

MATH 3705A
Test 1 Solutions
February 4, 2014

[Marks]

[4] 1. $\mathcal{L}\{e^{-2t}\cos(3t)\} =$
 (a) $\frac{s-2}{(s-2)^2+9}$ (b) $\frac{s}{(s+2)^2+9}$ (c) $\frac{s+2}{(s+2)^2+9}$ (d) $\frac{s+2}{s^2+9}$ (e) None of these
Answer: (c)

[4] 2. $\mathcal{L}^{-1}\left\{\frac{5s+2}{s^2-4}\right\} =$
 (a) $5\cos(2t) + \sin(2t)$ (b) $3e^{-2t} + 2e^{2t}$ (c) $2e^{-2t} + 3e^{2t}$ (d) $3e^{2t} + 2e^{-2t}$ (e) None of these
Answer: (c) or (d)

[4] 3. $\mathcal{L}\{u(t-2)e^{3t}\} =$
 (a) $\frac{e^{-2s}}{s-3}$ (b) $\frac{e^{2(s-3)}}{s-3}$ (c) $\frac{e^6e^{-2s}}{s+3}$ (d) $\frac{e^6e^{-2s}}{s-3}$ (e) None of these
Answer: (d)

[4] 4. $\mathcal{L}^{-1}\left\{\frac{se^{-2s}}{s^2+2s+10}\right\} =$
 (a) $u(t-2)e^{-t}\left[\cos(3t) - \frac{1}{3}\sin(3t)\right]$
 (b) $u(t-2)e^{-(t-2)}\{\cos[3(t-2)] - \sin[3(t-2)]\}$
 (c) $u(t-2)e^{-(t-2)}\left\{\cos[3(t-2)] - \frac{1}{3}\sin[3(t-2)]\right\}$
 (d) $u(t-2)e^{-(t-2)}\{\cos[3(t-2)] - 3\sin[3(t-2)]\}$
 (e) None of the above
Answer: (c)

[7] 5. Solve the initial-value problem $y'' - y' - 6y = 0$, $y(0) = 3$, $y'(0) = -1$.
Solution:
 $s^2Y(s) - sy(0) - y'(0) - [sY(s) - y(0)] - 6Y(s) = 0 \Rightarrow (s^2 - s - 6)Y(s) - 3s + 4 = 0$
 $\Rightarrow Y(s) = \frac{3s-4}{(s-3)(s+2)} = \frac{1}{s-3} + \frac{2}{s+2} \Rightarrow y(t) = e^{3t} + 2e^{-2t}$.

[7] 6. Solve the initial-value problem $y'' - 4y' + 20y = 0$, $y(0) = 3$, $y'(0) = -18$.
Solution:
 $s^2Y(s) - sy(0) - y'(0) - 4[sY(s) - y(0)] + 20Y(s) = 0 \Rightarrow (s^2 - 4s + 20)Y(s) - 3s + 30 = 0$
 $\Rightarrow Y(s) = \frac{3s-30}{s^2-4s+20} = 3\frac{s-10}{(s-2)^2+16} = 3\frac{(s-2)-8}{(s-2)^2+16}$
 $= 3\frac{s-2}{(s-2)^2+16} - 6\frac{4}{(s-2)^2+16} \Rightarrow y(t) = 3e^{2t}\cos(4t) - 6e^{2t}\sin(4t)$.