UNIVERSITY OF BRITISH COLUMBIA
DEPARTMENT OF MECHANICAL ENGINEERING

FINAL EXAMINATIONS, APRIL 2014

MECH 360 – Mechanics of Materials

Instructions:

- Time allowed = 2hours. Answer 4 questions. Total = 50 marks.
- A Mech 360 formula sheet is attached. No other materials are allowed.
- Calculators with memory capacity for formulas or information, programmable or with communication ability are not permitted.
- Turn off and put away all cell phones, pagers, alarms, etc. before starting the exam. Marks penalties may be assessed for disturbances that they cause.
- Clearly show all needed free-body diagrams.
- Credit will be given for orderly presentation of work, and conversely for disorderly work.

NAME:______________________________SIGNATURE:______________________________

SECTION:____________________STUDENT NUMBER:______________________________

Student Conduct During Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:

   i. speaking or communicating with other examination candidates, unless otherwise authorized;
   ii. purposely exposing written papers to the view of other examination candidates or imaging devices;

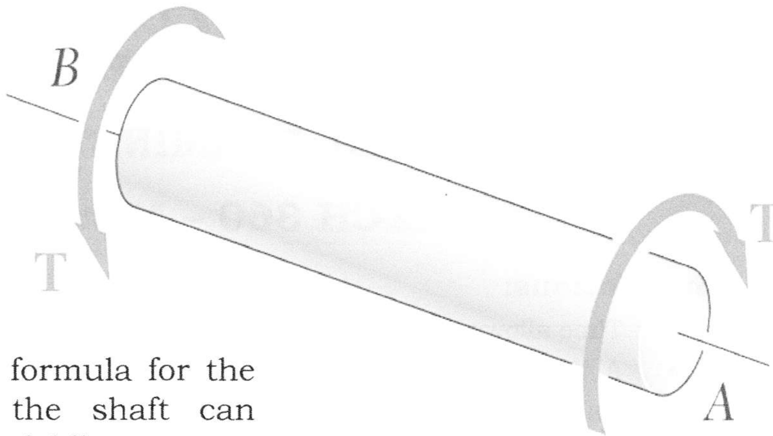
   iii. purposely viewing the written papers of other examination candidates;

   iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s) — (electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

CANDIDATES MUST IMMEDIATELY STOP WRITING WHEN THE INVIGILATOR ANNOUNCES THE EXAM IS OVER.

Question	Score
1	/13
2	/13
3	/13
4	/11
Orderly Presentation	/ 5
TOTAL	

1. A solid circular shaft of outside radius c and length L is made of a perfectly plastic ductile metal of shear yield stress τ_Y.



(a) Describe the ways in which the mechanics of shafts is analogous to the mechanics of beams.

(b) Using that analogy, derive a formula for the maximum torque T_Y that the shaft can support without any material yielding.

(c) Derive a formula for the fully plastic torque T_P.

(d) Determine and plot the residual stresses that remain after the fully plastic torque has been unloaded.

(e) Determine the shape factor $k = T_P / T_Y$.

(f) What does the shape factor indicate ? Compare your result from (e) with the shape factor of a solid circular beam in bending = 1.7. Explain the cause and significance of the difference.

(a) The basic theory of shafts and beams is very similar.

For shafts: plane sections remain plane and perpendicular.
shear strain created that is proportional to radius

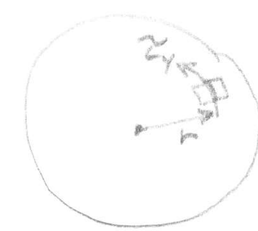
For beams: plane sections remain plane and perpendicular
longitudinal strain created that is proportional to distance from neutral plane.

These similar origins give rise to similar governing equations

For shafts: $\frac{\tau}{r} = \frac{T}{J} = \frac{G\phi}{L}$ For beams: $\frac{\sigma}{-y} = \frac{M}{I} = \frac{E}{R}$

(b) For $T_Y \rightarrow \tau = \tau_Y \quad r = c \rightarrow T_Y = \frac{J\tau_Y}{c} = \frac{\pi c^3 \tau_Y}{2}$

(c) For the fully plastic case, the shear stress $\tau = \tau_Y$ everywhere.

