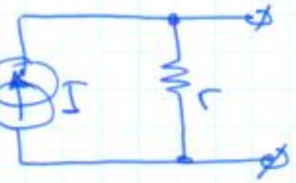
Midterm exam answer key

Version 2

S1a) Electrical resistance: opposition exerted by a conductor to an electric current flowing through it.

S1b)



I : source of current : it determines the electric current flowing throu the external circuit

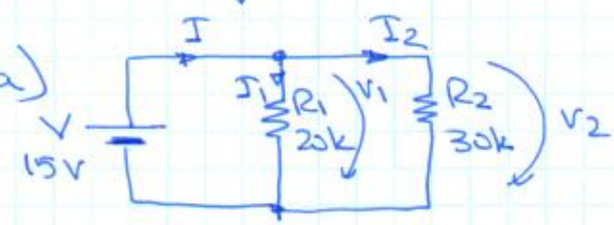
r : internal resistance : models the dependence of the output current of the load

S1c) Reactive power in the power reversibly stored in the magnetic field of the inductor.

S1d) Volt-Ampere (VA)

S1e) $G = \frac{I}{V}$; G = conductance, I = current, V = voltage

S2a)



S2b) $V_1 = V_2 = V$ $\quad V_1 = 15\,V, \quad V_2 = 15\,V$

S2c) We need to determine the current I first.

$$I = \frac{V}{R_1 \| R_2} \qquad I = \frac{V}{\frac{R_1 R_2}{R_1 + R_2}} = V \cdot \frac{R_1 + R_2}{R_1 R_2}$$

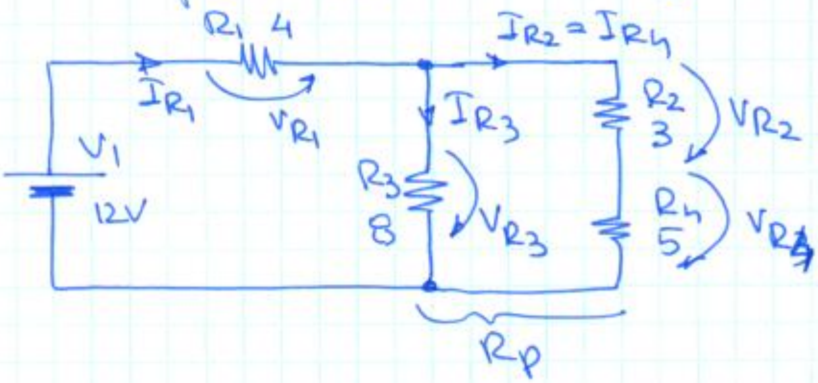
$$I = 15 \cdot \frac{50 \cdot \cancel{10^3}}{20 \cdot 10^3 \cdot 30 \cdot \cancel{10^3}} \qquad I = 1.25\ mA$$

$$I_1 = \frac{V}{R_1} \; ; \; I_1 = \frac{15}{20 \cdot 10^3} \qquad I_1 = 0.75\ mA$$

$I_2 = \frac{V}{R_2}$ or $I_2 = I - I_1$; either give $I_2 = 0.5\,mA$

Note: Computing I is optional, but helpful to confirm values obtained for I_1 and I_2

S3 ab) Upon inspecting the schematic, R_2 and R_4 are connected in series, and their group is in parallel with R_3



Let $R_p = R_3 \parallel (R_2 + R_4)$

$$R_p = \frac{R_3(R_2+R_4)}{R_3+R_2+R_4}$$

$$I_{R_1} = \frac{V_1}{R_1 + R_p} \Leftrightarrow I_{R_1} = \frac{V_1}{R_1 + \frac{R_3(R_2+R_4)}{R_3+R_2+R_4}}$$

$$I_{R_1} = \frac{12}{4 + \frac{8(3+5)}{8+3+5}} = \frac{12}{4 + \frac{\cancel{48}\cdot\cancel{8}}{\cancel{16}_{2}}} = \frac{\cancel{12}^{3}}{\cancel{8}_{2}} \quad I_{R_1} = \underline{1.5\,A}$$

$$V_{R_1} = R_1 \cdot I_{R_1} \Rightarrow V_{R_1} = 4 \times 1.5 \quad V_{R_1} = 6\underline{V}$$

$$V_{R_3} = V_{R_2} + V_{R_4} = V_1 - V_{R_1} \Rightarrow V_{R_3} = 12 - 6 = 6\underline{V}$$

parallel

$$I_{R_3} = \frac{V_{R_3}}{R_3} \quad I_{R_3} = \frac{6}{8} \quad I_{R_3} = 0.75\underline{A}$$

$$I_{R_2} = I_{R_1} - I_{R_3}\ (KCL) \Rightarrow I_{R_2} = 0.75\underline{A}$$

$$I_{R_4} = I_{R_2}\ (\text{series}) \Rightarrow I_{R_4} = 0.75\underline{A}$$

$$V_{R_2} = R_2 \cdot I_{R_2} \Rightarrow V_{R_2} = 3 \cdot \frac{3}{4} \quad V_{R_2} = 2.\underline{25V}$$

$$V_{R_4} = R_4 \cdot I_{R_4} \Rightarrow V_{R_4} = 5 \cdot \frac{3}{4} \quad V_{R_4} = 3.7\underline{5V}$$

