}{2}\right)$ $$\frac{Y_D(s)}{D(s)} = \frac{\frac{10}{s+3}}{1+\left(\frac{50}{s+2}\right)\left(\frac{10}{s+3}\right)} = \frac{10(s+2)}{(s+2)(s+3)+500}$$

$\left(\tfrac{1}{2}\right)$ $$Y_D\left(\begin{smallmatrix}\text{steady}\\ \text{state}\end{smallmatrix}\right) = \lim_{s\to 0} s\left[\frac{10(s+2)}{(s+2)(s+3)+500}\right]\cdot\frac{1}{s} = \frac{20}{506} = 0.0395$$

(1) $$\begin{cases} y = y_R + y_D = 0.9881 + 0.0395 = 1.0277 \\ e_{ss} = R - y = 1 - 1.0277 = -0.0277 \end{cases}$$

Problem 2. [1→ 7, 2→ 7, 3→ 2, 4→ 7, 5→ 7 = 30 marks]



For the feedback control system shown above:

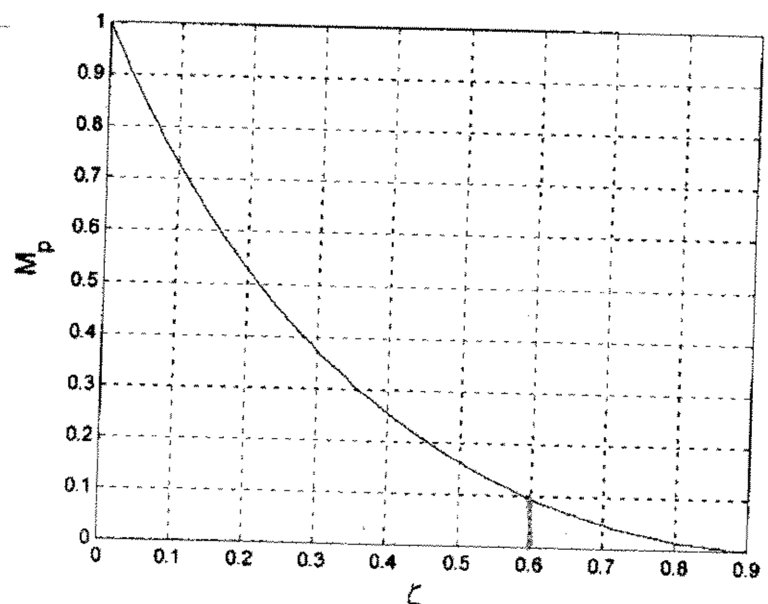
(1) Determine the closed-loop transfer function T(s) = Y(s)/R(s).

(2) Determine for which values of K_1 and K_2 the closed-loop system is stable. Plot the stability region in the $K_1 - K_2$ plane.

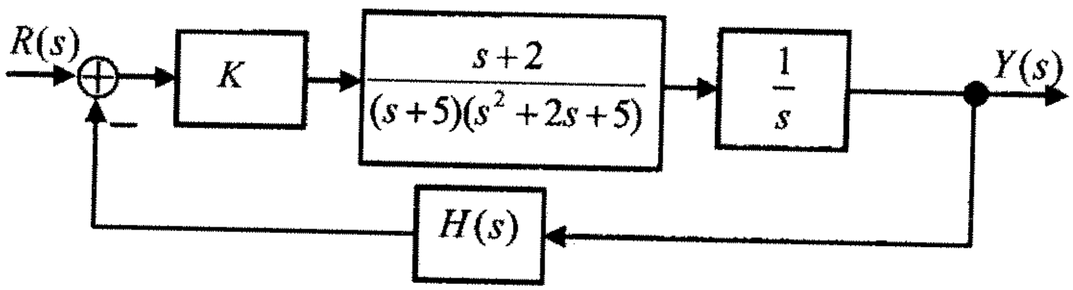
(3) What is the natural frequency of the system as a function of K_1 and K_2? And the damping ratio ?

(4) Determine K_1 and K_2 such that the closed-loop system is stable and such that the response of the closed-loop system to a step input has an overshoot of less than or equal to 10% and a 5% settling time of less than or equal to 3 seconds. Plot the allowed region for the roots in the s-plane.

(5) Determine the sensitivities of the transfer function T(s) with respect to K_1 and K_2, ie, $S_{K_1}^T$ and $S_{K_2}^T$. Find the relation between K_1 and K_2 so that the steady state (DC) gains of $S_{K_1}^T$ and $S_{K_2}^T$ are equal.



Problem 3. [1→ 7, 2a→ 7, 2b→ 8, 2c→ 7, 2d→ 7 = 36 marks]



For the feedback control system shown above:

(1) Determine the closed-loop transfer function T(s) = Y(s)/R(s)

(2) Make H(s)=1

a. Give necessary conditions on the values of K for stability

b. Give necessary and sufficient conditions on the value of K for stability.
HINT: $x^2 + 43x - 2000 = (x + 71.12)(x - 28.12)$

c. Compute K_p, K_v, K_a.

d. Using the values of K_p, K_v, K_a determined in c, compute the steady-state error to a step input, to a ramp input and to a parabola input for the closed-loop system. What is the type number of the system ?

(3) **BONUS** (10 marks) Make $H(s) = \frac{1}{s+1}$

a. Compute 1-T(s)

b. Compute K such that the steady state error of the closed-loop system to a unit ramp (velocity error) is zero.

c. Is the closed-loop system stable for this value of K ?

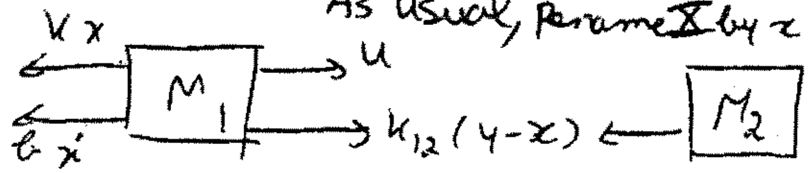
MECH 371/4

2.56

Midterm Solutions - Section T

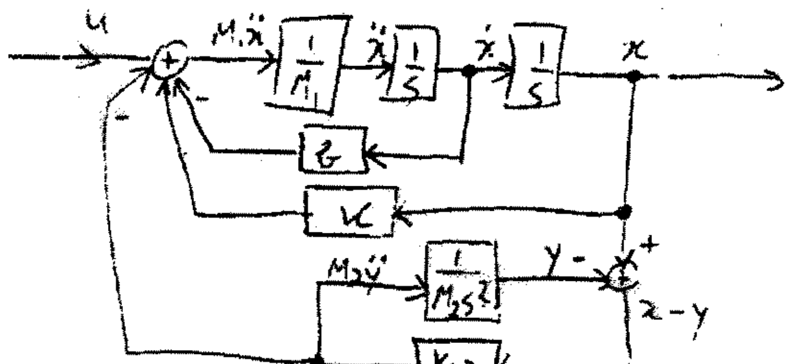
Problem 1 Equilibrium position for mass 1 is x_0
Equilibrium position for mass 2 is y_0
Let $X = x - x_0$, $Y = y - y_0$
As usual, rename X by x and Y by y

(1)

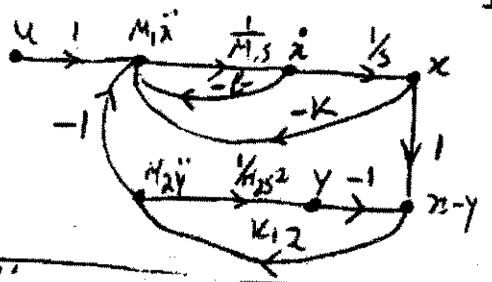


$$\begin{cases} u + k_{12}(y-x) - kx - b\dot{x} = M_1 \ddot{x} \\ -k_{12}(y-x) = M_2 \ddot{y} \end{cases}$$

(2)



(3)



Forward Path: $1\left(\frac{1}{M_1 s}\right)\left(\frac{1}{s}\right)$

Loop 1 ($M_1\ddot{x} \to \dot{x} \to M_1\ddot{x}$)
$-\frac{b}{M_1 s}$

Loop 2 ($M_1\ddot{x} \to \dot{x} \to x \to M_1\ddot{x}$)
$-\frac{k}{M_1 s^2}$

Loop 3 ($M_2\ddot{y} \to y \to x-y \to M_2\ddot{y}$): $-\frac{k_{12}}{M_2 s^2}$

Loop 4 ($M_1\ddot{x} \to \dot{x} \to x \to x-y \to M_2\ddot{y} \to M_1\ddot{x}$): $-\frac{k_{12}}{M_1 s^2}$

$\Delta_1(s) = 1 - \Sigma$ loops not touching forward path

$$\Delta_1(s) = 1 - \left(-\frac{K_{12}}{M_2 s^2}\right) = \frac{s^2 + K_{12}/M_2}{s^2}$$

$\Delta(s) = 1 - \Sigma$ all loops $+ \Sigma$ gain products 2 loops not touching.

$$\Delta(s) = 1 - \left(-\frac{B}{M_1 s} - \frac{K}{M_1 s^2} - \frac{K_{12}}{M_2 s^2} - \frac{K_{12}}{M_1 s^2}\right) +$$

$$+ \left(-\frac{B}{M_1 s}\right)\left(-\frac{K_{12}}{M_2 s^2}\right) + \left(-\frac{K}{M_1 s^2}\right)\left(-\frac{K_{12}}{M_2 s^2}\right)$$

Mason's Rule: $$\frac{X(s)}{U(s)} = \frac{1\left(\frac{1}{M_1 s}\right)\left(\frac{1}{s}\right)\frac{s^2 + K_{12}/M_2}{s^2}}{1 + \left[\frac{K + K_{12}}{M_1 s^2} + \frac{K_{12}}{M_2 s^2} + \frac{B}{M_1 s}\right] + \frac{BK_{12}}{M_1 M_2 s^3} + \frac{K}{}}$$

$$\frac{X(s)}{U(s)} = \frac{s^2 + \frac{K_{12}}{M_2}}{M_1 s^4 + M_1 s^4\left[\frac{K + K_{12} + Bs}{M_1 s^2} + \frac{K_{12}}{M_2 s^2}\right] + \frac{K_{12}(K + Bs)\cancel{M_1 s^4}}{\cancel{M_1} M_2 \cancel{s^4}}}$$

$$\frac{X(s)}{U(s)} = \frac{s^2 + \frac{K_{12}}{M_2}}{M_1 s^4 + \cancel{M_1} s^4\left[\frac{M_2(K + K_{12} + Bs) + K_{12} M_1}{\cancel{M_1} M_2 s^2}\right] + \frac{K_{12}(K + Bs)}{M_2}}$$

$$\boxed{\frac{X(s)}{U(s)} = \frac{s^2 + \frac{K_{12}}{M_2}}{M_1 s^4 + \left[(K + K_{12} + Bs) + K_{12}\frac{M_1}{M_2}\right]s^2 + \frac{K_{12}K + K_{12}Bs}{M_2}}}$$

$$\tilde{\Delta}(s) = M_1 s^4 + \left[(K_1 + K_{12} + Bs) + \frac{K_{12}}{M_2} M_1\right]s^2 + \frac{K_{12}}{M_2}K + \frac{K_{12}}{M_2}Bs$$

(5) For no vibration of M_1 when input is $u(t) = \sin(w_0 t) \neq 0$ we need the zeros of the transfer function to be located at the same frequency of the input.

<u>Zeros:</u> $s^2 + w_n^2 = 0 \Leftrightarrow s = \pm j w_n$, $w_n = \sqrt{\frac{K_{12}}{M_2}}$

$$w_n = w_0 \Leftrightarrow \boxed{\frac{K_{12}}{M_2} = w_0^2}$$

If we apply Final Value Theorem,

$$X(s) = \frac{X(s)}{U(s)} \cdot U(s) = \frac{s^2 + \frac{K_{12}}{M_2}}{\tilde{\Delta}(s)} \cdot \frac{w_0}{s^2 + w_0^2}$$

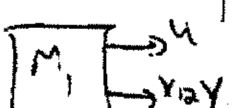
$$x(\infty) = \lim_{s \to 0} s X(s) = \lim_{s \to 0} \frac{s w_0}{\tilde{\Delta}(s)} \cdot \cancel{\frac{s^2 + \frac{K_{12}}{M_2}}{s^2 + w_0^2}} =$$

$$\frac{K_{12}}{M_2} = w_0^2$$

$$= \frac{0 \cdot w_0}{\tilde{\Delta}(0)} = 0$$

This only shows that Final Value is 0. It does not show that M_1 does not vibrate. That is guaranteed because of the placement of zeros at input frequency w_0.

<u>Free Body Diagram:</u> $\left.\begin{matrix} x = 0 \\ \dot{x} = 0 \end{matrix}\right\} \Rightarrow$



$\boxed{u + K_{12} y = 0}$

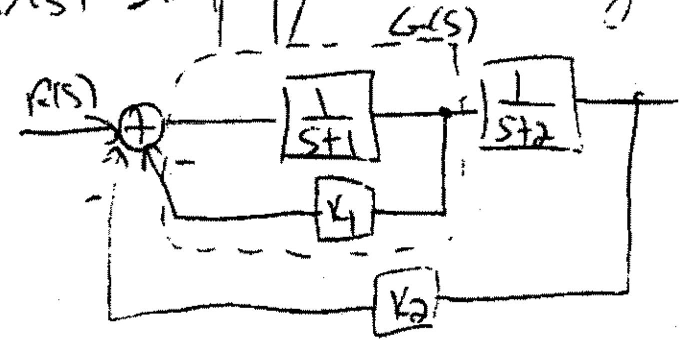
$\Downarrow$

No Vibration

3

Problem 2

(1) First simplify block diagram



Simplify G(S) to get

$$G(S) = \frac{\frac{1}{S+1}}{1+\frac{K_1}{S+1}} = \frac{1}{S+1+K_1}$$

Finally

$$T(S) = \frac{G(S)\frac{1}{S+2}}{1+G(S)\frac{K_2}{S+2}} = \frac{\frac{1}{S+1+K_1}\frac{1}{S+2}}{1+\frac{1}{S+1+K_1}\frac{K_2}{S+2}}$$

$$T(S) = \frac{1}{(S+1+K_1)(S+2)+K_2}$$

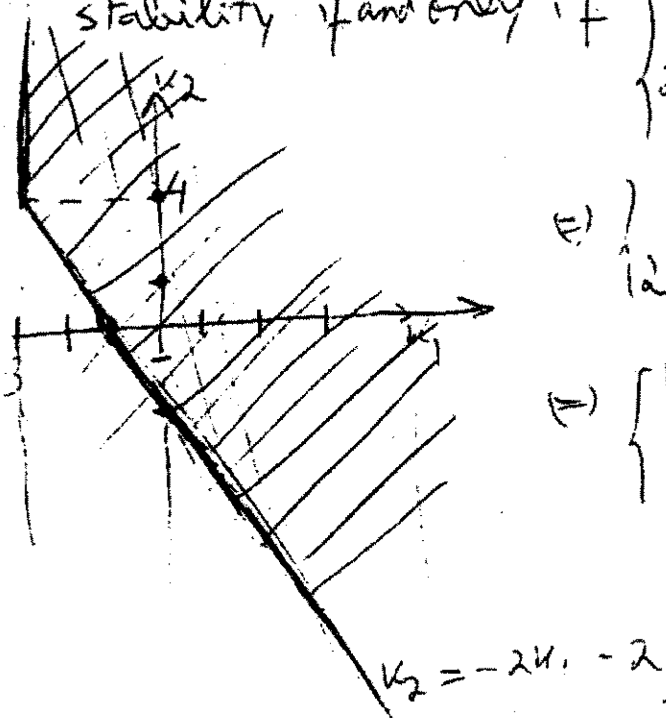
$$\boxed{T(S) = \frac{1}{S^2+(3+K_1)S+2(1+K_1)+K_2}}$$

(4)

(2)

$$\begin{array}{c|cc} s^2 & 1 & 2(1+k_1)+k_2 \\ s^1 & 3+k_1 & 0 \\ s^0 & 2(1+k_1)+k_2 & \end{array}$$

stability if and only if $\begin{cases} 3+k_1 > 0 \\ 2(1+k_1)+k_2 > 0 \end{cases} \iff \begin{cases} k_1 > -3 \\ 2k_1+k_2+2 > 0 \end{cases} \iff \begin{cases} k_1 > -3 \\ k_2 > -2(k_1+1) \end{cases}$



(3) $T(s) = \dfrac{1}{2(1+k_1)+k_2} \cdot \dfrac{2(1+k_1)+k_2}{\underbrace{s^2+(3+k_1)s+2(1+k_1)+k_2}_{s^2+2\xi\omega_n s+\omega_n^2}}$

$$\begin{cases} \omega_n = \sqrt{2(1+k_1)+k_2} \\ \xi = \dfrac{3+k_1}{2\sqrt{2(1+k_1)+k_2}} \end{cases}$$

(4) $M_p \leq 10\% \Rightarrow \xi \geq 0.6$ (from plot)

$t_s \leq 3 \Leftrightarrow \frac{3}{\xi \omega_n} \leq 3 \Leftrightarrow \xi \omega_n \geq 1$

Pick, for example, $\xi = 0.6$ and $\xi \omega_n = 1$, which implies $\boxed{\xi = 0.6}$, $\omega_n = \frac{10}{6} \Leftrightarrow \boxed{\frac{5}{3} = \omega_n}$

From $2\underbrace{\xi \omega_n}_{=1} = 3 + K_1$ we get $\boxed{K_1 = -1}$

From $\omega_n^2 = \frac{5^2}{3^2} = 2(1+K_1) + K_2$ and $K_1 = -1$ we get $\frac{25}{9} = 2\cancel{(1-1)}^{0} + K_2 \Leftrightarrow \boxed{K_2 = \frac{25}{9}}$

