COEN 231 (Fall 2013) Assignment # 1 (Due 3 October)

Instructor: Dr. Nizar Bouguila

****You can form a group of two for this assignment.*

Exercise 1: (4 points)

Construct a truth table for each of these compound propositions

- $(p \wedge q) \vee r$
- $p \rightarrow (\neg q \vee r)$
- $(p \rightarrow q) \wedge (\neg p \rightarrow r)$
- $((p \rightarrow q) \rightarrow r) \rightarrow s$

Exercise 2: (4 points)

Show that the following statements are tautology without using truth tables:

- $[\neg p \wedge (p \vee q)] \rightarrow q$
- $[p \wedge (p \rightarrow q)] \rightarrow q$

Exercise 3: (4 points)

Show, without using truth tables, that

- $p \rightarrow q$ and $\neg q \rightarrow \neg p$ are logically equivalent.
- $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent.

Exercise 4: (4 points)

State the converse, the contrapositive and inverse of each of these statements:

- If it snows tonight, then I will stay at home.
- I go to the beach whenever it is a sunny summer day.

Exercise 5: (3 points)

Let $Q(x)$ be the statement "$x + 1 > 2x$". If the domain consists of all integers, what are the truth values of the following expressions?
$Q(0)$, $Q(-1)$, $\exists x Q(x)$, $\forall x Q(x)$, $\exists x \neg Q(x)$, $\forall x \neg Q(x)$

Exercise 6: (2 points)

Let $P(x, y)$ be the statement "student x has taken class y" where the domain of x consist of all students in your class and the domain of y consists of all computer engineering courses at your school. Express in English the following two quantifications and their negations.
$\exists x \exists y P(x, y)$ and $\exists x \forall y P(x, y)$

Exercise 7: (3 points)

Let $F(x, y)$ be the statement "x can fool y" where the domain consists of all people in the world. Use quantifiers to express each of these statements.

- Everybody can fool Fred
- Everybody can fool somebody
- There is no one who can fool everybody
- Everyone can be fooled by somebody
- There is exactly one person whom everyone can fool.

Exercise 8: (12 points)

- Use a direct proof to show that the sum of two even integers is even.
- Prove that if n is a perfect square, then $n + 2$ is not a perfect square.
- Prove that if x is rational and $x \neq 0$, the 1/x is rational.
- Prove that if n is an integer and $3n+2$ is even, then n is even
- Prove that if n is a positive integer, then n is even if and only if $7n+4$ is even.
- Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$.

Exercise 9: (4 points)

Let x and y be real numbers. Prove that if $x \leq y + \varepsilon$ for every positive real number ε, then $x \leq y$