CARLETON UNIVERSITY

FINAL EXAMINATION
MATH 1004 A, B, C, D, E, G
December 2011

DURATION: 3 HOURS

Department Name and Course Number: School of Mathematics and Statistics, MATH 1004 A, B, C, D, E.
Course Instructor(s): Dr. A.B. Mingarelli (Sect. A), Dr. P.K. Chan (Sect. B), Dr. Z. Montazeri (Sect. C), Mr. L. Bourbonnais (Sect. D), Dr. M. Sadeghi (Sect. E), G

AUTHORIZED MEMORANDA
NON-PROGRAMMABLE CALCULATOR PERMITTED.
BLANK SHEETS OF PAPER FOR ROUGH WORK

This exam may be released to the Library and may be taken away by the student.

1. Please verify that you are in possession of a Scantron FORM
2. Please **fill in your COURSE CODE** (e.g., MATH 1004) and **COURSE SECTION** (e.g., A, B, C, D), **YOUR NAME** and **YOUR STUDENT NUMBER** where required on the Scantron form.
3. **The examination is out of a total of 100** and consists of 20 multiple choice questions each worth 5 marks **Please fill in only one answer on your Scantron sheets with a pencil** as there is only one answer to any given question. Circling two or more answers to any question invalidates that question (*i.e.*, you get 0 marks for that question).

Return only the duly completed Scantron form, not the examination nor your work.

1. **[5 marks] Let $f(x) = \ln(\text{Arctan}\, x)$. Evaluate $f'(1)$. In other words, find the derivative of f at $x = 1$.**
   (a) $f'(1) = \frac{2}{\pi}$ (b) $f'(1) = \frac{1}{\pi}$ (c) $f'(1) = \pi$ (d) $f'(1) = 2\pi$

2. **[5 marks] Let $f(x) = |x-2| + 2$. Calculate $L = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$.**
   (a) $L = 0$ (b) $L = 1$ (c) $L = -1$ (d) This limit does not exist

3. **[5 marks] Let $f(x) = 3^{x^2+1} \log_3(x^3)$. Evaluate $f'(1)$. In other words, find the derivative of f at $x = 1$.**
   (a) $f'(1) = 0$ (b) $f'(1) = 81$ (c) $f'(1) = 27/\ln 3$ (d) $f'(1) = 3\ln 3$

4. **[5 marks] Evaluate $L = \lim_{x\to\infty} x^2 \sin\frac{1}{x^2}$.**
   (a) 1 (b) 0 (c) 2 (d) This limit does not exist

5. **[5 marks] A differentiable function f has the property that $f(2) = 3$, $f'(3) = 2$, $f'(2) = 1$, and $f(3) = 5$, which is the value of the derivative of $f(f(x))$ at $x = 2$?**
   (a) 3 (b) 10 (c) 5 (d) 2

6. **[5 marks] Let $f(x) = 4^{3\sin x}$. Evaluate $f'(x)$. In other words, find the derivative of f at x.**
   (a) $f'(x) = 4^{-3\sin x}$ (b) $f'(x) = (3\cos x)\, 4^{3\sin x - 1}$ (c) $f'(x) = (3\cos x)(\ln 4)\, 4^{3\sin x}$
   (d) $f'(x) = (3\sin x)(\ln 4)\, 4^{3\sin x}$

7. **[5 marks] Evaluate the limit: $L = \lim_{x\to\infty} \{3 + x^3 e^{-x}\}$.**
   (a) $L = 1$ (b) $L = 3$ (c) $L = 0$ (d) $L = \infty$

8. [5 marks] Evaluate $L = \dfrac{d}{dx}\displaystyle\int_0^{\frac{x^4}{4}} \cos(16\,t^2)\,dt$.

(a) $L = x^3\cos(x^8)$ (b) $L = \cos(x^8)$ (c) $L = \cos(x^8) - 1$ (d) $L = 32x\sin(x^8)$

9. [5 marks] Let y be given implicitly as a differentiable function of x by $x^3 + y^2 + e^y = 2$. Calculate the value of the derivative $\dfrac{dy}{dx}$ at the point (x, y) where $x = 1$, $y = 0$:

(a) -4, (b) 2, (c) 0, (d) -3

10. [5 marks] Consider the function f whose rule is defined by $f(x) = x^{3/5}(4 - x)$. Its critical points (numbers) are:

(a) $x = 0,\ x = 4$ (b) $x = 3/2$ only (c) $x = 0,\ x = 3/2,\ x = 4$ (d) $x = 0,\ x = 3/2$.