K_1, K_2 inside region of stability

(5) $$\frac{\partial T}{\partial K_1} = \frac{-(s+2)}{[s^2 + (3+K_1)s + 2(1+K_1) + K_2]^2}$$

$$\frac{\partial T}{\partial K_2} = -\frac{1}{[s^2 + (3+K_1)s + 2(1+K_1) + K_2]^2}$$

$$S_{K_1}^T = \frac{\partial T}{\partial K_1} \cdot \frac{K_1}{T} \Leftrightarrow S_{K_1}^T = -\frac{\frac{(s+2)K_1}{(s^2 + (3+K_1)s + 2(1+K_1) + K_2)^{\cancel{2}}}}{\cancel{s^2 + (3+K_1)s + 2(1+K_1) + K_2}}$$

$$\Leftrightarrow \boxed{S_{K_1}^T = -\frac{(s+2)K_1}{s^2 + (3+K_1)s + 2(1+K_1) + K_2}}$$

$$S_{K_2}^T = \frac{\partial T}{\partial K_2} \cdot \frac{K_2}{T} \Leftrightarrow \boxed{S_{K_2}^T = -\frac{K_2}{s^2 + (3+K_1)s + 2(1+K_1) + K_2}}$$

DC gain: $S_{K_1}^T(0) = S_{K_2}^T(0) \Rightarrow \boxed{K_1 = \frac{K_2}{2}}$

$\boxed{\text{Problem 3}}$

(1) $$\frac{Y(s)}{R(s)} = \frac{K\dfrac{s+2}{(s+5)(s^2+2s+5)}\cdot\dfrac{1}{s}}{1 + H(s)\,K\dfrac{s+2}{(s+5)(s^2+2s+5)}\cdot\dfrac{1}{s}}$$

$$\frac{Y(s)}{R(s)} = \frac{K(s+2)}{s(s+5)(s^2+2s+5) + K(s+2)H(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{K(s+2)}{s^4+7s^3+15s^2+25s + K(s+2)H(s)}$$

(2) $H(s) = 1 \Rightarrow \Delta(s) = s^4+7s^3+15s^2+(25+K)s+2K$

(a) $\begin{cases} (25+K) > 0 \\ 2K > 0 \end{cases} \Leftrightarrow \begin{cases} K > -25 \\ K > 0 \end{cases} \Leftrightarrow \boxed{K > 0}$

(b)

$$\begin{array}{c|ccc} s^4 & 1 & 15 & 2K \\ s^3 & 7 & 25+K & 0 \\ s^2 & \frac{1}{7}(105-25-K) & 2K & 0 \\ s^1 & a & 0 & \\ s^0 & 2K & & \end{array}$$

$a = -\dfrac{7}{80-K}\begin{vmatrix} 7 & 25+ \\ \frac{80-K}{7} & 2K \end{vmatrix}$

$a = -\dfrac{7}{80-K}\left(14K - \dfrac{(25+K)(8}{7}\right.$

$a = -\dfrac{\cancel{7}}{80-K}\,\dfrac{98K-(25+K)(8}{\cancel{7}}$

$a = -\dfrac{98K-2000+25K-80K}{80-K}$

$a = -\dfrac{K^2+43K-2000}{30-K}$

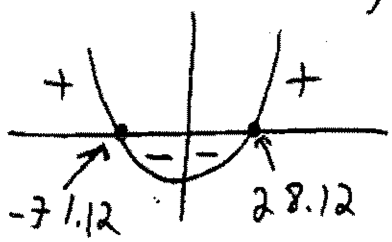
$\boxed{\text{Hint}}$

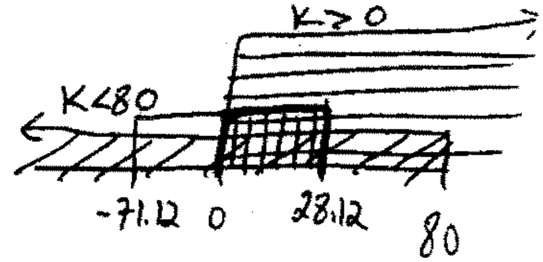
$a = -\dfrac{(K+71.12)(K-}{80-K}$

$80 - K > 0 \Leftrightarrow K < 80$

$2K > 0 \Leftrightarrow K > 0$

$(K + 71.12)(K - 28.12) < 0 \Leftrightarrow K \in (-71.12, 28.12)$





Therefore: $K \in (0, 28.12)$

(c) $K_p = \lim_{s \to 0} K \frac{s+2}{(s+5)(s^2+2s+5)} \frac{1}{s} = \infty$

$K_v = \lim_{s \to 0} \cancel{s} K \frac{s+2}{(s+5)(s^2+2s+5)} \cdot \frac{1}{\cancel{s}} = \frac{2K}{25} \neq$

$K_a = \lim_{s \to 0} s^{\cancel{2}} K \frac{s+2}{(s+5)(s^2+2s+5)} \cdot \frac{1}{\cancel{s}} = 0$

(d) $e_p(\infty) = \frac{1}{1+K_p} = 0$

$e_v(\infty) = \frac{1}{K_v} = \frac{25}{2K}$

$e_a(\infty) = \frac{1}{K_a} = \frac{1}{0} = \infty$

$\left\{ \text{Type 1 system for any finite } K \right.$

(3) $H(s) = \frac{1}{s+1}$

(a) $1 - T(s) = 1 - \dfrac{k(s+2)}{s^4+7s^3+15s^2+25s+k(s+2)H(s)}$

$$1 - T(s) = \frac{s^4+7s^3+15s^2+25s+k(s+2)\left[-1+H(s)\right]}{s^4+7s^3+15s^2+25s+k(s+2)H(s)}$$

$$1 - T(s) = \frac{s^4+7s^3+15s^2+25s+k(s+2)\left[\frac{\cancel{1}-s\cancel{-1}}{s+1}\right]}{s^4+7s^3+15s^2+25s+k(s+2)\frac{1}{s+1}}$$

$$1 - T(s) = s\,\frac{s^3+7s^2+15s+25 - k\frac{s+2}{s+1}}{s^4+7s^3+15s^2+25s+k\frac{s+2}{s+1}}$$

(b) $R(s) = \frac{1}{s^2}$

$$e_v(\infty) = \lim_{s\to 0} s\left[1-T(s)\right]R(s) =$$

$$= \lim_{s\to 0} \cancel{s}(\cancel{s})\,\frac{s^3+7s^2+15s+25-k\frac{s+2}{s+1}}{s^4+7s^3+15s^2+25s+k\frac{s+2}{s+1}}\cdot\frac{1}{\cancel{s^2}}$$

$e_v(\infty) = \dfrac{25-2K}{2K}$; $e_v(\infty) = 0 \Leftrightarrow K = \dfrac{25}{2}$

(c) $T(s) = \dfrac{k(s+2)(s+1)}{(s+1)(s^4+7s^3+15s^2+25s)+k(s+2)}$

$\Delta(s) = s^5+8s^4+22s^3+40s^2+25\frac{3}{2}s+25$

s^5	1	22	75/2
s^4	8	40	25
s^3	17	$\frac{275}{8}$	
s^2	$\frac{405}{17}$	25	
s^1	$\frac{53575\,(8)}{405}$		
s^0	25		

All positive entries in first column

$\Downarrow$

stable

9

.68

CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
ENGR 372/4 sec. Y FUNDAMENTALS OF CONTROL SYSTEMS

Mid-Term | Instructor: Dr. H. Hong | Date: March 9, 2000

Answer all three (3) questions. Closed book exam. Total time: 1 hour 15 minutes

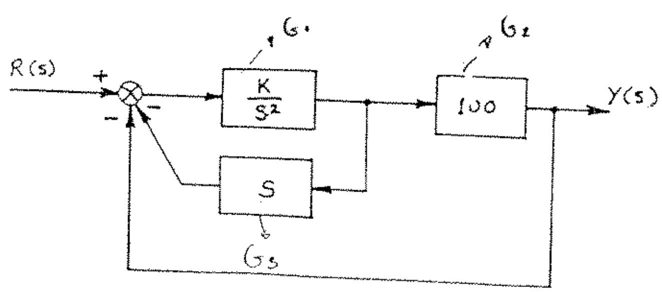
Problem 1.→23; Problem 2.→7; Problem 3.→5; = 35 marks total.

Problem 1. [a→1, b→7, c→3, d→2, e→1, f→3, g→3, h→3 = 23 marks]

A negative feedback control system has a block diagram as shown below.

Ch.5

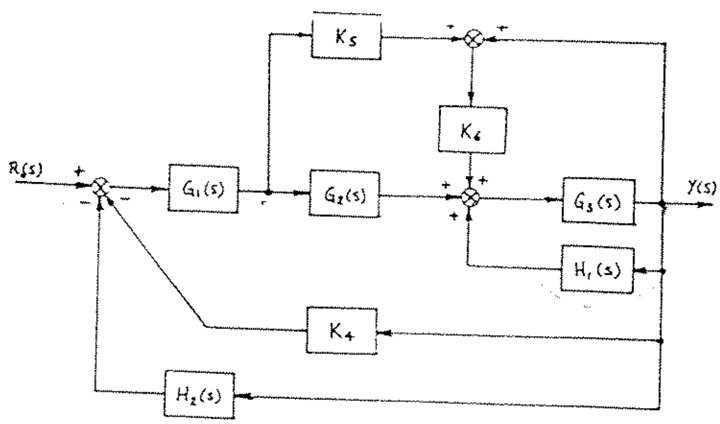
(a) Determine the closed-loop transfer function T(s) = Y(s)/R(s).
(b) For a unit step input, find the value for the gain K so that the system has 20% overshoot.
What is the natural frequency ω_n of the system?
What is the settling time Ts of the system?
(c) Sketch the closed-loop poles on the s-plane. Clearly indicate ω_n (natural frequency) and θ (angular representation of the damping ratio ζ), and their numerical values, on the figure.
(d) Using the Routh-Hurwitz stability criterion determine the range of gain K for the system to be stable?
(e) What is the "type number" of the system?
(f) For a unit step input, determine the position error constant Kp, and the steady-state error. Show all mathematical steps leading to your answer.
(g) For a unit ramp input, determine the velocity error constant Kv, and the steady-state error. Show all mathematical steps leading to your answer.
(h) For a unit acceleration input, determine the acceleration error constant Ka, and the steady-state error. Show all mathematical steps leading to your answer.



Problem 2. [a→1½, b→5½ = 7 marks]

For the system represented in the block diagram below, find:

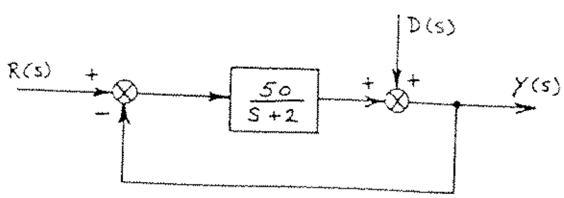
(a) the signal flow graph representation.
(b) the transfer function T(s) = Y(s)/R(s), by using (only) Mason's signal-flow gain formula.

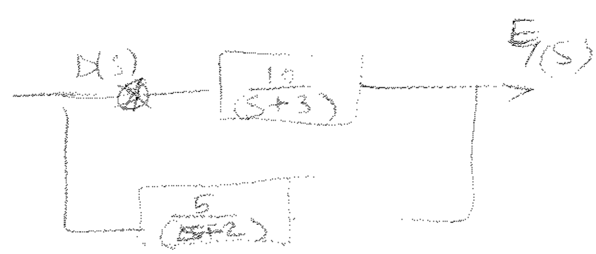


Problem 3. [5 marks]

For the system shown below find the total steady-state error due to a unit step input R(s) and a unit step disturbance D(s).

E(s) = R(s) − Y(s)





From equation 2

$$2\xi\sqrt{100K} = K$$

$$4\xi^2(100K) = K^2$$

$$\therefore K = 400\xi^2 = 400(0.4559)$$

$$K = 83.1379$$

From equation 1

$$\omega_n = \sqrt{100K} = \sqrt{100 \times 83.1379}$$

$$\omega_n = 91.18 \text{ rad/sec}$$

$$\therefore S^2 + KS + 100K = S^2 + 83.1379S + 8313.7924$$

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{(0.4559)(91.18)}$$

$$T_s = 0.0962 \text{ sec.}$$

$s^2 + KS + 100K$

c) $S^2 + 83.1379S + 8313.7924 = 0$

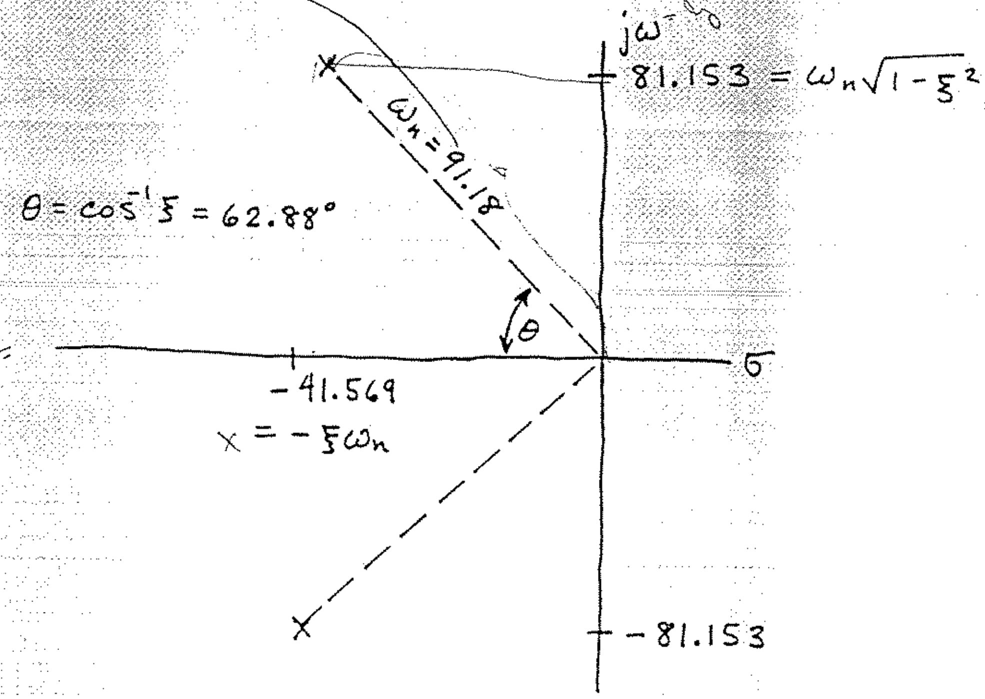
$$S_1, S_2 = \frac{-83.1379 \pm \sqrt{(83.1379)^2 - 4(8313.7924)}}{2}$$

$$= \frac{-83.1379 \pm 162.3061}{2}$$

$$-\xi\omega_n \pm j\omega$$

$$S_1, S_2 = -41.569 \pm j81.153$$

$\sqrt{81.153^2 + 41.569^2}$



$$\frac{-b \pm \sqrt{b^2 - 4(ac)}}{2a}$$

6911.91

d) From the Routh-Hurwitz Stability Criterion

$s^2 + Ks + 100K = 0$

$$\begin{array}{c|cc} s^2 & 1 & 100K \\ s^1 & K & 0 \\ s^0 & 100K & \end{array}$$

from row s^1, $K > 0$ for the system to be stable.

e) From the reduced block diagram, in unity feedback form, the system is "type 1".

f) $e_{ss} = \dfrac{1}{1+K_p}$ for $R(s) = \dfrac{A}{s} = \dfrac{1}{s}$

$K_p = \lim_{s \to 0} G(s)$ where $G(s) = \dfrac{100K}{s(s+K)}$

$= \lim_{s \to 0} \left[\dfrac{100K}{s(s+K)} \right]$

$K_p = \infty$

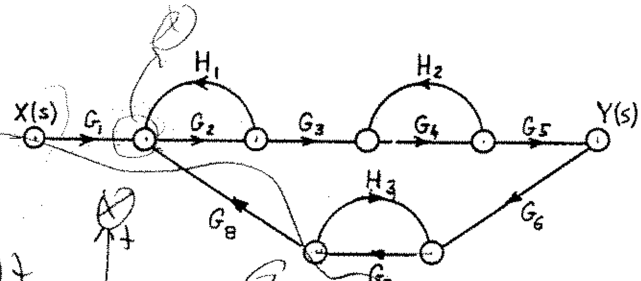
$\therefore e_{ss} = \dfrac{1}{1+\infty} = 0$

CONCORDIA UNIVERSITY
DEPARTMENT OF MECHANICAL ENGINEERING
ENGR 372/4 SEC X FUNDAMENTALS OF CONTROLS SYSTEMS

CLASS TEST Instructor: Dr. J. Svoboda Date: 11 March 1999

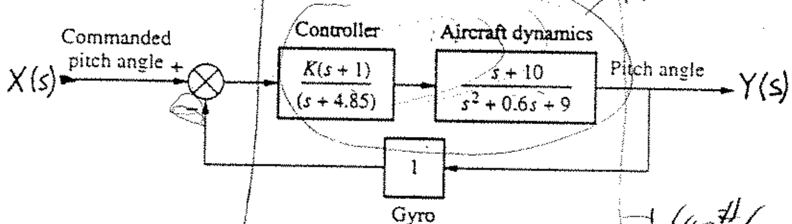
Problem 1 For the system represented by the signal flow graph shown below determine:

(a) block diagram representation
(b) transfer function $T(s) = Y(s)/X(s)$
(use method of your choice)

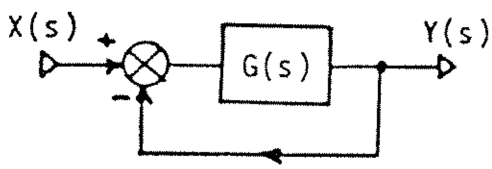


$\frac{G}{1+GH}$ $\frac{G}{1-GH}$

Problem 2 For the aircraft flight control system shown below, determine:
(a) transfer function $T(s) = Y(s)/X(s)$
(b) range of gain K for stability (use R.-H. criterion)
(c) value of gain K for marginal stability and the corresponding oscillatory frequency ω_{osc}



Problem 3 Shown below is a unity feedback system.