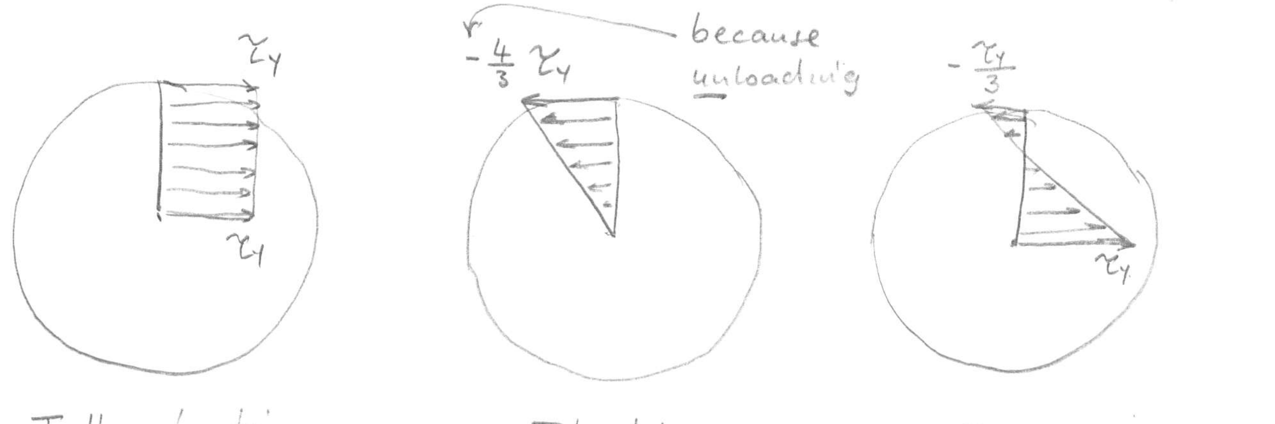


$$T_P = \int_0^c \tau r \, dA = \int_0^c \tau_Y r \cdot 2\pi r \, dr$$

$$T_p = \int_0^c 2\pi \tau_Y r^2 dr = 2\pi \tau_Y \left[\frac{r^3}{3}\right]_0^c = \underline{\frac{2\pi c^3 \tau_Y}{3}}$$

(d) Unloading occurs elastically, so we can use elastic formula

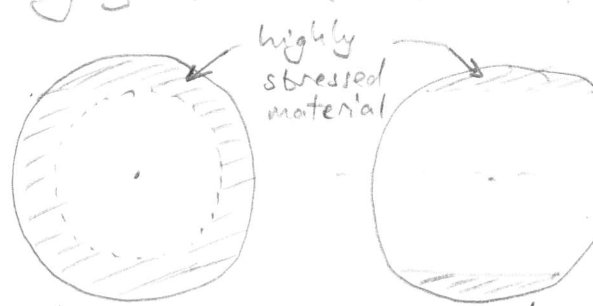
$$\tau_p = \frac{T_p c}{J} = \frac{2\pi c^3 \tau_Y}{3} \cdot \frac{c}{\frac{\pi}{2} c^4} = \frac{4}{3} \tau_Y \leftarrow \text{max stress on surface}$$