11. [5 marks] Consider the function f whose rule is given by $f(x) = 3x^5 - 80x^3 + 100$. This function is DECREASING AND CONCAVE DOWN on the interval(s):

(a) $(-\infty, -\sqrt{8}) \cup (0, \sqrt{8})$ (b) $(-4, -\sqrt{8}) \cup (0, \sqrt{8})$ (c) $(-\sqrt{8}, 0) \cup (\sqrt{8}, 4)$ (d) $(-4, 4)$

12. [5 marks] Determine the asymptotes (vertical and horizontal) of the function f whose rule is given by $f(x) = \dfrac{x - 3}{5x^2 - 11x - 12}$.

(a) Vertical asymptotes at $x = -4/5, x = 3$ only. Horizontal asymptote at $y = 0$. (b) Vertical asymptotes at $x = -4/5$. Horizontal asymptote given by $y = 1/5$. (c) Vertical asymptote at $x = -4/5$ only. Horizontal asymptote given by $y = 0$. (d) Vertical asymptotes at $x = -4/5$. No Horizontal asymptote.

13. [5 marks] An antiderivative of the function $f(x) = \tan(\frac{x}{2} + 1)$ is given by

(a) $\ln|\sec^2(\frac{x}{2} + 1)| - 1$ (b) $\ln|\sec(\frac{x}{2} + 1)|$ (c) $\ln|2\csc(\frac{x}{2} + 1)|$ (d) $2\ln|\csc(\frac{x}{2} + 1)|$.

14. [5 marks] Let $f : \mathbf{R} \to \mathbf{R}$ be a function whose second derivative is such that $f''(x) = (x - 1)(x - 2)^2(x - 3)^3(x - 4)^4$. Find the point(s) of inflection of the function f.

(a) $x = 1,\ x = 2$ only (b) $x = 1,\ x = 3$ only (c) $x = 2,\ x = 3$ only
(d) $x = 1,\ x = 2,\ x = 3$ and $x = 4$

15. [5 marks] Evaluate $\displaystyle\int_{-1}^{0} (x + 2)\ \sqrt[3]{x + 1}\ dx$

(a) $\dfrac{3}{11}$ (b) $\dfrac{33}{25}$ (c) $\dfrac{33}{28}$ (d) $\dfrac{11}{28}$

16. [5 marks] The improper integral $\displaystyle\int_0^{\infty} x^2\,e^{-x}\,dx$ is given by

(a) 1 (b) 3 (c) $\dfrac{1}{3}$ (d) 2

17. [5 marks] The value of $\displaystyle\int t^2\ln t\ dt$ is given by

(a) $-t^2(\ln t)^2 - \dfrac{t^2}{9} + C$ (b) $\dfrac{1}{3}t^3\ln t - \dfrac{t^3}{9} + C$ (c) $2(\ln t)^2 + \dfrac{1}{9} + C$ (d) $(\ln t)^2 + C$

18. [5 marks] Evaluate and simplify the definite integral: $\displaystyle\int_0^1 \frac{2x}{(x + 1)(x + 2)}\ dx$.

(a) $\ln 3$ (b) $\ln(81/64)$ (c) $\ln(3/2)$ (d) $\ln(1/2)$

19. [5 marks] Find an expression for the area between the curves defined by $y = 2$ and $y = 2x^2$ in the first quadrant.

(a) $2\displaystyle\int_0^1 (1 - x^2)\ dx$ (b) $\displaystyle\int_0^1 (x - \sqrt{x})\ dx$ (c) $\displaystyle\int_0^1 (x - 2x^2)\ dx$ (d) $\displaystyle\int_0^1 (x - x^2)\ dx$

20. [5 marks] Find an expression for the volume of the solid of revolution obtained by rotating the region in the first quadrant bounded by the curve defined by $y = \cos x$ between $x = 0$ and $x = \pi/2$ about the $y-$axis.

(a) $\pi\displaystyle\int_0^{\pi/2} x^2\cos x\,dx$ (b) $\displaystyle\int_0^{\pi/2} \cos x\,dx$ (c) $\displaystyle\int_0^{\pi/2} x\sin x\,dx$ (d) $2\pi\displaystyle\int_0^{\pi/2} x\cos x\,dx$