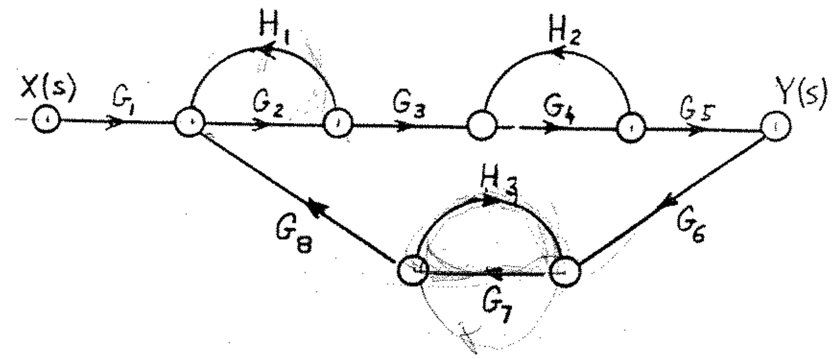


(a) Assuming $G(s) = K/(s^2 + s)$, determine for the feedback system:
(i) undamped natural frequency ω_n, damping ratio ξ, percent overshoot %OS, peak time T_P, and settling time T_S (choose K=4)
(ii) sensitivity function S_K^T

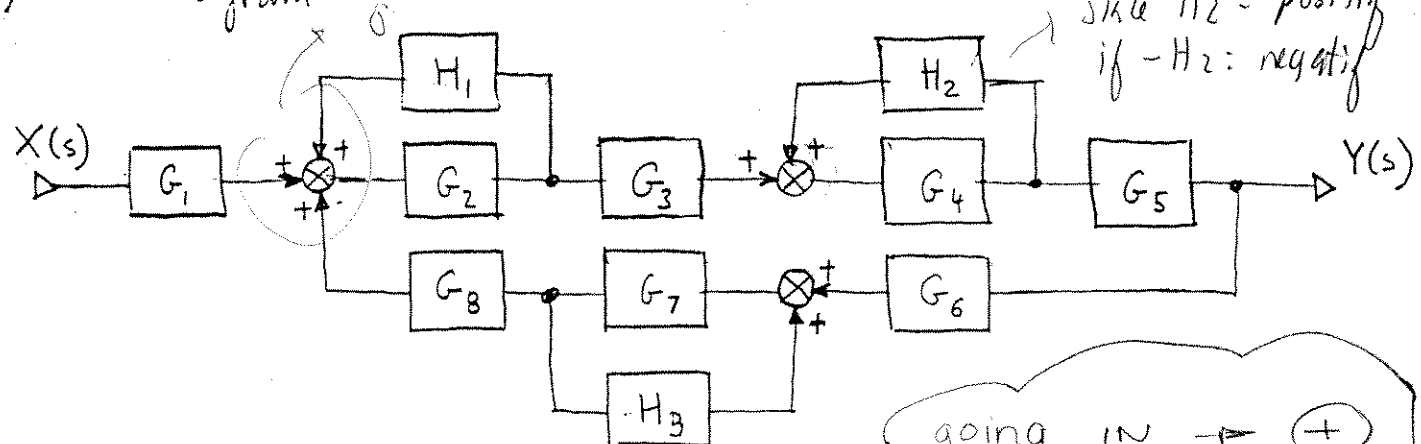
(b) Assuming $G(s) = 8/(s^3+4s^2)$, determine for the feedback system:
(i) the error constants K_P , K_V , K_A
(ii) the steady-state error resulting from the input $X(s) = 3/s^3 + 2/s^2 + 1/s$

$\frac{8}{s^2(s+4)}$ $e_{ss} = \frac{1}{K_A}$

oblem 1

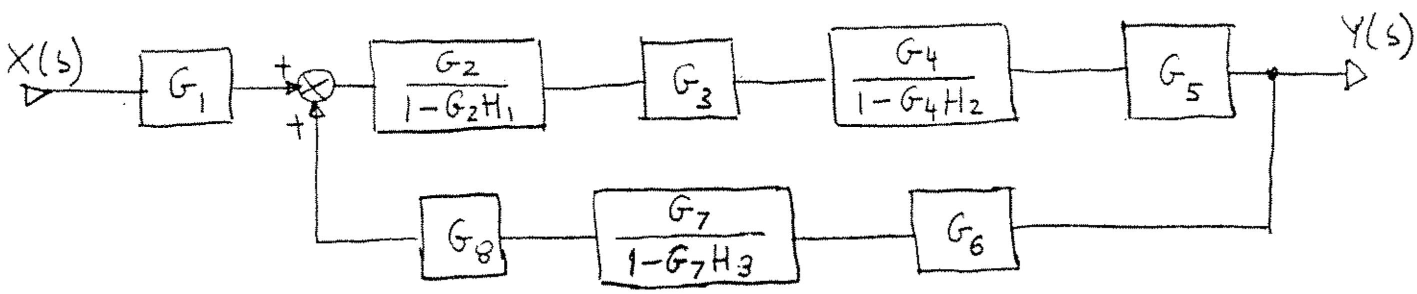


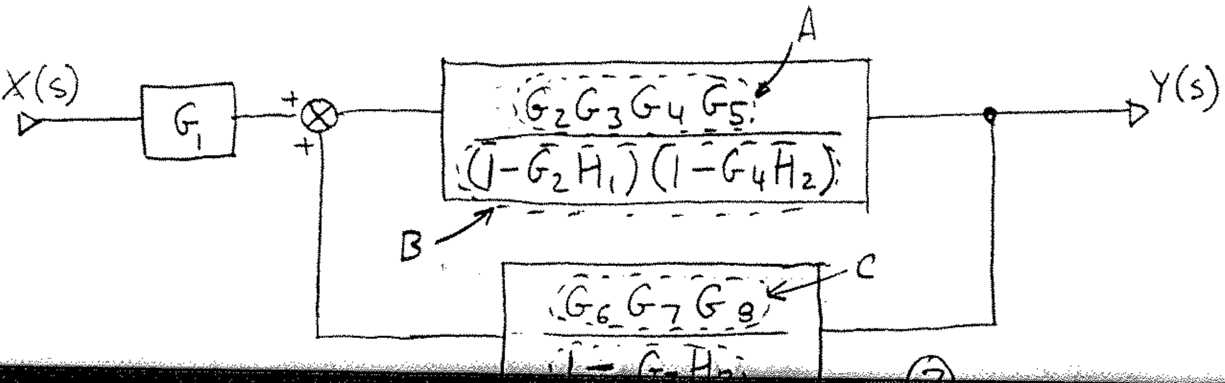
(a) block diagram



going IN → +
going OUT → −

(b) transfer function
(i) by block diagram





then:

$$\underline{\underline{T(s)}} = \frac{Y(s)}{X(s)} = G_1 \frac{\frac{A}{B}}{1-\frac{A}{B}\cdot\frac{C}{D}} = \frac{G_1 A D}{BD - AC} = \cdots =$$

$$= \frac{G_1G_2G_3G_4G_5\,(1-G_7H_3)}{1-G_2H_1-G_4H_2-G_7H_3+G_2G_4H_1H_2+G_2G_7H_1H_3+G_4G_7H_2H_3-G_2G_4G_7H_1H_2H_3-G_2G_3G_4G_5G_6G_7G_8}$$

(ii) by Mason's rule

1. forward path's

$K=1$; $T_1 = G_1G_2G_3G_4G_5$

2. loops

- singles

$L_1 = G_2H_1$; $L_2 = G_4H_2$; $L_3 = G_7H_3$; $L_4 = G_2G_3G_4G_5G_6G_7G_8$

- non-touching two's

$L_1L_2 = G_2G_4H_1H_2$; $L_1L_3 = G_2G_7H_1H_3$; $L_2L_3 = G_4G_7H_2H_3$

- non-touching three's

$L_1L_2L_3 = G_2G_4G_7H_1H_2H_3$

3. determinant

$$\Delta = 1-(L_1+L_2+L_3+L_4)+(L_1L_2+L_1L_3+L_2L_3)-(L_1L_2L_3) = \cdots =$$

$$1-G_2H_1-G_4H_2-G_7H_3+G_2G_4H_1H_2+G_2G_7H_1H_3+G_4G_7H_2H_3-G_2G_4G_7H_1H_2H_3-G_2G_3G_4G_5G_6G_7G_8$$

4. co-factors

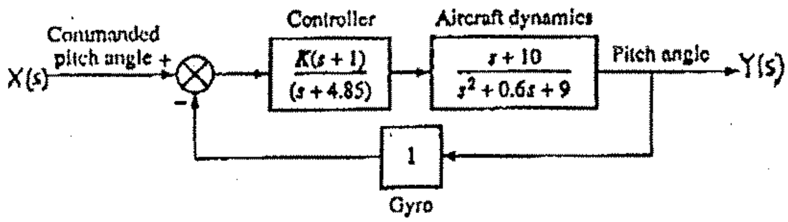
$T_1\Delta_1 = G_1G_2G_3G_4G_5(1-G_7H_3)$

where $\Delta_1 = \Delta$ without $(L_1, L_2, L_4) = 1 - L_3 = 1 - G_7H_3$

5. transfer function

$$T(s) = \frac{T_1 \Delta_1}{\Delta} = \ldots \text{ same as in } b)(i)$$

Problem 2



(a)

$$T(s) = \frac{\frac{K(s+1)}{(s+4.85)} \cdot \frac{s+10}{s^2+0.6s+9}}{1+\frac{K(s+1)}{(s+4.85)} \cdot \frac{s+10}{s^2+0.6s+9}} = \ldots = \frac{K(s+1)(s+10)}{s^3+(5.45+K)s^2+(11.91+11K)s+(43.65+10K)}$$

(b) Routh-Hurwitz

s^3	1	$(11.91+11K)$
s^2	$(5.45+K) > 0$	$(43.65+10K)$
s^1	$\frac{11K^2+61.96K+21.26}{5.45+K}$	
s^0	$\Rightarrow 43.65+10K > 0$	—

$-b \pm \sqrt{b^2-4ac}$ / $2a$

satisfy all the condition

Smallest from negative side.

- from conditions:

  $5.45+K > 0 \Rightarrow K > -5.45$

  $-11K^2+61.86K+21.26 > 0$;

  $43.65+10K > 0 \Rightarrow K > -4.37$

  $\Rightarrow K > \begin{cases} -0.368 \\ -5.26 \end{cases}$

- thus stability condition is:

  $-0.369 < K < +\infty$

Range

Sole pair K

(c). for marginal stability $\underline{K = -0.369}$ lowest K stable

- auxiliary equation $(5.45-0.369)s^2 + (43.65-10\times 0.369) = 0$

$5.45+K$

$s^2 + 7.87 = 0 \quad = s^2 + \omega_{osc}^2 = 0$

p 299

$s^2 + \omega_{osc}^2 = 0$

$K = -0.369$

$\Rightarrow$ oscillatory frequency

$$\omega_{osc} = \sqrt{7.87} = 2.81 \text{ rad/sec}$$

$90 = 9.291$

sole pairs - find when $K = 0$

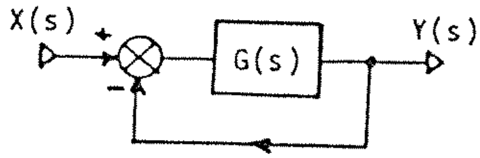
if $K = 4.37$ $\quad 5.082\, s^2 - 39.47 = 0$

$5.45 + K$

$5.082\, s^2 = 39.47$

take the s^1 ...

Problem 3



$$\frac{G}{1+GH} = \frac{\frac{K}{s^2+s}}{1+\frac{K}{s^2+s}}$$

(a) $G(s) = K/(s^2+s) \Rightarrow T(s) = \dfrac{K}{s^2+s+K}$

3 (i) $T(s) = \dfrac{4}{s^2+s+4} = \dfrac{K\cdot\omega_n^2}{s^2+2\xi\omega_n+\omega_n^2}$

$\Rightarrow \underline{\underline{\omega_n = 2\text{ rad/sec}}}$, $\underline{\underline{\xi = 0.25}}$

$\underline{\underline{\%OS}} = 100e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} = \underline{\underline{44.5\%}}$; $\underline{\underline{T_p}} = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} = \underline{\underline{1.62\text{ sec}}}$

$\underline{\underline{T_s}} = \dfrac{4}{\xi\omega_n} = \underline{\underline{8\text{ sec}}}$

3 (ii) $\underline{\underline{S_K^T}} = \dfrac{K}{T}\cdot\dfrac{\partial T}{\partial K} = \cdots = \underline{\underline{\dfrac{s^2+s}{s^2+s+K}}}$

(b) $G(s) = GH(s) = 8/s^2(s+4)$; type-2 system

2 (i) $\underline{\underline{K_p = \infty}}$; $\underline{\underline{K_v = \infty}}$; $\underline{\underline{K_A}} = \lim\limits_{s\to 0} s^2GH(s) = \dfrac{8}{4} = \underline{\underline{2}}$

2 (ii) $\underline{\underline{e_{ss}}} = \dfrac{3}{K_A} + 0 + 0 = \underline{1.5}$

FUNDAMNETALS OF CONTROL SYSTEMS Section X

Mid-Term Thursday March 7, 2002

Answer All 4 Questions (total 40 marks)

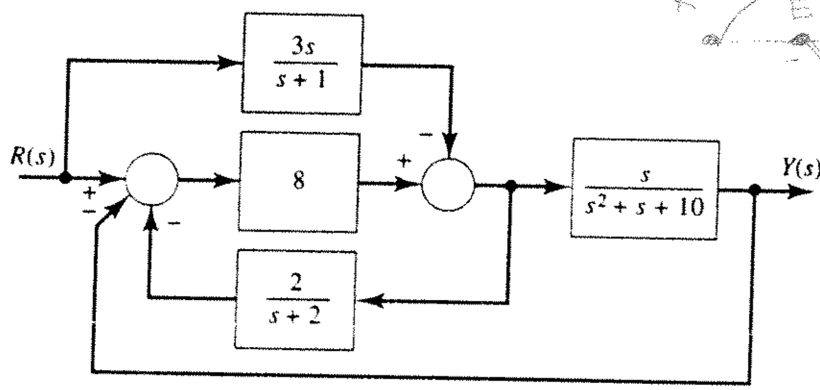
Question #1 (10 marks)

For the system shown in the block diagram:

a) Draw the signal flow graph.

b) Using Mason's loop rule, determine the overall system transfer function $\frac{Y(s)}{R(s)}$.

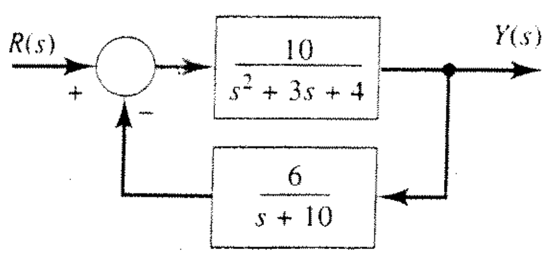
Show all mathematical steps.



Question #2 (10 marks)

a) Determine the type number of the system.
b) Using the concept of system type number, determine the steady state error to a unit step input. Kp
c) Using the concept of system type number, determine the steady state error to a unit ramp input. Kv
d) Using the concept of system type number, determine the steady state error to a unit parabola input. Ka

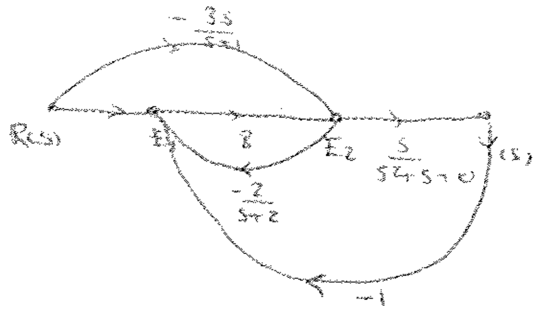
Show all equations and mathematical steps.
Answers without mathematical proofs will be given a mark of zero.



Midterm, March 2002

Q1

(a) sfg



(b) Mason's rule $\frac{Y(s)}{R(s)}$

$$P_1 = \frac{8\ s}{s^2+s+10}$$

$$P_2 = -\frac{3s}{s+1}\cdot\left(\frac{s}{s^2+s+10}\right)$$

$$L_1 = -\frac{16}{s+2}$$

$$L_2 = -\frac{8\ s}{s^2+s+10}$$

$$T(s) = \frac{P_1+P_2}{1-(L_1+L_2)} =$$

cont.

steady state error.

(b) $\boxed{2}$ $K_p = \lim_{s \to 0} \bar{G}$; $e_{ss} = \frac{1}{1+K_p}$

(c) $K_v = \lim_{s \to 0} s\bar{G}$; $e_{ss} = \frac{1}{K_v}$.

(d) $K_a = \lim_{s \to 0} s^2\bar{G}$; $e_{ss} = \frac{1}{K_a}$.

2B ??
Not understood.

Q3. $T_s \leq 4$ sec.

