UNIVERSITY OF OTTAWA
Principles of Physics
PHY1321/1331 FALL 2013
Dr. A. Czajkowski

CLASS: PHY_____

STUDENT #: ________________

NAME: ______________________

Assignment 2: KINEMATICS

Assigned: Sept 13 Due: September 20 18:00

1 By algebraic manipulation of the first two kinematic equations for one-dimensional motion:

a) $v_f = v_i + at$ b) $x_f = x_i + v_i t + \frac{1}{2} at^2$

Obtain the other two kinematic equations:

c) $v_f^2 - v_i^2 = 2a(x_f - x_i)$ d) $x_f - x_i = \frac{1}{2}(v_f + v_i)t$

SOLUTION:

$$1)\, v_f = v_i + at \Rightarrow t = \frac{(v_f - v_i)}{a}$$

$$2)\, x_f = x_i + v_i \frac{(v_f - v_i)}{a} + \frac{1}{2} a \frac{(v_f - v_i)^2}{a^2} \Rightarrow x_f - x_i = v_i \frac{(v_f - v_i)}{a} + \frac{1}{2a}\left(v_f^2 - 2v_i v_f + v_i^2\right) \Rightarrow$$

$$\Rightarrow 2a(x_f - x_i) = 2v_i(v_f - v_i) + \left(v_f^2 - 2v_i v_f + v_i^2\right) \Rightarrow 2a(x_f - x_i) = 2v_i v_f - 2v_i v_i + v_f^2 - 2v_i v_f + v_i^2 \Rightarrow 2a(x_f - x_i) = v_f^2 - v_i^2$$

$$1)\, v_f = v_i + at \Rightarrow t = \frac{(v_f - v_i)}{a}$$

$$2)\, x_f = x_i + v_i t + \frac{1}{2} a \frac{(v_f - v_i)}{a} t \Rightarrow x_f = x_i + v_i t + \frac{1}{2}(v_f - v_i)t \Rightarrow x_f = x_i + v_i t + \frac{1}{2} v_f t - \frac{1}{2} v_i t \Rightarrow x_f = x_i + \frac{1}{2}(v_f + v_i)t$$

2 An artillery shell is fired with an initial velocity of 400 m/s at 60.0° above the horizontal. It explodes on a mountainside 42.0 s after firing. If x is horizontal and y vertical, find the (x, y) coordinates where the shell explodes.

Note: originally there was a typo in this question which resulted v=200m/s. TA was informed for it and it will not affect the mark.

If x is horizontal and y vertical, find the (x, y) coordinates where the shell explodes.

$$x_f = x_i + v_x t = x_i + v_0(\cos\theta)t = 0 + 400(\cos 55)42 = 9636m$$

$$y_f = y_i + v_y t + \frac{1}{2} a_y t^2 = 0 + v_0(\sin\theta)t - \frac{1}{2} g t^2 = 0 + 400(\sin 55)42 - 4.9(42)^2 = 5118m$$

ANS: 9.63 km, 5.12 km

3 Find the angle of the projection for which the maximum height is equal to twice of the range.

We will use the equation for projectile motion trajectory y(x) for x=R/2 / 2R=y(R/2)

$$y = (\tan\theta)x - \frac{g}{2(v_0\cos\theta)^2}x^2 \Rightarrow 2R = (\tan\theta)(\frac{R}{2}) - \frac{g}{2(v_0\cos\theta)^2}(\frac{R}{2})^2 \Rightarrow 2 = (\tan\theta)(\frac{1}{2}) - \frac{g}{2(v_0\cos\theta)^2}\frac{R}{4}$$

$$2 = (\tan\theta)(\frac{1}{2}) - \frac{g}{2(v_0\cos\theta)^2}\frac{v_0^2\sin 2\theta}{4g} \Rightarrow 2 = \frac{1}{2}\frac{\sin\theta}{\cos\theta} - \frac{1}{8}\frac{\sin 2\theta}{\cos^2\theta} \Rightarrow 2 = \frac{1}{2}\frac{\sin\theta}{\cos\theta} - \frac{1}{8}\frac{2\sin\theta\cos\theta}{\cos^2\theta}$$

$$2 = \frac{1}{2}\frac{\sin\theta}{\cos\theta} - \frac{1}{4}\frac{\sin\theta}{\cos\theta} \Rightarrow 8 = \tan\theta \Rightarrow \theta = 82.9^o$$

$$\boxed{\theta = 82.9^o}$$

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Assignment 2: Cont.

4 A tire 0.500 m in radius rotates at a constant rate of 200 rev/min. Find the speed and acceleration of a small stone lodged in the tread of the tire (on its outer edge)

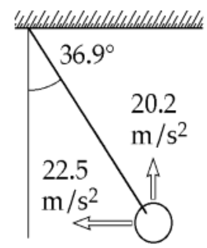
$. r = 0.500\text{ m}$;

$$v_t = \frac{2\pi r}{T} = \frac{2\pi(0.500\text{ m})}{\frac{60.0\text{ s}}{200\text{ rev}}} = 10.47\text{ m/s} = \boxed{10.5\text{ m/s}}$$

$$a = \frac{v^2}{R} = \frac{(10.47)^2}{0.5} = \boxed{219\text{ m/s}^2\text{ inward}}$$

5 A ball swings in a vertical circle at the end of a rope 1.50 m long. When the ball is 36.9° past the lowest point on its way up, its total acceleration $-22.5\vec{\imath} + 20.2\vec{\jmath}(\frac{m}{s^2})$. At that instant, (a) sketch a vector diagram showing the components of its acceleration, (b) determine the magnitude of its radial acceleration, and (c) determine the speed and velocity of the ball.

(a) See figure to the right.



(b) The components of the 20.2 and the 22.5 m/s^2 along the rope together constitute the centripetal acceleration:

$$a_c = (22.5\text{ m/s}^2)\cos(90.0° - 36.9°) + (20.2\text{ m/s}^2)\cos 36.9° = \boxed{29.7\text{ m/s}^2}$$

(c) $a_c = \frac{v^2}{r}$ so

$$\mathbf{v} = \boxed{6.67\text{ m/s at }36.9°\text{ above the horizontal}}$$

6 A river has a steady speed of 0.500 m/s. A student swims upstream a distance of 1.00 km and swims back to the starting point. If the student can swim at a speed of 1.20 m/s in still water, how long does the trip take? Compare this with the time the trip would take if the water were still.

Total time in still water $t = \frac{d}{v} = \frac{2\,000}{1.20} = \boxed{1.67 \times 10^3\text{ s}}$.

Total time = time upstream plus time downstream:

$$t_{\text{up}} = \frac{1\,000}{(1.20 - 0.500)} = 1.43 \times 10^3\text{ s}$$

$$t_{\text{down}} = \frac{1\,000}{1.20 + 0.500} = 588\text{ s}.$$

Therefore, $t_{\text{total}} = 1.43 \times 10^3 + 588 = \boxed{2.02 \times 10^3\text{ s}}$.