Fully plastic loading | Elastic unloading | Residual stresses

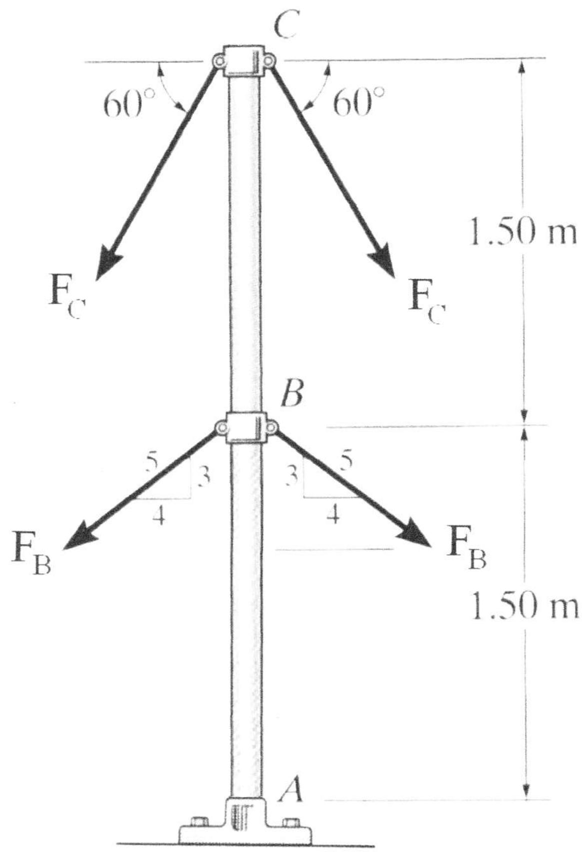
(e) Shape factor $k = \frac{T_p}{T_Y} = \dfrac{\frac{2\pi c^3 \tau_Y}{3}}{\frac{\pi c^3 \tau_Y}{2}} = \frac{4}{3} = \underline{1.33}$

(f) The shape factor indicates how much load carrying capacity remains after initial yielding has occurred. For efficient cross-sections, most of the load carrying capacity is in the elastic range, so the shape factor is only slightly greater than 1. When loaded in torsion, a circular shaft is fairly efficient because the large fraction of material near the surface is highly stressed. However, when loaded in bending, only a small fraction of the material is highly stressed, giving a much larger shape factor.



torsion | bending

2. A hollow steel column 30mm OD and 26mm ID is supported by stiff guy-wires, as shown in the diagram. There are four guy wires equally spaced at 90° intervals at each level (only two wires at each level are shown, the other two are in the plane "out of the paper"). The column is built-in at the base and each span between guy wire connections is 1.5m. For the steel, E = 210 GPa, ν = 0.3, σ_Y = 250 MPa. You are asked to determine the forces within the guy-wires that will cause the column to buckle along its entire length. *(Hint: the guy-wires prevent sideways displacements only, not rotations.)*



(a) Comment on the significant issues that need to be considered in this question.

(b) Describe the computational procedure you plan to use.

(c) Implement your method described in (b) and determine the required guy-wire forces.

(a) The column has two sections and the question asks about buckling over the "entire length". Thus, both parts must buckle simultaneously. Because the guy-wires do not provide rotational constraint, the section BC is a classic pinned-pinned column. But A is built-in, so section AC is fixed-pinned. The sections are not extremely slender, so it would be prudent to check the Johnson formula. Also, it can be seen that the axial force on section AB combines the force from BC and the lower guy-wires.

(b) 1. Determine J, r, Le, $\frac{Le}{r}$ and transition $\frac{Le}{r}$ for Johnson for sections AB and BC

2. Decide whether to use Euler or Johnson.

3. Find buckling loads using appropriate formula

4. Find corresponding F_C and F_B, noting that F_B need only provide the difference between the needed axial forces in AB and BC. Take into account the angles and the four wires in each set.

(c) $I = \frac{\pi}{4}(c_2^4 - c_1^4) = \frac{\pi}{4}(15^4 - 13^4) = 17330 \text{ mm}^4$

$A = \pi(c_2^2 - c_1^2) = \pi(15^2 - 13^2) = 176 \text{ mm}^2$ $\rightarrow r = \sqrt{\frac{I}{A}} = 9.92 \text{ mm}$

AB is fixed-pinned $\rightarrow L_e = 0.7L \rightarrow \frac{L_e}{r} = \frac{0.7 \times 1500}{9.92} = 105.8$

BC is pinned-pinned $\rightarrow L_e = L \rightarrow \frac{L_e}{r} = \frac{1500}{9.92} = 151.1$