$P.O\% \leq 4.3\%$

(a)
$$P.O = 100 e^{\left(\frac{-\zeta \pi}{\sqrt{1-\zeta^2}}\right)} \leq 4.3$$

$$\zeta \geq 1/\sqrt{2}$$

$$T_s = \frac{4}{\zeta \omega_n} \leq 4$$

$$\omega_n \geq \frac{1}{\zeta} = \sqrt{2}.$$

$$T s = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

.80

CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE

DEPARTMENT OF MECHANICAL AND INDUSTRIAL ENGINEERING
MECH 371/4 sec. X FUNDAMENTALS OF CONTROL SYSTEMS

Mid-Term **Instructor: H. Hong** **Date: March 4, 2003**

Answer all four (4) questions. Closed book exam. Total time: 1 hour 15 minutes

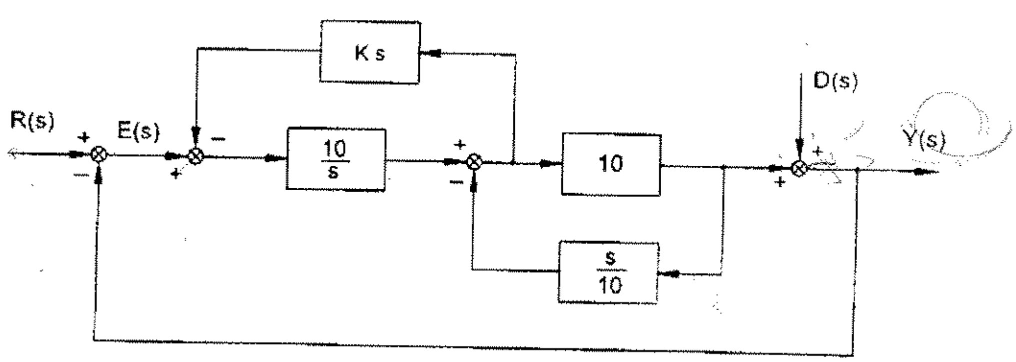
For all questions:
Clearly show all equations and mathematical steps.
Answers without mathematical steps will be given a mark of zero.

Problem 1.→ 23; Problem 2.→ 10; Problem 3.→ 10; Problem 4.→ 12 = 55 marks total.

Problem 1. [1→ 3, 2→ 2, 3→ 6, 4→ 4, 5→ 6, 6→ 2 = 23 marks]

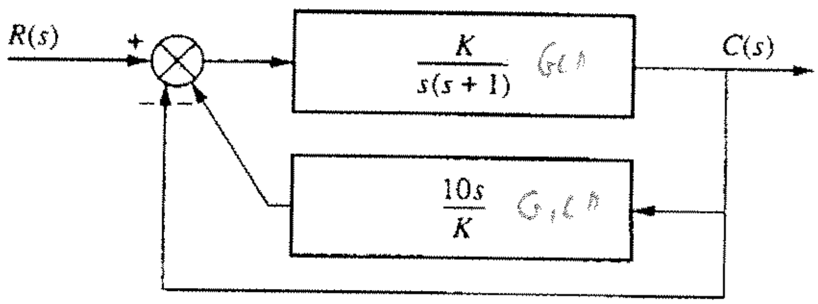
A negative feedback control system has a block diagram as shown below.

(1) Draw the signal flow graph for the system.
(2) Determine the closed-loop transfer function T(s) = Y(s)/R(s).
(3) For a unit step input, find the value for the gain K so that the system has 20% overshoot.
What is the natural frequency ω_n of the system?
What is the settling time T_S of the system?
(4) Sketch the closed-loop poles on the s-plane. Clearly indicate ω_n (natural frequency), ω_d (damped natural frequency), and θ (angular representation of the damping ratio ζ), and their numerical values, on the figure.
(5) Find the total steady-state response due to a unit step input R(s) and a unit step disturbance D(s).
(6) Determine the sensitivity of the transfer function T(s) with respect to gain K, ie: S_K^T



Problem 2. [10 marks]

For the feedback control system shown below:
(1) Determine the type number of the system.
(2) Using the concept of system type number, determine the steady state error to a unit step input.
(3) Using the concept of system type number, determine the steady state error to a unit ramp input.
(4) Using the concept of system type number, determine the steady state error to a unit parabola input.



Problem 3. [10 marks]

Using the Routh-Hurwitz test, determine the stability of the closed-loop transfer function:

$$T(s) = \frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56}$$

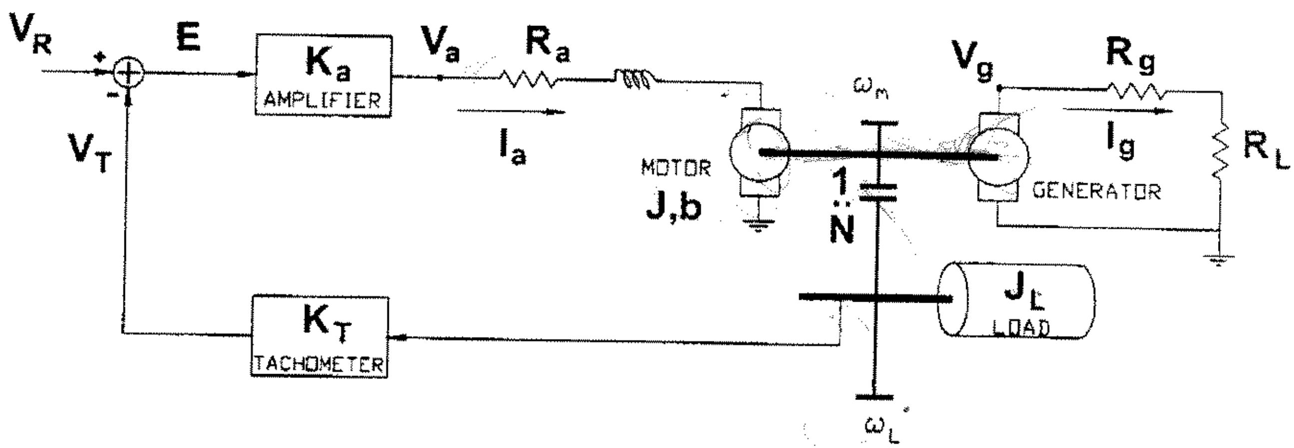
Determine, if any,
(1) the marginally stable roots, and their numerical values.
(2) the total number of unstable roots.

HINT: The negative number -4 is a vector written as $-4+0j$ or as $4e^{+180j}$.

Problem 4. [12 marks]

A permanent magnet DC-motor drives a rotational load through a speed reduction gear ratio. The motor also directly drives a generator that supplies current to a resistive load. The load speed is measured by a tachometer for velocity feedback control. The error between the reference and tachometer voltage drives an amplifier that supplies power to the armature of the DC-motor.

Draw the block diagram for the feedback control system shown in the schematic diagram below. Provide justification to any assumptions made.



<u>Given equations with respect to schematic diagram:</u>

$E(s) = V_R(s) - V_T(s)$	error = reference voltage minus tachometer voltage
$V_T(s) = K_T\omega_L(s)$	tachometer voltage = tachometer gain times load speed
$V_a(s) = K_a E(s)$	amplifier voltage = amplifier gain times error voltage
$V_a(s) = R_a I_a(s) + V_b(s)$	amplifier (or armature) voltage = armature resistance times armature current plus back EMF [armature inductance neglected]
$V_b(s) = K_b\omega_m(s)$	back EMF = back EMF gain times motor speed
$T(s) = K_m I_a(s)$	motor generated torque = motor constant times armature current
$T_m(s) = T(s) - T_d(s)$	torque driving motor = motor generated torque less torque driving disturbances
$T_m(s) = J\dot{\omega}_m(s) + b\omega_m$	torque driving motor = armature inertia times motor speed plus damping times motor speed
$T_d(s) = T_{d1}(s) + T_{d2}(s)$	disturbance torque = torque driving inertia load plus torque driving generator

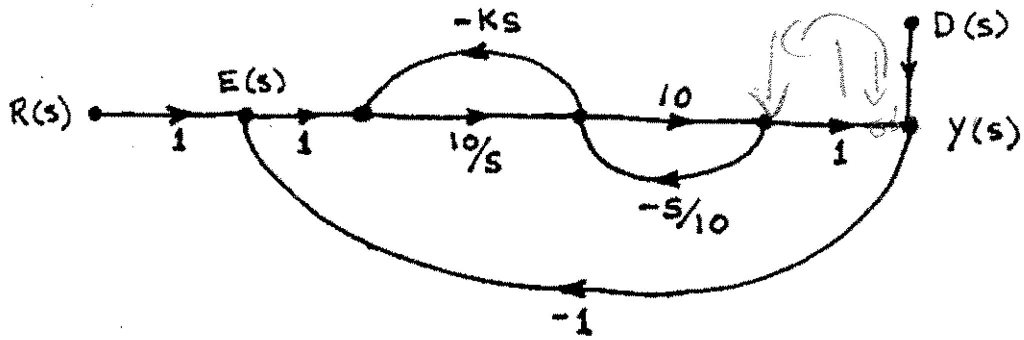
Equations that must be derived from schematic diagram:

$\omega_L(s) = ??$	✓	load speed $\omega_L(s) \Rightarrow$ motor speed $\omega_m(s)$ with gear reduction. Let gear ratio be N>1
$T_{d1}(s) = ??$	✓	load torque $T_{d1}(s) \Rightarrow$ rotational load inertia J_L; omit damping; load angular acceleration $\dot{\omega}_L(s)$; gear ratio N>1
$V_g(s) = ??$		generator voltage $V_g(s) \Rightarrow$ generator voltage gain K_g; generator (or motor) speed $\omega_m(s)$
$I_g(s) = ??$		V_g - [illegible] generator current $I_g(s) \Rightarrow$ generator armature resistance R_g; resistive load R_L; generator voltage $V_g(s)$; neglect generator inductance
$T_{d2}(s) = ??$		generator torque $T_{d2}(s) \Rightarrow$ generator constant K_{GG}; generator current $I_g(s)$

$$I_g = \frac{1}{R_g + R_L} V_g$$

Problem # 1

(1)



(2) $$T(s) = \frac{Y(s)}{R(s)} = \frac{\left(\frac{10}{s}\right)(10)}{1 - \left(-[Ks]\left[\frac{10}{s}\right] - [10]\left[\frac{s}{10}\right] - \left[\frac{10}{s}\right][10]\right)}$$

$$T(s) = \frac{10^2/s}{1 + 10K + s + 10^2/s}$$

$$T(s) = \frac{100}{s^2 + s(10K+1) + 100}$$

(3) $$s^2 + s(10K+1) + 100 = s^2 + 2\xi\omega_n s + \omega_n^2$$

$\omega_n = 10$ rad/sec

$2\xi\omega_n = 10K + 1$

$$P.O. = 20 = 100 \exp\left[\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right]$$

$$\ln\left[\frac{20}{100}\right] = -\frac{\xi\pi}{\sqrt{1-\xi^2}} = -1.6094$$

1/6

$$1 - \xi^2 = 3.8104\,\xi^2$$

$$\therefore\ \xi = 0.4559$$

$$K = \frac{2\xi\omega_n - 1}{10} = \frac{(2)(0.4559)(10) - 1}{10}$$

$$K = 0.8119$$

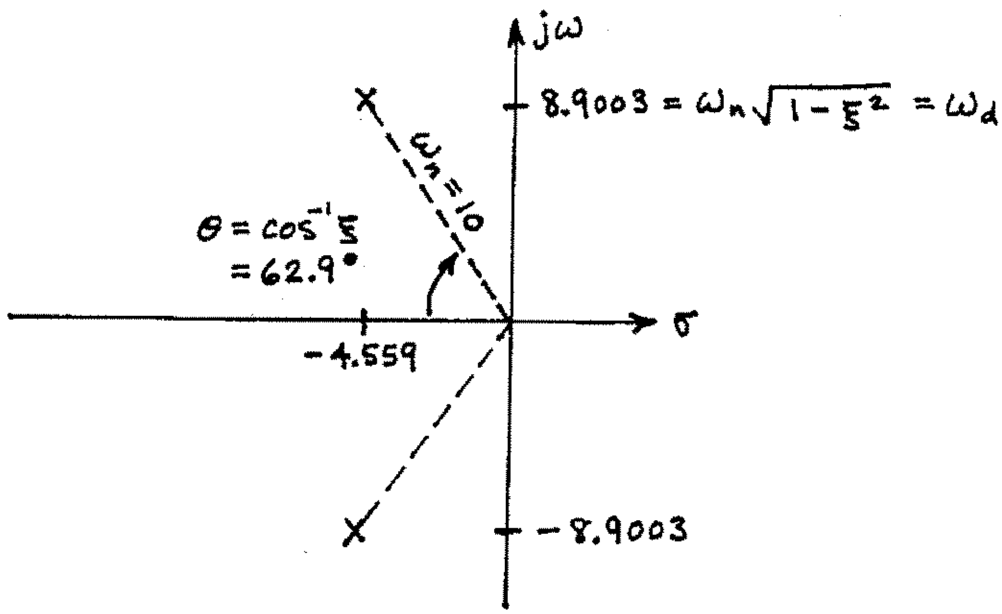
$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{(0.4559)(10)}$$

$$T_s = 0.8774 \text{ sec.}$$

(4) $s^2 + 9.118s + 100$

$$\frac{-9.118 \pm \sqrt{(9.118)^2 - 4\times 100}}{2} = \frac{-9.118 \pm 17.8006j}{2}$$

$$S_1, S_2 = \underbrace{-4.559}_{\xi\omega_n} \pm \underbrace{8.9003}_{\omega_d}\,j$$



2/6

(5) $Y(s) = T_1 R(s) + T_2 D(s)$

$$T_1 = \frac{100}{S^2 + S(10K+1) + 100}$$

$$T_2 = \frac{1\cdot(1+10K+S)}{1+10K+S+\frac{100}{S}} = \frac{S(1+10K+S)}{S^2+S(10K+1)+100}$$

for $R(s) = \frac{1}{S}$ and $D(s) = \frac{1}{S}$

$$y(\infty) = \lim_{s\to 0} \cancel{S}\left[\frac{100}{S^2+S(10K+1)+100}\right]\cdot\frac{1}{\cancel{S}}$$

$$+ \lim_{s\to 0} \cancel{S}\left[\frac{S(1+10K+S)}{S^2+S(10K+1)+100}\right]\cdot\frac{1}{\cancel{S}}$$

$$Y(\infty) = 1 + 0 = 1$$

(6) $S_K^T = \frac{\partial T}{\partial K}\cdot\frac{K}{T}$

$$\frac{\partial T}{\partial K} = -\frac{10S}{[S^2+S(10K+1)+100]^2}$$

$$S_K^T = \frac{-10S}{[S^2+S(10K+1)+100]^2}\cdot K\cdot\frac{[S^2+S(10K+1)+100]}{100}$$

$$S_K^T = \frac{10\ -KS}{10[S^2+S(10K+1)+100]}$$

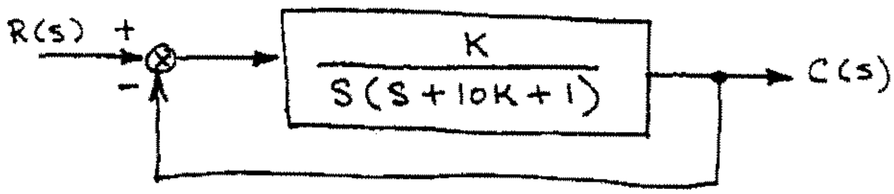
3/6

7

Problem #2

Reduce inner loop
$$\frac{\frac{K}{s(s+1)}}{1+\left(\frac{\cancel{K}}{\cancel{s(s+1)}}\right)\left(\frac{10\cancel{s}}{\cancel{K}}\right)}$$

$$= \frac{K}{s(s+1)+10Ks} = \frac{K}{s[s+10K+1]}$$



(1) Type 1 system, $G(s) = \frac{K}{s(s+10K+1)}$

(2) $K_p = \lim_{s \to 0} G(s) = \infty$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

(3) $K_v = \lim_{s \to 0} s\,G(s) = \frac{K}{10K+1}$

$$e_{ss} = \frac{1}{K_v} = \frac{10K+1}{K}$$

(4) $K_a = \lim_{s \to 0} s^2 G(s) = 0$

$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$, ie: cannot track acceleration input

4/6

<u>Problem #3</u> $S^5 + 7S^4 + 6S^3 + 42S^2 + 8S + 56$

S^5	1	6	8
S^4	7	42	56
S^3	$\frac{6\times7-1\times42}{7}$	$\frac{7\times8-1\times56}{7}$	0
	$= 0$	$= 0$	0

Since S^3 row are all zero, from S^4 row (auxilary equation)

$$F(s) = 7S^4 + 42S^2 + 56$$

$$\frac{dF(s)}{ds} = 28S^3 + 84S \Longrightarrow \text{new } S^3 \text{ row}$$

S^5	1	6	8
S^4	7	42	56
S^3	28	84	0
S^2	$\frac{28\times42-7\times84}{28} = 21$	$\frac{28\times56-0}{28} = 56$	0
S^1	$\frac{21\times84-28\times56}{21} = 9.33$	0	0
	9.33		

(2) First column has no changes in sign, $\therefore$ no roots in RHP.

