Final Examination

Math1339 (C) Calculus and Vectors

December 22, 2010

09:30-12:30

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Final Examination

MAT 1339 C

Instructor: Sanghoon Baek

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Name: _____

Student Number: _____

- There are eight questions worth 100 points in total.
- Only non-programmable, non-graphic calculators are permitted.
- Please answer the questions in the provided space, clearly stating which question you are answering. Use the back of the pages if necessary, but please indicate you are doing so.

Good Luck!

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Total
/10	/15	/15	/10	/15	/10	/15	/10	/100

Problem 1: (10 points) Let $f(x) = \begin{cases} \frac{3x^2-3}{x+1}, & \text{for } x < -1 \\ 2x^2 + A, & \text{for } x \geq -1. \end{cases}$

(a) (5 points) For what value of A is $f(x)$ continuous at $x = -1$? Justify your answer.

(b) (5 points) Use the definition of the derivative to find the derivative of the function $g(x) = 2x^2 + A$. Even though you do not know the exact value of A, you can still do this problem.

Problem 2: (15 points)

Find the derivative of the following functions. Do not simplify.

(a) (5 points) $y = 8^x \ln(x^2)$

(b) (5 points) $y = \cos(2x)e^{x^2+x+1}$

(c) (5 points) $y = \dfrac{\log_2 x}{\sin(3x) + 2}$

Problem 3: (15 points)
Let $f(x) = \dfrac{x}{x^2 - 9}$.

(a) (3 points) Determine the domain of the function, i.e., find (if any) the values of x for which $\frac{x}{x^2-9}$ is not defined.

(b) (3 points) Find the vertical asymptotes (if any).

(c) (3 points) Determine all local minimums and maximums, if any exists.

(d) (3 points) Find the intervals where $f(x)$ is concave up, concave down.

(e) (3 points) Sketch the graph of the function. You may write a table including x, $f'(x)$, $f''(x)$, $f(x)$ and the important data found in (a) — (d).

Space for problem 3

Problem 4: (10 points)

Determine two positive integers such that their sum is 20 and the product of the cube of one and the square of the other is maximal.

Problem 5: (15 points)
Let $\vec{u} = [2, -2, 1]$, $\vec{v} = [0, 3, -4]$ and $\vec{w} = [3, 4, -5]$. Let θ be the angle between $\vec{u}$ and $\vec{v}$

(a) (3 points) Find the dot product $\vec{u} \cdot \vec{v}$.

(b) (3 points) Find the cross product $\vec{u} \times \vec{v}$.

(c) (3 points) Find $\cos\theta$ and $\sin\theta$.

(d) (3 points) Find the volume of the parallelepiped defined by $\vec{u}$, $\vec{v}$ and $\vec{w}$.

(e) (3 points) Decompose the vector $\vec{u}$ into a sum of two orthogonal vectors, one of which is collinear with the vector $\vec{v}$.

Problem 6: (10 points)

(a) (4 points) Find the parametric equation of the line in three-space which is parallel to the z-axis and passes through the point $(1, 2, 3)$.

(b) (6 points) Find the distance between two parallel planes $H_1 : 2x + y + 3z = 0$ and $H_2 : 2x + y + 3z = 1$.

Problem 7: (15 points)

(a) (7 points) Find a vector equation of the plane that contains the points $A(1, 2, -3)$, $B(4, -5, 6)$ and $C(7, 0, 1)$.

(b) (8 points) Does the plane given by $x + y + z + 10 = 0$ intersect the plane given by $[x, y, z] = [0, 1, 0] + t[1, 1, -1] + s[-1, 0, 1]$? If the answer is affirmative, find the solution.

Problem 8: (10 points)

Indicate whether the statement is true or false. Justify your answer.

(a) (2 points) The plane $3x - 42y - 5z - 2010 = 0$ is parallel to the plane $-x + 14y + z + 1 = 0$.

(b) (2 points) The line $[x, y, z] = [2, 1, 2] + t[1, 3, 4]$ is parallel to the line $[x, y, z] = [1, 2, 3] + s[-1, -3, -4]$.

(c) (2 points) $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$.

(d) (2 points) For a given vector $\vec{v}$, $\frac{\vec{v}}{|\vec{v}|}$ is a unit vector.

(e) (2 points) In three-space, two different lines that are not intersecting should be parallel.

Rough Work

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