Check: (optional) $V_{R_1} + V_{R_3} = V_1 \quad 12 = 12$ ✓

$V_{R_2} + V_{R_4} = V_{R_3} \quad 2.25 + 3.75 = 6$ ✓

S3c) Let R2 resistance be x. The text tells us that the resistance is x Ohms, and the conductance is x Siemens. Thus

$$x = \frac{1}{x} \quad \left(\text{because } G = \frac{1}{R} \text{ or } R = \frac{1}{G}\right) \Leftrightarrow$$

$\Leftrightarrow$ $x^2 = 1 \Rightarrow x = \pm 1$. We only retain the positive value (resistance can't be negative in this context), so $R_2 = 1\,\Omega$.

We need to recalculate the currents/voltages:

$$I_{R_1} = \frac{V_1}{R_1 + \frac{R_3(R_2+R_4)}{R_3+R_2+R_4}} \qquad I_{R_1} = \frac{12}{4 + \frac{8\cdot(1+5)}{8+1+5}} = \frac{12}{4+\frac{8\cdot 6}{14}}$$

$$I_{R_1} = \frac{12\cdot 14}{4\cdot 14 + 48} = \frac{168}{104} \qquad I_{R_1} = 1.615\,A$$

$$\left.\begin{array}{l} V_{R_1} = R_1 \cdot I_{R_1} \\ V_{R_3} = V_{R_2} + V_{R_4} = V_1 - V_{R_1} \\ \qquad\qquad (KVL) \end{array}\right] \Rightarrow$$

$$\left.\begin{array}{l} \Rightarrow V_{R_2} + V_{R_4} = V_1 - R_1 \cdot I_{R_1} \\ V_{R_2} + V_{R_4} = \text{voltage across the series group} \\ I_{R_2} = I_{R_4} = \frac{V_{R_2}+V_{R_4}}{R_2+R_4} \end{array}\right] \Rightarrow$$

$$\left.\begin{array}{l} \Rightarrow I_{R_4} = \frac{V_1 - R_1\cdot I_{R_1}}{R_2+R_4} \\ P_{R_4} = R_4 \cdot I_{R_4}^2 \end{array}\right] \Rightarrow P_{R_4} = R_4\left[\frac{V_1 - R_1 I_{R_1}}{R_2+R_4}\right]^2$$

So $P_{R_4} = 5 \cdot \left(\frac{12 - 4 \times 1.615}{1 + 5} \right)^2$ $\quad P_{R_4} = 4.26\,W$ $\quad 4/4$ $\quad v2$

S4) L_i = initial inductance $\quad L_i = \frac{\mu_m N^2 A}{l_i}$

L_f = final inductance $\quad L_f = \frac{\mu_m N^2 A}{l_f}$

We're told that $\underbrace{L_f = 0.9 \times L_i}_{\text{10\% decrease}}$

$$\frac{L_f}{L_i} = \frac{9}{10} \Leftrightarrow \frac{\cancel{\mu_m N^2 A}}{l_f} \cdot \frac{l_i}{\cancel{\mu_m N^2 A}} = \frac{9}{10} \Rightarrow$$

$$\Rightarrow l_f = \frac{10}{9} \cdot l_i \quad \Big| - l_i \Rightarrow l_f - l_i = l_i \left(\frac{10}{9} - 1 \right)$$

$$l_f - l_i = \frac{l_i}{9} \; ; \; l_i = 9 \times \underbrace{r}_{\text{radius}} \qquad r = \frac{d}{2} \quad \cdot \; d = 6 \cdot 10^{-2}\,m$$

$$l_f - l_i = \frac{1}{\cancel{9}} \cdot \cancel{9} \cdot \frac{d}{2} \qquad l_f - l_i = \frac{6 \cdot 10^{-2}}{2}$$

$$l_f - l_i = 3\ cm = 0.03\ m$$

So the coil needs to be extended with 3 cm to achieve a 10% decrease in inductance.

Note: the formula $L = \frac{\mu_m N^2 A}{l}$ is valid for $l > 10 \cdot d$ – however, in the context of this problem we assume that the L formula is valid regardless of the coil length / diameter ratio.