(1) From auxilary equation, $F(s) = S^4 + 6S + 8 = 0$

$$S_1^2, S_2^2 = \frac{-6 \pm \sqrt{36 - 4\times8}}{2} = -3 \pm 1$$

$$S_1^2 = -4 = 4e^{180j} \quad \text{and} \quad S_2^2 = -2 = 2e^{180j}$$

$$\therefore S_1 = 2e^{\pm 90j} \quad \text{and} \quad S_2 = \sqrt{2}\, e^{\pm 90j}$$

5/6

Problem # 4

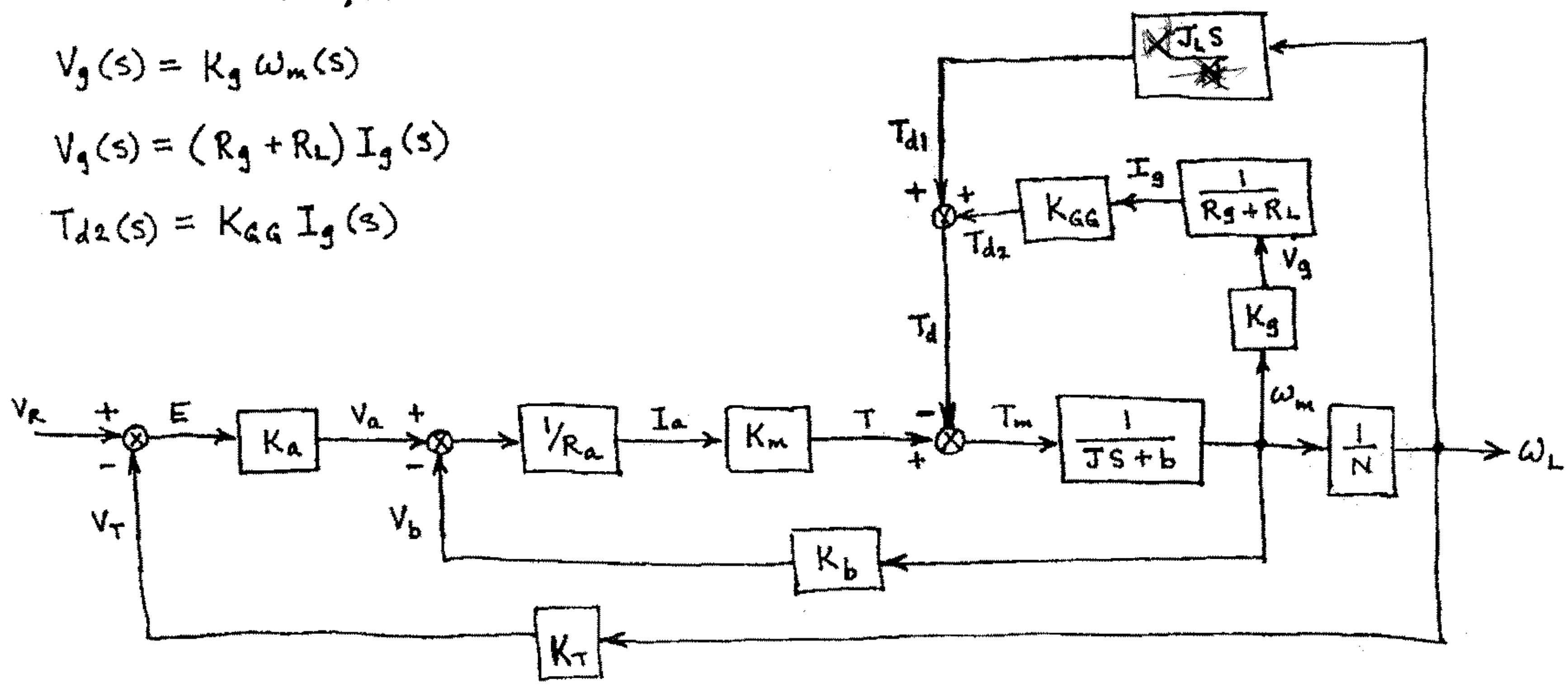
$\omega_L(s) = \omega_m(s)/N$

$T_{d1}(s) = J_L \dot{\omega}_L(s)/N$

$V_g(s) = K_g\,\omega_m(s)$

$V_g(s) = (R_g + R_L)\,I_g(s)$

$T_{d2}(s) = K_{GG}\,I_g(s)$



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MECH 371/4

1.12

FUNDAMNETALS OF CONTROL SYSTEMS Section X

Mid-Term

Thursday March 7, 2002

Answer All 4 Questions (total 45 marks)

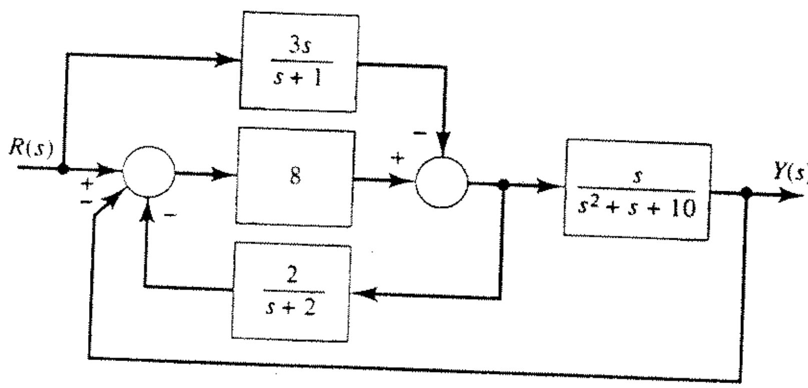
Question #1 (10 marks)

For the system shown in the block diagram:

a) Draw the signal flow graph.

b) Using Mason's loop rule, determine the overall system transfer function $\dfrac{Y(s)}{R(s)}$.

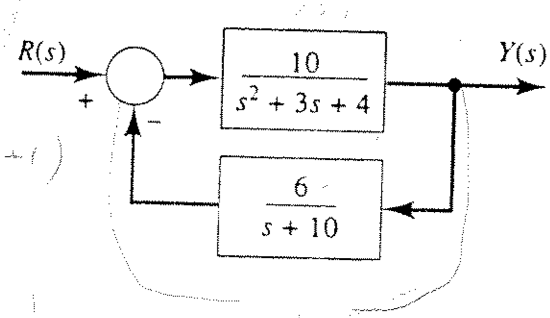
Show all mathematical steps.



Question #2.A (10 marks)

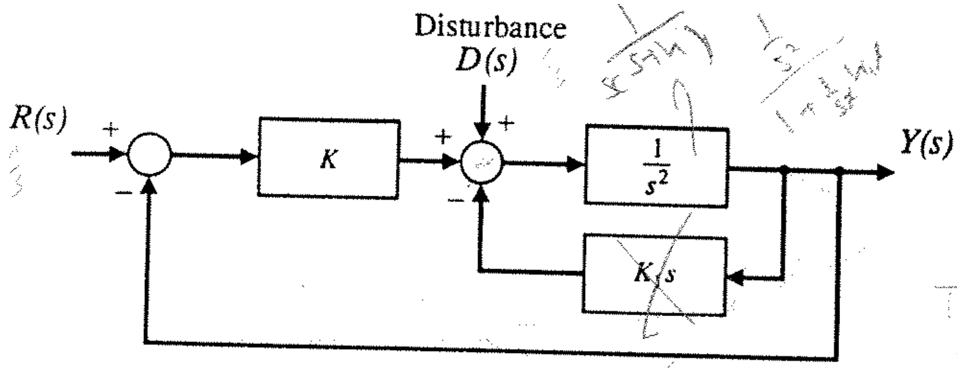
a) Determine the type number of the system.
b) Using the concept of system type number, determine the steady state error to a unit step input.
c) Using the concept of system type number, determine the steady state error to a unit ramp input.
d) Using the concept of system type number, determine the steady state error to a unit parabola input.

Show all equations and mathematical steps.
Answers without mathematical proofs will be given a mark of zero.



Question #2.B (5 marks)

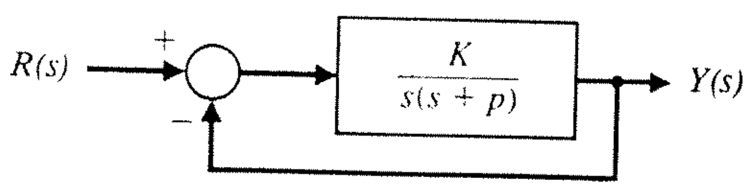
For the system shown, the command signal $R(s)$ and disturbance signal $D(s)$ are both unit step inputs. Determine the gain K so that the steady state output $y(t)$ is 1.01 .



Question #3 (10 marks)

For the system shown, it is required that the settling time $T_S \leq 4$ seconds and the percent overshoot $P.O. \leq 4.3\%$. Determine:

a) The gain K and parameter p necessary to satisfy the above performance requirements.
b) On the s-plane, plot the region of valid characteristic root locations.
c) Determine the equation for the sensitivity of the overall system transfer function with respect to parameter p. ie: determine S_p^T



Question #4 (10 marks)

The characteristic equation of a system is given by: $s^6 + s^5 + 5s^4 + s^3 + 2s^2 - 2s - 8$

Using the Routh-Hurwitz test, determine:

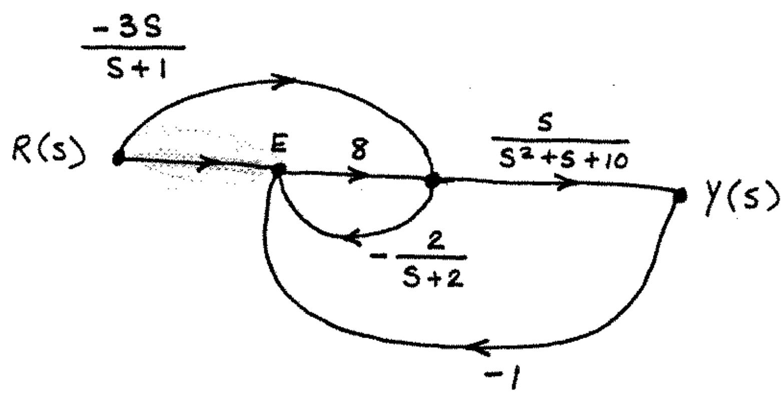
a) If any, the marginally stable roots, and their numerical values.
b) If any, the total number of unstable roots.

$= \frac{1}{s^2 + 4s + 1}$

HINT: The negative number -4 is a vector written as $-4 + 0j$ or as $4e^{+180j}$.

#1

a)



b) $P_1 = -\frac{3S}{S+1}\left(\frac{S}{S^2+S+10}\right)$

$P_2 = \frac{8S}{S^2+S+10}$

$L_1 = -\frac{8\times 2}{S+2} \qquad L_2 = -\frac{8S}{S^2+S+10}$

$$T(s) = \frac{-\frac{3S}{S+1}\left(\frac{S}{S^2+S+10}\right) + \frac{8S}{S^2+S+10}}{1 + \frac{16}{S+2} + \frac{8S}{S^2+S+10}}$$

$$= \frac{-3S^2\,\frac{S+2}{S+1} + 8S(S+2)}{(S+2)(S^2+S+10) + 16(S^2+S+10) + 8S(S+2)}$$

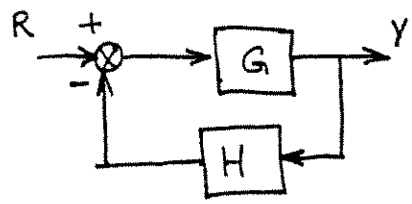
$$\begin{array}{r} S^3+S^2+10S \\ +2S^2+2S+20 \\ +16S^2+16S+160 \\ +8S^2+16S \\ \hline S^3+27S^2+44S+180 \end{array}$$

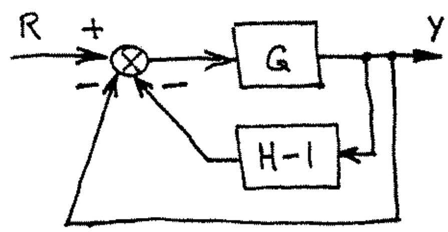
$$= \frac{-3S^2(S+2) + 8S(S+2)(S+1)}{(S+1)(S^3+27S^2+44S+180)}$$

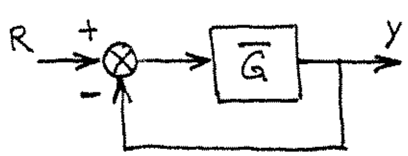
$$= \frac{5S^3+18S^2+16S}{S^4+28S^3+71S^2+224S+180}$$

$$\begin{array}{r} S^4+27S^3+44S^2+180S \\ +\quad S^3+27S^2+44S+180 \\ \hline S^4+28S^3+71S^2+224S+180 \end{array}$$

#2.A







where

$$\overline{G} = \frac{G}{1+G(H-1)}$$

$$\overline{G} = \frac{\frac{10}{s^2+3s+4}}{1+\frac{10}{s^2+3s+4}\cdot\left(\frac{6}{s+10}-1\right)}$$

$$= \frac{\frac{10}{s^2+3s+4}}{1+\frac{10}{s^2+3s+4}\cdot\left(\frac{-s-4}{s+10}\right)}$$

$$= \frac{10(s+10)}{(s^2+3s+4)(s+10)-10(s+4)}$$

$$\begin{array}{r} s^3+3s^2+4s \\ +10s^2+30s+40 \\ -10s-40 \\ \hline s^3+13s^2+24s \end{array} = s(s^2+13s+24)$$

$$\overline{G} = \frac{10(s+10)}{s(s^2+13s+24)} \Rightarrow \text{Type 1}$$

b) $k_p = \lim_{s\to 0} \overline{G} = \frac{100}{0} = \infty$

$$e_{ss} = \frac{1}{1+k_p} = \frac{1}{\infty} = 0$$

c) $k_v = \lim_{s\to 0} s\overline{G} = \lim_{s\to 0} \frac{10(s+10)}{s^2+13s+24} = \frac{100}{24}$

$$e_{ss} = \frac{1}{k_v} = \frac{24}{100} = \frac{6}{25}$$

d) $k_a = \lim_{s\to 0} s^2\overline{G} = \lim_{s\to 0} \frac{s(10)(s+10)}{s^2+13s+24} = 0$

$$e_{ss} = \frac{1}{k_a} = \frac{1}{0} = \infty$$ cannot track acceleration input

#2.B

$$\frac{Y_R}{R} = \frac{K/s^2}{1 + \left(K/s^2 + K_1/s\right)} = \frac{K}{s^2 + K_1 s + K}$$

$$T(s) = \frac{Y_D}{D} = \frac{1/s^2}{1 + \left(K/s^2 + K_1/s\right)} = \frac{1}{s^2 + K_1 s + K}$$

$$Y_{ssR} = \lim_{s \to 0} s \underbrace{\left(\frac{K}{s^2 + K_1 s + K}\right)}_{T(s)} \cdot \underbrace{\frac{1}{s}}_{R(s)} = 1 \qquad y_{ss} = \lim_{s \to 0} s \overbrace{T(s)\,R(s)}^{Y(s)}$$

$$Y_{ssD} = \lim_{s \to 0} s \left(\frac{1}{s^2 + K_1 s + K}\right) \frac{1}{s} = \frac{1}{K}$$

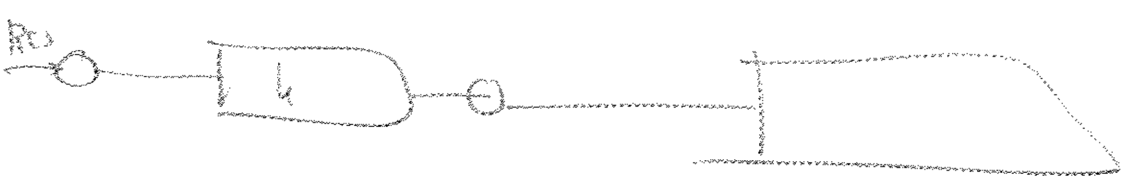
$$Y_{ss} = 1.01 = Y_{ssR} + Y_{ssD} = 1 + \frac{1}{K}$$

$$\therefore K = 100$$

Steady state response $= y_{ss} =$

$$\frac{\frac{1}{s^2}}{1 + \frac{K_1}{s}} = \frac{\frac{1}{s^2}}{\frac{s + K_1}{s}}$$

$$Y = T_1 R + T_2 D(s)$$



$$\Rightarrow \frac{\frac{1}{s^2}}{\frac{1}{s(s+K_1)}} \qquad = \frac{1}{s(s+K_1)}$$

#3

$$\text{P.O.} \Rightarrow \quad 100 \exp\left[\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right] \le 4.3$$

$$-\frac{\xi\pi}{\sqrt{1-\xi^2}} \le \ln \frac{4.3}{100} = -\pi$$

$$\xi \ge \sqrt{1-\xi^2} \quad \text{or} \quad \xi^2 \ge 1-\xi^2$$

$$\therefore \quad \xi \ge \frac{1}{\sqrt{2}} \quad (0.707)$$

$$T_s \Rightarrow \frac{4}{\xi\omega_n} \le T_s = 4$$

$$\therefore \quad \omega_n \ge \frac{1}{\xi} = \sqrt{2}$$

$$(1.414)$$

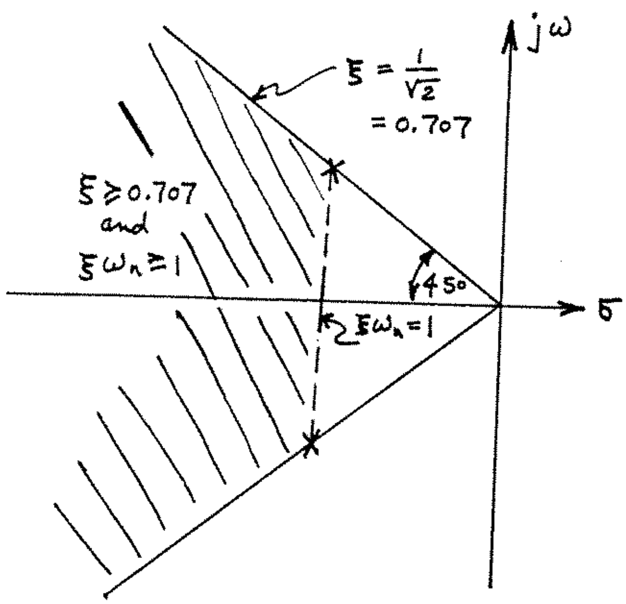
a)
$$T(s) = \frac{\frac{K}{s(s+p)}}{1+\frac{K}{s(s+p)}} = \frac{K}{s^2+ps+K} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\therefore \quad K = \omega_n^2 = 2 \quad \text{and} \quad p = 2\xi\omega_n = 2\cdot\frac{1}{\sqrt{2}}\cdot\sqrt{2} = 2$$

$$s^2+ps+K = s^2+2s+2$$

b) $\xi\omega_n \ge 1$

$\theta = \cos^{-1}\xi = 45°$



c)
$$T(s) = \frac{K}{s^2+ps+K}$$

$$S_p^T = \frac{\partial T}{\partial p}\cdot\frac{p}{T}$$

$$= \frac{-sK}{(s^2+ps+K)^2}\cdot\frac{s^2+ps+K}{K}\cdot p$$

$$\therefore \quad S_p^T = \frac{-ps}{s^2+ps+K}$$

#4

s^6	1	5	2	−8
s^5	1	1	−2	
s^4	4	4	−8	
s^3	0	0		

Because row s^3 are all zero, there are roots on the imaginary axis.
From the auxiliary equation (row s^4):

$$4s^4 + 4s^2 - 8 = 0 \implies s^4 + s^2 - 2 = 0$$

$$\therefore \; s_{1,2}^2 = \frac{-1 \pm \sqrt{1-4(-2)}}{2} = \frac{-1 \pm 3}{2} \implies s_1^2 = +1 \text{ and } s_2^2 = -2$$

$$\therefore \; s_1 = \pm 1$$

Since $-2 = 2e^{180j}$

$$\therefore \; s_2 = \sqrt{2}\, e^{\pm 90j} \implies \text{marginally stable roots}$$

To fill the Routh-Hurwitz table, replace row s^3 by:

$$\frac{d}{ds}(4s^4 + 4s^2 - 8) = 16s^3 + 8s$$

s^6	1	5	2	−8
s^5	1	1	−2	
s^4	4	4	−8	
s^3	16	8		
s^2	2	−8		$\leftarrow \frac{16\times4 - 4\times8}{16} = 2$
s^1	72			$\leftarrow \frac{2\times8 - 16\times(-8)}{2} = 72$
s^0	−8			

change in sign indicates 1 root in RHP

Imaginary Roots = 2 Roots in the RHP (unstable) = 1

85%

CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
ENGR 372/4 sec. Y FUNDAMENTALS OF CONTROL SYSTEMS

Mid-Term **Instructor: Dr. H. Hong** **Date: March 8, 2001**

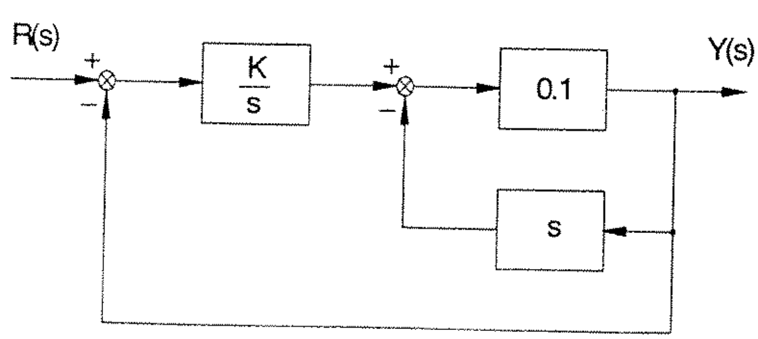
Answer all three (3) questions. **Closed book exam.** **Total time: 1 hour 15 minutes**

Problem 1.→23; Problem 2.→7; Problem 3.→5; = 35 marks total.

Problem 1. [a→1, b→7, c→3, d→2, e→1, f→3, g→3, h→3 = 23 marks]

A negative feedback control system has a block diagram as shown below.

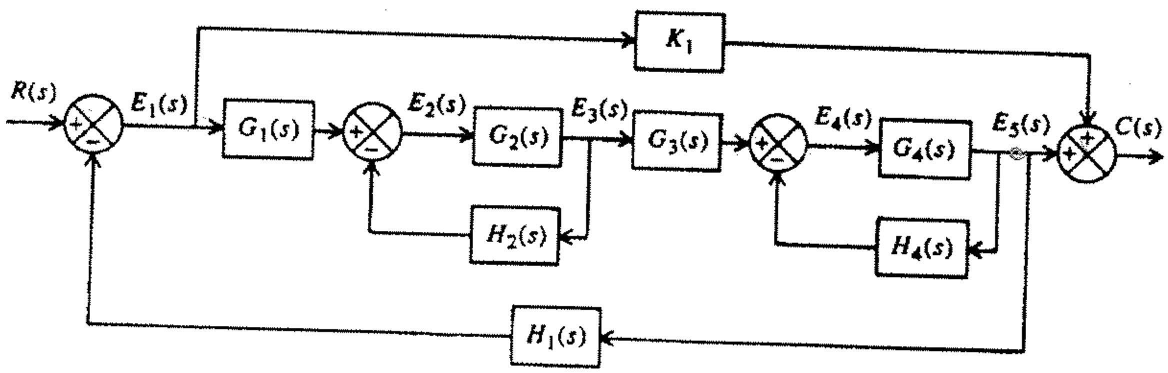
(1) Determine the closed-loop transfer function T(s) = Y(s)/R(s).
(2) For a unit step input, find the value for the gain K so that the system has 20% overshoot.
What is the natural frequency ω_n of the system?
What is the settling time T_s of the system?
(3) Sketch the closed-loop poles on the s-plane. Clearly indicate ω_n (natural frequency) and θ (angular representation of the damping ratio ζ), and their numerical values, on the figure.
(4) Using the Routh-Hurwitz stability criterion determine the range of gain K for the system to be stable?
(5) What is the "type number" of the system?
(6) For a unit step input, determine the position error constant K_p , and the steady-state error. Show all mathematical steps leading to your answer.
(7) For a unit ramp input, determine the velocity error constant K_v , and the steady-state error. Show all mathematical steps leading to your answer.
(8) For a unit acceleration input, determine the acceleration error constant K_a , and the steady-state error. Show all mathematical steps leading to your answer.



Problem 2. [a→1½, b→5½ = 7 marks]

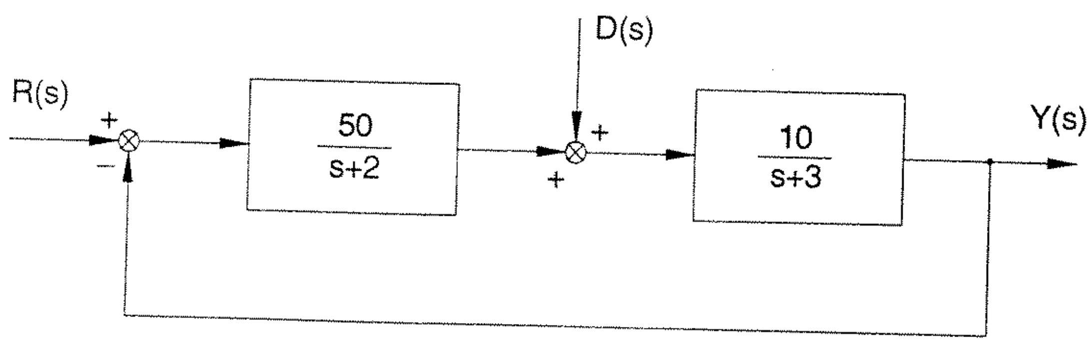
For the system represented in the block diagram below, find:

(1) the signal flow graph representation.
(2) the transfer function T(s) = C(s)/R(s), by using (only) Mason's signal-flow gain formula.

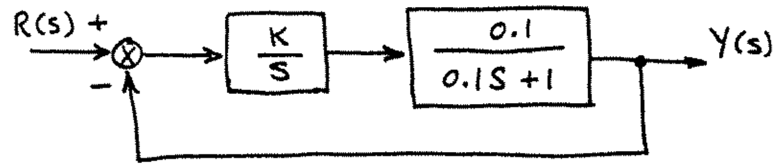


Problem 3. [5 marks]

For the system shown below find the total steady-state error due to a unit step input R(s) and a unit step disturbance D(s).



Problem # 1



inner loop: $\dfrac{0.1}{1+0.1S}$

(1) $$\frac{Y(s)}{R(s)} = \frac{G}{1+G} = \frac{\dfrac{0.1K}{S(0.1S+1)}}{1+\dfrac{0.1K}{S(0.1S+1)}} = \frac{0.1K}{S(0.1S+1)+0.1K}$$

$$= \frac{0.1K}{0.1S^2+S+0.1K} = \frac{K}{S^2+10S+K}$$

(2) From the characteristic equation

$$S^2+10S+K = S^2+2\xi\omega_n S+\omega_n^2$$

$\omega_n = \sqrt{K}$ ⟶ equation (a)

$2\xi\omega_n = 10$ ⟶ equation (b)

given 20% overshoot

$$P.O. = 100 \exp\left[\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right]$$

$$-\frac{\xi\pi}{\sqrt{1-\xi^2}} = \ln\left[\frac{20}{100}\right] = -1.6094$$

$$1-\xi^2 = 3.8102\,\xi^2$$

$$\therefore\ \xi = 0.4559$$

From equation (b)

$$2\xi\omega_n = 10$$

1) $$\therefore \omega_n = \frac{10}{2\times 0.4559} = 10.9661 \ \frac{rad}{sec}$$

From equation (a)

) $$K = \omega_n^2 = (10.9661)^2 = 120.2558$$

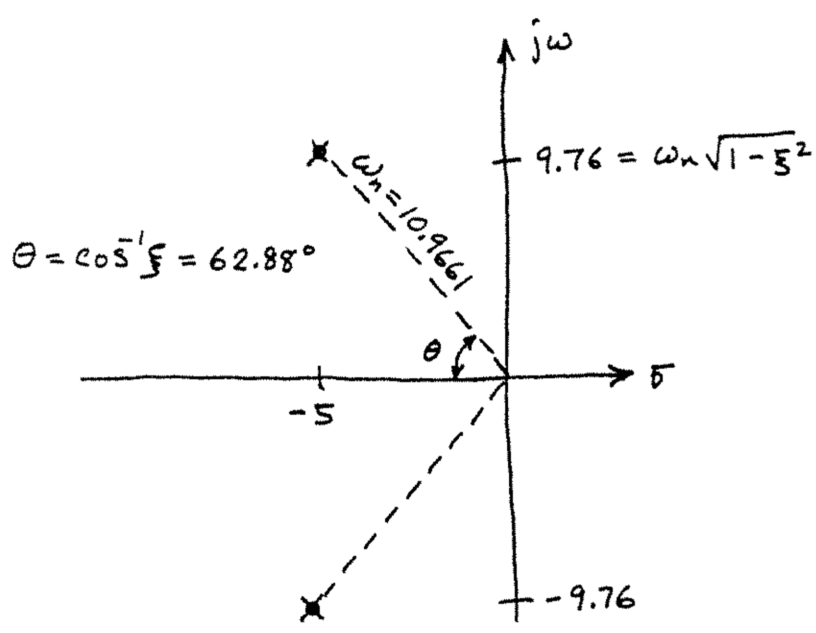
) $$T_s = \frac{4}{\xi\omega_n} = \frac{4}{0.4559\times 10.9661} = 0.8 \text{ sec}$$

(3) $$s^2 + 10s + 120.2558 = 0$$

$$s_1, s_2 = \frac{-10 \pm \sqrt{(10)^2 - 4\times 120.2558}}{2}$$

$$= \frac{-10 \pm j19.52}{2} = -5 \pm j9.76$$

(3)



(4) $s^2 + 10s + K = 0$

(1)
$$\begin{array}{c|cc} s^2 & 1 & K \\ s^1 & 10 & 0 \\ s^0 & K & \end{array}$$

(1) From row s^0, $K > 0$ for the system to be stable.

(1) (5) From the reduced block diagram, in unity feedback form, the system is "Type 1".

3) (6) $e_{ss} = \dfrac{1}{1+K_p}$ for $R(s) = \dfrac{1}{s}$

$1\frac{1}{2}$ $K_p = \lim\limits_{s \to 0} G(s) = \lim\limits_{s \to 0} \left[\dfrac{0.1K}{s(0.1s+1)} \right] = \infty$

$\frac{1}{2}$ $e_{ss} = \dfrac{1}{1+\infty} = 0$

(3) (7) $e_{ss} = \dfrac{1}{K_v}$ for $R(s) = \dfrac{1}{s^2}$

$K_v = \lim\limits_{s \to 0} s\,G(s) = \lim\limits_{s \to 0} s \left[\dfrac{0.1K}{s(0.1s+1)} \right] = 0.1K$

For $K = 120.2558$ $K_v = 12.026$

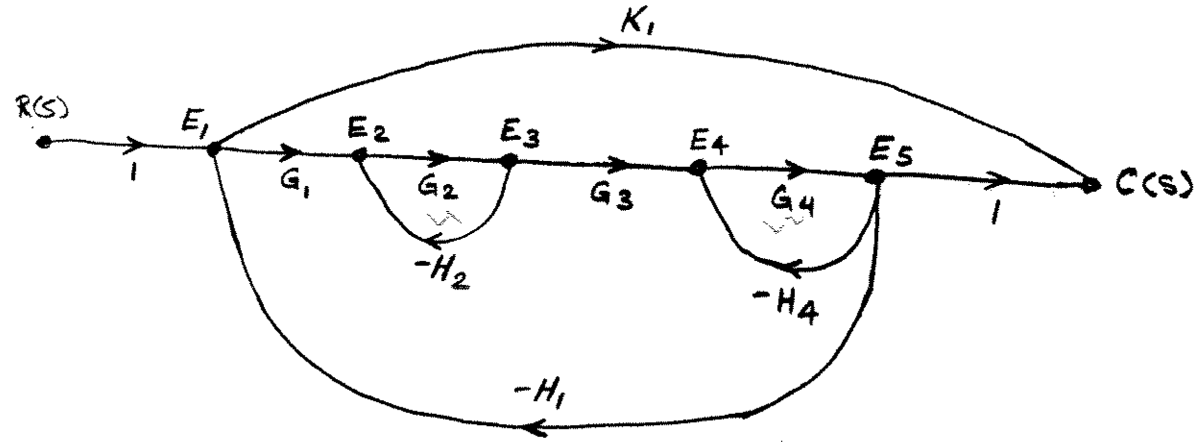
$e_{ss} = \dfrac{1}{0.1K} = 0.0832$

3) (8) $e_{ss} = \dfrac{1}{K_a}$ for $R(s) = \dfrac{1}{s^3}$

$1\frac{1}{2}$ $K_a = \lim\limits_{s \to 0} s^2 G(s) = \lim\limits_{s \to 0} s^2 \left[\dfrac{0.1K}{s(0.1s+1)} \right] = 0$

$e_{ss} = \dfrac{1}{0} = \infty$ system cannot track a parabola input

Problem # 2



$\frac{1}{4}$ $L_1 = -G_2H_2$

$\frac{1}{4}$ $L_2 = -G_4H_4$

$\frac{1}{4}$ $L_3 = -G_1G_2G_3G_4H_1$

$P_1 = G_1G_2G_3G_4$ $\frac{1}{4}$

$P_2 = K_1$ $\frac{1}{4}$

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1L_2)$$

$\frac{1}{2}$ $\frac{1}{2}$

$$= 1 + G_2H_2 + G_4H_4 + G_1G_2G_3G_4H_1 + G_2H_2G_4H_4$$

$\frac{1}{2}$ $\Delta_1 = 1$ $\frac{1}{2}$ $P_1\Delta_1 = G_1G_2G_3G_4$

$\frac{3}{4}$ $\Delta_2 = 1 - (L_1 + L_2) + L_1L_2 = 1 + G_2H_2 + G_4H_4 + G_2H_2G_4H_4$ $\frac{1}{2}$

$\frac{1}{2}$ $P_2\Delta_2 = K_1(1 + G_2H_2 + G_4H_4 + G_2H_2G_4H_4)$

$\frac{1}{2}$ $$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3G_4 + K_1(1 + G_2H_2 + G_4H_4 + G_2H_2G_4H_4)}{1 + G_2H_2 + G_4H_4 + G_1G_2G_3G_4H_1 + G_2H_2G_4H_4}$$

Problem #3

$$E(s) = E_R(s) + E_D(s)$$

$(1\tfrac{1}{2})$ $$E_R(s) = \frac{1}{1+\left(\frac{50}{s+2}\right)\left(\frac{10}{s+3}\right)} \cdot R(s) = \frac{(s+2)(s+3)}{(s+2)(s+3)+500} \cdot \frac{1}{s}$$

$(1\tfrac{1}{2})$ $$E_D(s) = -\frac{\frac{10}{s+3}}{1+\left(\frac{50}{s+2}\right)\left(\frac{10}{s+3}\right)} \cdot D(s) = \frac{-10(s+2)}{(s+2)(s+3)+500} \cdot \frac{1}{s}$$

$(\tfrac{1}{2})$ $$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\left[E_R(s) + E_D(s)\right]$$

$(1\tfrac{1}{2})$ $$= \lim_{s\to 0} s\left[\frac{(s+2)(s+3) - 10(s+2)}{(s+2)(s+3)+500}\right] \cdot \frac{1}{s}$$

$$= \frac{6-20}{6+500} = 0.0119 - 0.0395 = -0.0276$$

OR

$(1\tfrac{1}{2})$ $$\frac{Y_R(s)}{R(s)} = \frac{\left(\frac{50}{s+2}\right)\left(\frac{10}{s+3}\right)}{1+\left(\frac{50}{s+2}\right)\left(\frac{10}{s+3}\right)} = \frac{500}{(s+2)(s+3)+500}$$

$(\tfrac{1}{2})$ $$Y_R\left(\begin{smallmatrix}\text{steady}\\ \text{state}\end{smallmatrix}\right) = \lim_{s\to 0} s\left[\frac{500}{(s+2)(s+3)+500}\right] \cdot \frac{1}{s} = \frac{500}{506} = 0.9881$$

$(1\tfrac{1}{2})$ $$\frac{Y_D(s)}{D(s)} = \frac{\frac{10}{s+3}}{1+\left(\frac{50}{s+2}\right)\left(\frac{10}{s+3}\right)} = \frac{10(s+2)}{(s+2)(s+3)+500}$$

$(\tfrac{1}{2})$ $$Y_D\left(\begin{smallmatrix}\text{steady}\\ \text{state}\end{smallmatrix}\right) = \lim_{s\to 0} s\left[\frac{10(s+2)}{(s+2)(s+3)+500}\right] \cdot \frac{1}{s} = \frac{20}{506} = 0.0395$$

(1) $$\left\{\begin{aligned} &Y = Y_R + Y_D = 0.9881 + 0.0395 = 1.0277 \\ &\boxed{e_{ss} = R - Y} = 1 - 1.0277 = -0.0277 \end{aligned}\right.$$

mech 371/4

1.12

FUNDAMNETALS OF CONTROL SYSTEMS Section X

Mid-Term Thursday March 7, 2002

Answer All 4 Questions (total 45 marks)

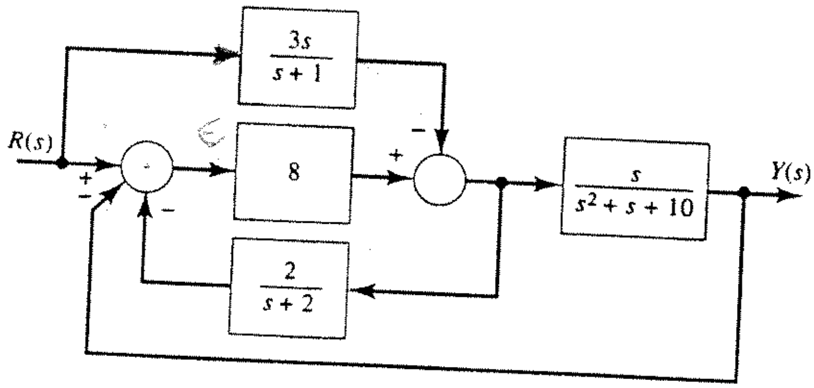
Question #1 (10 marks)

For the system shown in the block diagram:

a) Draw the signal flow graph.

b) Using Mason's loop rule, determine the overall system transfer function $\frac{Y(s)}{R(s)}$.

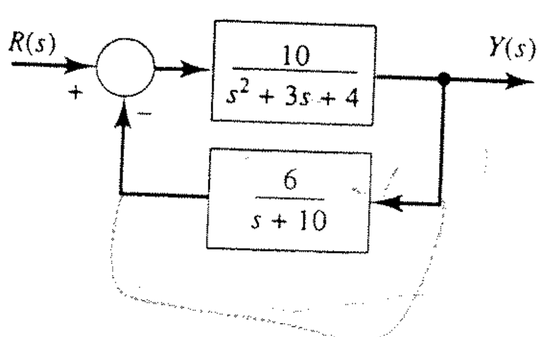
Show all mathematical steps.



Question #2.A (10 marks)

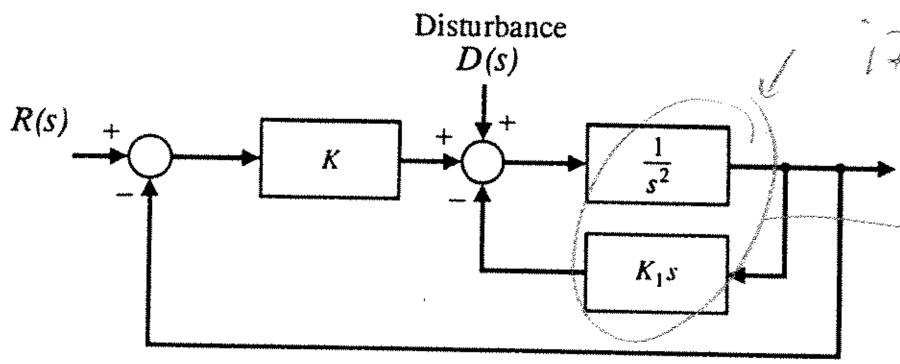
a) Determine the type number of the system.
b) Using the concept of system type number, determine the steady state error to a unit step input.
c) Using the concept of system type number, determine the steady state error to a unit ramp input.
d) Using the concept of system type number, determine the steady state error to a unit parabola input.

Show all equations and mathematical steps.
Answers without mathematical proofs will be given a mark of zero.



Question #2.B (5 marks)

For the system shown, the command signal $R(s)$ and disturbance signal $D(s)$ are both unit step inputs. Determine the gain K so that the steady state output $y(t)$ is 1.01 .

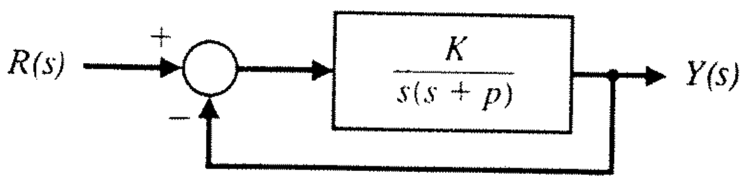


$$\frac{1/s^2}{1+\frac{k}{s}} = \frac{1/s^2}{\frac{s+k}{s}} = \frac{1}{s(s+k)}$$

Question #3 (10 marks)

For the system shown, it is required that the settling time $T_S \le 4$ seconds and the percent overshoot $P.O. \le 4.3\%$. Determine:

a) The gain K and parameter p necessary to satisfy the above performance requirements.
b) On the s-plane, plot the region of valid characteristic root locations.
c) Determine the equation for the sensitivity of the overall system transfer function with respect to parameter p. ie: determine S_p^T



Question #4 (10 marks)

The characteristic equation of a system is given by: $s^6 + s^5 + 5s^4 + s^3 + 2s^2 - 2s - 8$

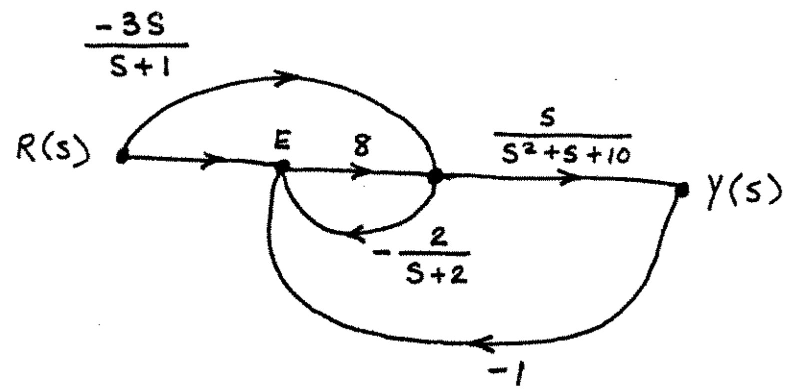
Using the Routh-Hurwitz test, determine:

a) If any, the marginally stable roots, and their numerical values.
b) If any, the total number of unstable roots.

HINT: The negative number -4 is a vector written as $-4+0j$ or as $4e^{+180j}$.

#1

a)



b) $P_1 = -\dfrac{3S}{S+1}\left(\dfrac{S}{S^2+S+10}\right)$

$P_2 = \dfrac{8S}{S^2+S+10}$

$L_1 = -\dfrac{8\times 2}{S+2} \qquad L_2 = -\dfrac{8S}{S^2+S+10}$

$$T(s) = \frac{-\dfrac{3S}{S+1}\left(\dfrac{S}{S^2+S+10}\right) + \dfrac{8S}{S^2+S+10}}{1 + \dfrac{16}{S+2} + \dfrac{8S}{S^2+S+10}}$$

$$= \frac{-3S^2\dfrac{S+2}{S+1} + 8S(S+2)}{(S+2)(S^2+S+10) + 16(S^2+S+10) + 8S(S+2)}$$

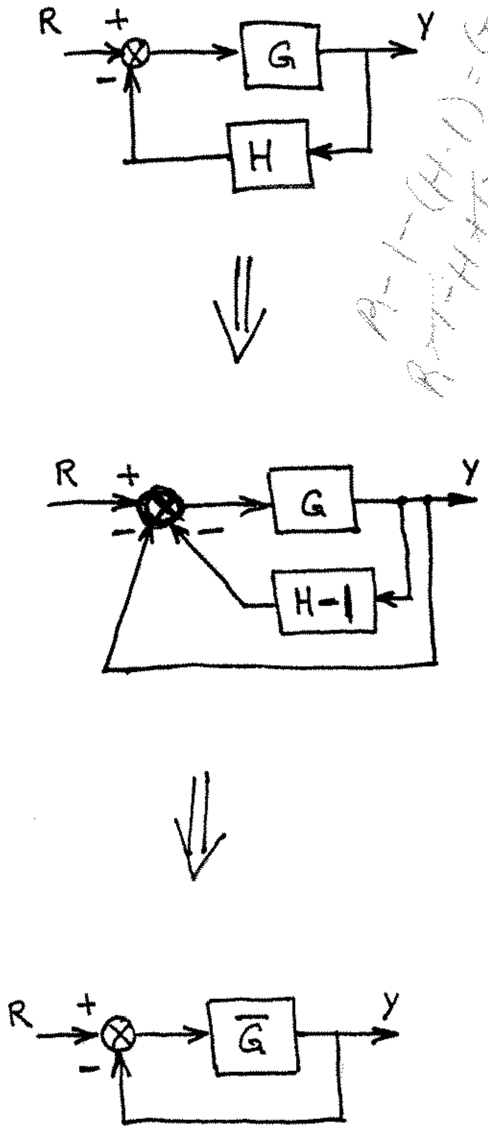
$$\begin{array}{r} S^3+S^2+10S \\ +2S^2+2S+20 \\ +16S^2+16S+160 \\ +8S^2+16S \\ \hline S^3+27S^2+44S+180 \end{array}$$

$$= \frac{-3S^2(S+2) + 8S(S+2)(S+1)}{(S+1)(S^3+27S^2+44S+180)}$$

$$= \frac{5S^3+18S^2+16S}{S^4+28S^3+71S^2+224S+180}$$

$$\begin{array}{r} S^4+27S^3+44S^2+180S \\ +\quad S^3+27S^2+44S+180 \\ \hline S^4+28S^3+71S^2+224S+180 \end{array}$$

#2.A



Where

$$\overline{G} = \frac{G}{1+G(H-1)}$$

$$\overline{G} = \frac{\dfrac{10}{s^2+3s+4}}{1+\dfrac{10}{s^2+3s+4}\cdot\left(\dfrac{6}{s+10}-1\right)}$$

$$= \frac{\dfrac{10}{s^2+3s+4}}{1+\dfrac{10}{s^2+3s+4}\cdot\left(\dfrac{-s-4}{s+10}\right)}$$

$$= \frac{10(s+10)}{(s^2+3s+4)(s+10)-10(s+4)}$$

$$\begin{array}{r} s^3+3s^2+4s \\ +10s^2+30s+40 \\ -10s-40 \\ \hline s^3+13s^2+24s \end{array} = s(s^2+13s+24)$$

$$\boxed{\overline{G} = \frac{10(s+10)}{s(s^2+13s+24)}} \Longrightarrow \text{Type 1}$$

b) $k_p = \lim_{s\to 0} \overline{G} = \dfrac{100}{0} = \infty$

$$e_{ss} = \frac{1}{1+k_p} = \frac{1}{\infty} = 0$$

c) $k_v = \lim_{s\to 0} s\overline{G} = \lim_{s\to 0} \dfrac{10(s+10)}{s^2+13s+24} = \dfrac{100}{24}$

$$e_{ss} = \frac{1}{k_v} = \frac{24}{100} = \frac{6}{25}$$

d) $k_a = \lim_{s\to 0} s^2\overline{G} = \lim_{s\to 0} \dfrac{s(10)(s+10)}{s^2+13s+24} = 0$

$$e_{ss} = \frac{1}{k_a} = \frac{1}{0} = \infty$$ cannot track acceleration input

#2.B

$$\frac{Y_R}{R} = \frac{K/S^2}{1+(K/S^2 + K_1/S)} = \frac{K}{S^2+K_1S+K}$$

$$\frac{Y_D}{D} = \frac{1/S^2}{1+(K/S^2 + K_1/S)} = \frac{1}{S^2+K_1S+K}$$

$$Y_{SSR} = \lim_{S\to 0} S\left(\frac{K}{S^2+K_1S+K}\right)\cdot\frac{1}{S} = 1$$

$$Y_{SSD} = \lim_{S\to 0} S\left(\frac{1}{S^2+K_1S+K}\right)\frac{1}{S} = \frac{1}{K}$$

$$Y_{SS} = 1.01 = Y_{SSR} + Y_{SSD} = 1 + \frac{1}{K}$$

$$\therefore K = 100$$

#3

$$\text{P.O.} \Rightarrow \quad 100 \exp\left[\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right] \le 4.3$$

$$-\frac{\xi\pi}{\sqrt{1-\xi^2}} \le \ln\frac{4.3}{100} = -\pi$$

$$\xi \ge \sqrt{1-\xi^2} \quad \text{or} \quad \xi^2 \ge 1-\xi^2$$

$$\therefore \quad \xi \ge \frac{1}{\sqrt{2}} \qquad (0.707)$$

$$T_s \Rightarrow \quad \frac{4}{\xi\omega_n} \le T_s = 4$$

$$\therefore \quad \omega_n \ge \frac{1}{\xi} = \sqrt{2}$$

$$(1.414)$$

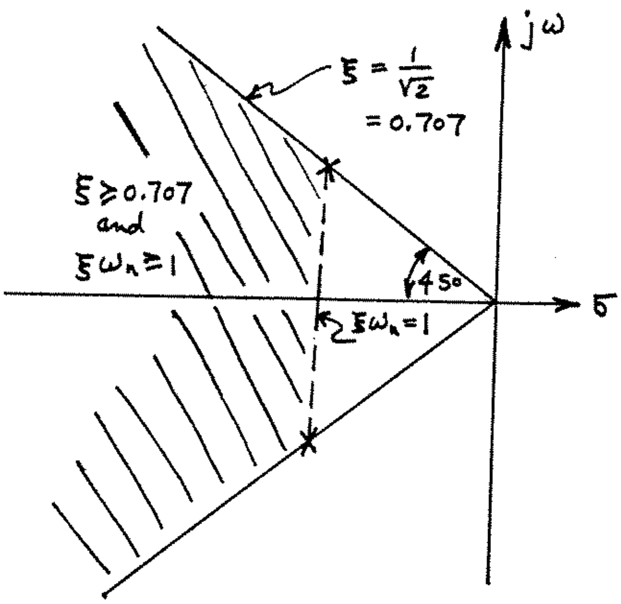
a) $$T(s) = \frac{\dfrac{K}{s(s+p)}}{1 + \dfrac{K}{s(s+p)}} = \frac{K}{s^2+ps+K} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\therefore \quad K = \omega_n^2 = 2 \quad \text{and} \quad p = 2\xi\omega_n = 2\cdot\frac{1}{\sqrt{2}}\cdot\sqrt{2} = 2$$

$$s^2+ps+K = s^2+2s+2$$

b) $\xi\omega_n \ge 1$

$\theta = \cos^{-1}\xi = 45^\circ$



c) $$T(s) = \frac{K}{s^2+ps+K}$$

$$S_p^T = \frac{\partial T}{\partial p}\cdot\frac{p}{T}$$

$$= \frac{-sK}{(s^2+ps+K)^2}\cdot\frac{s^2+ps+K}{K}\cdot p$$

$$\therefore \quad S_p^T = \frac{-ps}{s^2+ps+K}$$

#4

$$\begin{array}{c|cccc} S^6 & 1 & 5 & 2 & -8 \\ S^5 & 1 & 1 & -2 & \\ S^4 & 4 & 4 & -8 & \\ S^3 & 0 & 0 & & \end{array}$$

Because row S^3 are all zero, there are roots on the imaginary axis.
From the auxiliary equation (row S^4):

$$4S^4 + 4S^2 - 8 = 0 \implies S^4 + S^2 - 2 = 0$$

$$\therefore \quad S^2_{1,2} = \frac{-1 \pm \sqrt{1-4(-2)}}{2} = \frac{-1 \pm 3}{2} \implies S_1^2 = +1 \text{ and } S_2^2 = -2$$

$S = \sqrt{2}j$

$\therefore S_1 = \pm 1$

Since $-2 = 2e^{180j}$

2 $\therefore S_2 = \sqrt{2}\, e^{\pm 90j} \implies$ marginally stable roots

To fill the Routh-Hurwitz table; replace row S^3 by:

2 $$\frac{d}{dS}(4S^4 + 4S^2 - 8) = 16S^3 + 8S$$

$$\begin{array}{c|cccc} S^6 & 1 & 5 & 2 & -8 \\ S^5 & 1 & 1 & -2 & \\ S^4 & 4 & 4 & -8 & \\ S^3 & 16 & 8 & & \\ S^2 & 2 & -8 & & \leftarrow \dfrac{16\times 4 - 4\times 8}{16} = 2 \\ S^1 & 72 & & & \leftarrow \dfrac{2\times 8 - 16\times(-8)}{2} = 72 \\ S^0 & -8 & & & \end{array}$$

change in sign indicates 1 root in RHP

Imaginary Roots = 2 Roots in the RHP (unstable) = 1

1

CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
ENGR 372/4 sec. Y FUNDAMENTALS OF CONTROL SYSTEMS

Mid-Term **Instructor: Dr. H. Hong** **Date: March 8, 2001**

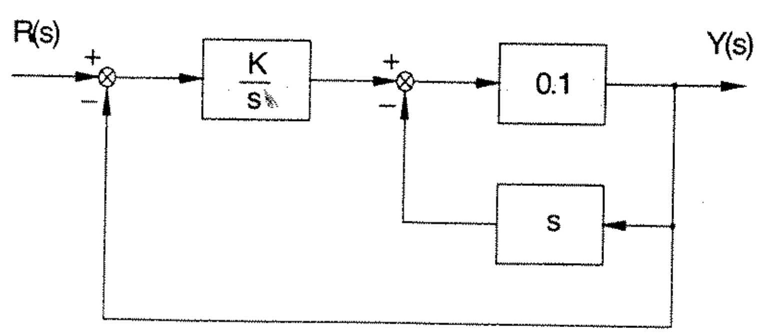
Answer all three (3) questions. **Closed book exam.** **Total time: 1 hour 15 minutes**

Problem 1.→23; Problem 2.→7; Problem 3.→5; = 35 marks total.

Problem 1. [a→1, b→7, c→3, d→2, e→1, f→3, g→3, h→3 = 23 marks]

A negative feedback control system has a block diagram as shown below.

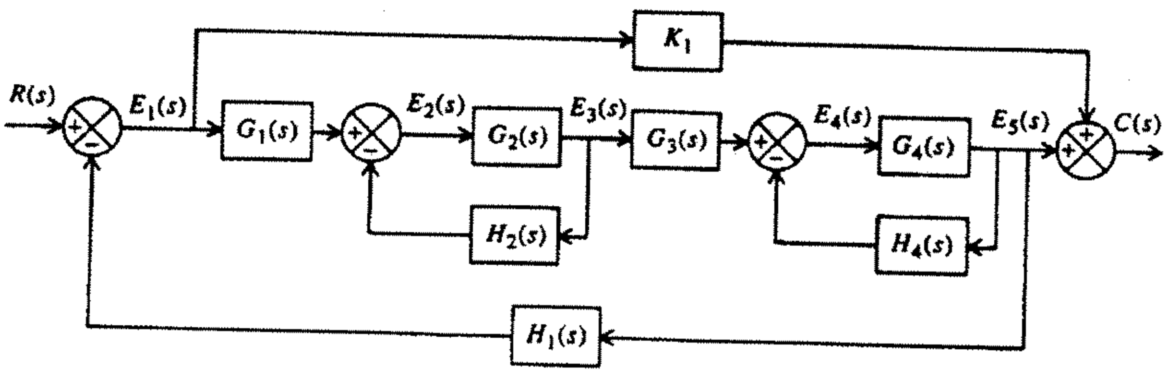
(1) Determine the closed-loop transfer function T(s) = Y(s)/R(s).
(2) For a unit step input, find the value for the gain K so that the system has 20% overshoot. What is the natural frequency ω_n of the system? What is the settling time T_s of the system?
(3) Sketch the closed-loop poles on the s-plane. Clearly indicate ω_n (natural frequency) and θ (angular representation of the damping ratio ζ), and their numerical values, on the figure.
(4) Using the Routh-Hurwitz stability criterion determine the range of gain K for the system to be stable?
(5) What is the "type number" of the system?
(6) For a unit step input, determine the position error constant K_p , and the steady-state error. Show all mathematical steps leading to your answer.
(7) For a unit ramp input, determine the velocity error constant K_v , and the steady-state error. Show all mathematical steps leading to your answer.
(8) For a unit acceleration input, determine the acceleration error constant K_a , and the steady-state error. Show all mathematical steps leading to your answer.



Problem 2. [a→1½, b→5½ = 7 marks]

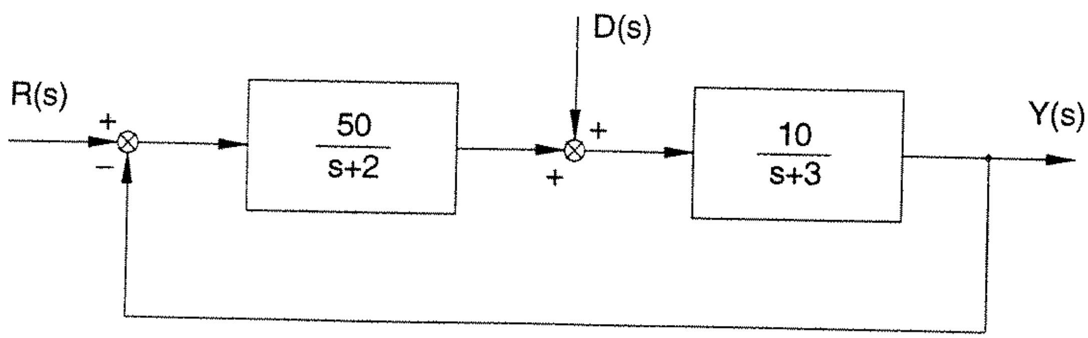
For the system represented in the block diagram below, find:

(1) the signal flow graph representation.
(2) the transfer function T(s) = C(s)/R(s), by using (only) Mason's signal-flow gain formula.



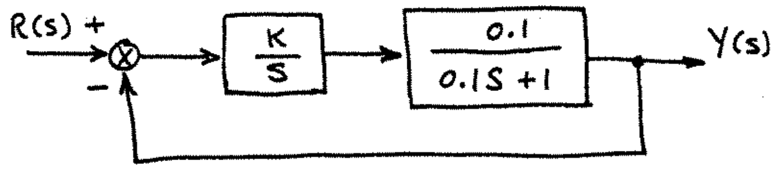
Problem 3. [5 marks]

For the system shown below find the total steady-state error due to a unit step input R(s) and a unit step disturbance D(s).



$$Y = R(s)\frac{50}{s+2} + D(s)\frac{10}{s+3}$$

Problem # 1



inner loop: $\dfrac{0.1}{1+0.1S}$

(1) $$\frac{Y(s)}{R(s)} = \frac{G}{1+G} = \frac{\dfrac{0.1K}{S(0.1S+1)}}{1+\dfrac{0.1K}{S(0.1S+1)}} = \frac{0.1K}{S(0.1S+1)+0.1K}$$

1) $$= \frac{0.1K}{0.1S^2+S+0.1K} = \frac{K}{S^2+10S+K}$$

(2) From the characteristic equation

1) $$S^2+10S+K = S^2+2\xi\omega_n S+\omega_n^2$$

1) $\omega_n = \sqrt{K}$ ⟶ equation (a)

1) $2\xi\omega_n = 10$ ⟶ equation (b)

given 20% overshoot

1) $$P.O. = 100\exp\left[\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right]$$

$$-\frac{\xi\pi}{\sqrt{1-\xi^2}} = \ln\left[\frac{20}{100}\right] = -1.6094$$

$$1-\xi^2 = 3.8102\,\xi^2$$

$$\therefore\ \xi = 0.4559$$

From equation (b)

$$2\xi\omega_n = 10$$

1) $\therefore \omega_n = \dfrac{10}{2\times 0.4559} = 10.9661 \ \dfrac{\text{rad}}{\text{sec}}$

From equation (a)

) $K = \omega_n^2 = (10.9661)^2 = 120.2558$

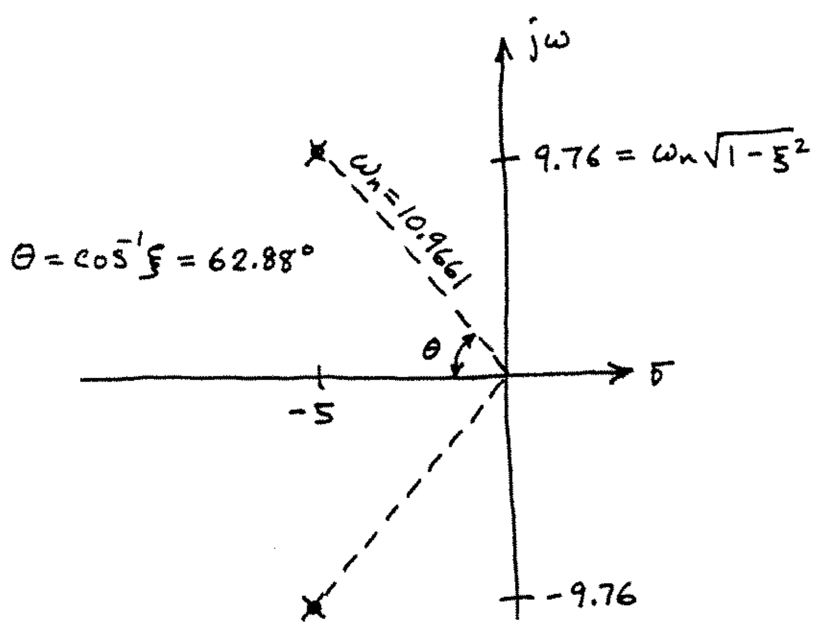
) $T_s = \dfrac{4}{\xi\omega_n} = \dfrac{4}{0.4559 \times 10.9661} = 0.8 \text{ sec}$

(3) $s^2 + 10s + 120.2558 = 0$

$$s_1, s_2 = \frac{-10 \pm \sqrt{(10)^2 - 4\times 120.2558}}{2}$$

$$= \frac{-10 \pm j\,19.52}{2} = -5 \pm j\,9.76$$

(3)



(4) $s^2 + 10s + K = 0$

(1)
$$\begin{array}{c|cc} s^2 & 1 & K \\ s^1 & 10 & 0 \\ s^0 & K & \end{array}$$

(1) From row s^0, $K > 0$ for the system to be stable.

(1) (5) From the reduced block diagram, in unity feedback form, the system is "Type 1".

(3) (6) 1 $e_{ss} = \dfrac{1}{1+K_p}$ for $R(s) = \dfrac{1}{s}$

$1\frac{1}{2}$ $K_p = \lim\limits_{s\to 0} G(s) = \lim\limits_{s\to 0}\left[\dfrac{0.1K}{s(0.1s+1)}\right] = \infty$

$\frac{1}{2}$ $e_{ss} = \dfrac{1}{1+\infty} = 0$

(3) (7) 1 $e_{ss} = \dfrac{1}{K_v}$ for $R(s) = \dfrac{1}{s^2}$

$1\frac{1}{2}$ $K_v = \lim\limits_{s\to 0} s\,G(s) = \lim\limits_{s\to 0} s\left[\dfrac{0.1K}{s(0.1s+1)}\right] = 0.1K$

$\frac{1}{2}$ For $K = 120.2558$ $K_v = 12.026$

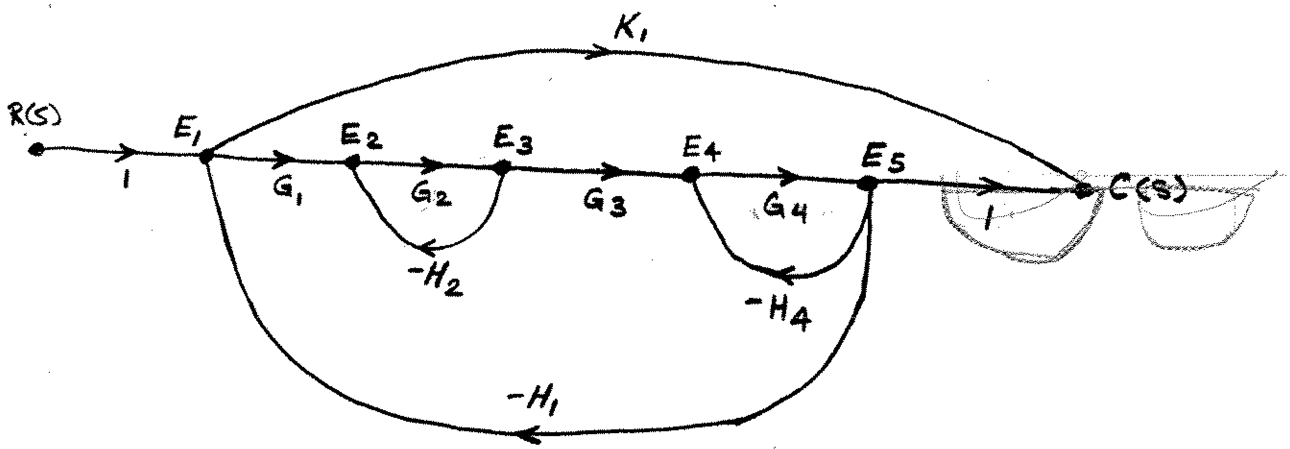
$e_{ss} = \dfrac{1}{0.1K} = 0.0832$

3) (8) 1 $e_{ss} = \dfrac{1}{K_a}$ for $R(s) = \dfrac{1}{s^3}$

$1\frac{1}{2}$ $K_a = \lim\limits_{s\to 0} s^2 G(s) = \lim\limits_{s\to 0} s^2\left[\dfrac{0.1K}{s(0.1s+1)}\right] = 0$

$\frac{1}{2}$ $e_{ss} = \dfrac{1}{0} = \infty$ system cannot track a parabola input

Problem # 2



$\frac{1}{4}$ $L_1 = -G_2 H_2$

$\frac{1}{4}$ $L_2 = -G_4 H_4$

$\frac{1}{4}$ $L_3 = -G_1 G_2 G_3 G_4 H_1$

$P_1 = G_1 G_2 G_3 G_4$ $\frac{1}{4}$

$P_2 = K_1$ $\frac{1}{4}$

$\frac{1}{2}$ $\frac{1}{2}$

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2)$$

$$= 1 + G_2 H_2 + G_4 H_4 + G_1 G_2 G_3 G_4 H_1 + G_2 H_2 G_4 H_4$$

$\frac{1}{2}$ $\frac{1}{2}$

$\Delta_1 = 1$ $\quad P_1 \Delta_1 = G_1 G_2 G_3 G_4$

$3/4$ $\frac{1}{2}$

$$\begin{cases} \Delta_2 = 1 - (L_1 + L_2) + L_1 L_2 = 1 + G_2 H_2 + G_4 H_4 + G_2 H_2 G_4 H_4 \\ P_2 \Delta_2 = K_1 (1 + G_2 H_2 + G_4 H_4 + G_2 H_2 G_4 H_4) \end{cases}$$

$\frac{1}{2}$

$\frac{1}{2}$ $$\frac{C(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 G_3 G_4 + K_1 (1 + G_2 H_2 + G_4 H_4 + G_2 H_2 G_4 H_4)}{1 + G_2 H_2 + G_4 H_4 + G_1 G_2 G_3 G_4 H_1 + G_2 H_2 G_4 H_4}$$

Problem #3

$$E(s) = E_R(s) + E_D(s)$$

$(1\frac{1}{2})$ $$E_R(s) = \frac{1}{1+\left(\frac{50}{s+2}\right)\left(\frac{10}{s+3}\right)} \cdot R(s) = \frac{(s+2)(s+3)}{(s+2)(s+3)+500} \cdot \frac{1}{s}$$

$(1\frac{1}{2})$ $$E_D(s) = -\frac{\frac{10}{s+3}}{1+\left(\frac{50}{s+2}\right)\left(\frac{10}{s+3}\right)} \cdot D(s) = \frac{-10(s+2)}{(s+2)(s+3)+500} \cdot \frac{1}{s}$$

$(\frac{1}{2})$ $$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\left[E_R(s) + E_D(s)\right]$$

$(1\frac{1}{2})$ $$= \lim_{s\to 0} s\left[\frac{(s+2)(s+3) - 10(s+2)}{(s+2)(s+3)+500}\right] \cdot \frac{1}{s}$$

$$= \frac{6-20}{6+500} = 0.0119 - 0.0395 = -0.0276$$

OR

$(1\frac{1}{2})$ $$\frac{Y_R(s)}{R(s)} = \frac{\left(\frac{50}{s+2}\right)\left(\frac{10}{s+3}\right)}{1+\left(\frac{50}{s+2}\right)\left(\frac{10}{s+3}\right)} = \frac{500}{(s+2)(s+3)+500}$$

$(\frac{1}{2})$ $$Y_R\begin{pmatrix}\text{steady}\\ \text{state}\end{pmatrix} = \lim_{s\to 0} s\left[\frac{500}{(s+2)(s+3)+500}\right] \cdot \frac{1}{s} = \frac{500}{506} = 0.9881$$

$(1\frac{1}{2})$ $$\frac{Y_D(s)}{D(s)} = \frac{\frac{10}{s+3}}{1+\left(\frac{50}{s+2}\right)\left(\frac{10}{s+3}\right)} = \frac{10(s+2)}{(s+2)(s+3)+500}$$

$(\frac{1}{2})$ $$Y_D\begin{pmatrix}\text{steady}\\ \text{state}\end{pmatrix} = \lim_{s\to 0} s\left[\frac{10(s+2)}{(s+2)(s+3)+500}\right] \cdot \frac{1}{s} = \frac{20}{506} = 0.0395$$

(1) $$\left\{\begin{aligned} y &= y_R + y_D = 0.9881 + 0.0395 = 1.0277 \\ e_{ss} &= R - y = 1 - 1.0277 = -0.0277 \end{aligned}\right.$$