Transition $\frac{L_e}{r}$ for Johnson $= \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 \times 210 \times 10^9}{250 \times 10^6}} = 128.7$

For AB, $\frac{L_e}{r} = 105.8 < 128.7 \longrightarrow$ use Johnson

For BC, $\frac{L_e}{r} = 151.1 > 128.7 \longrightarrow$ use Euler

For AB, $P_{AB} = A\left[\sigma_Y - \left(\frac{\sigma_Y^2}{4\pi^2 E}\right)\left(\frac{L_e}{r}\right)^2\right]$

$$= 176 \times 10^{-6}\left[250 \times 10^6 - \left(\frac{(250 \times 10^6)^2}{4\pi^2 \times 210 \times 10^9}\right) \times 105.8^2\right]$$

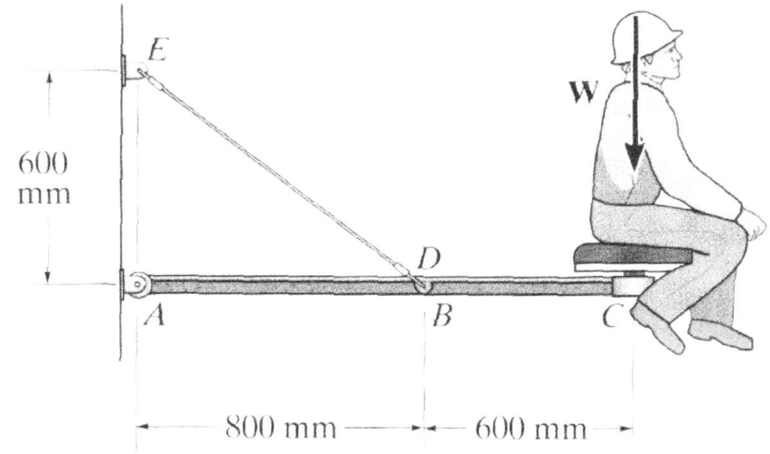
$= 29.15 \text{ kN}$

For BC, $P_{BC} = \frac{\pi^2 E I}{L_e^2} = \frac{\pi^2 \times 210 \times 10^9 \times 17330 \times 10^{-12}}{1.5^2} = 15.96 \text{ kN}$

At C: $4 \sin 60° F_C = P_{BC} \rightarrow F_C = \frac{15.96 \times 10^3}{4 \sin 60°} = \underline{4.6 \text{ kN}}$

At B: $4 \cdot \frac{3}{5} F_B = P_{AB} - P_{BC} \rightarrow F_B = \frac{(29.15 - 15.96) \times 10^3}{4 \times \frac{3}{5}} = \underline{5.5 \text{ kN}}$

3. An engineer, mass 80 kg, sits at point C on the S75 x 8.5 steel beam shown in the diagram. The beam is pinned at A and supported by a 4mm diameter steel wire at B. You are asked to determine how much point C moves downwards under load.



(a) Comment on the significant issues that need to be considered in this question.

(b) Describe the computational procedure you plan to use. Explain the thought process your used to make this plan.

(c) Implement your method described in (b) and determine the required downward displacement.

(d) Examine your result, is it reasonable ? Explain your conclusion.

(a) Typically, bending deformations are much bigger than axial deformations. This is true within the beam ABC, but since the cross-sectional area of DE is much less than of ABC, we cannot judge the relative deformations, and so must include both. Having both the beam and wire deform simultaneously makes the overall deformation rather complex.

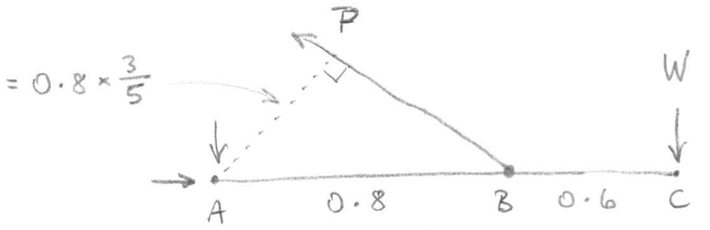
(b) Thus, we may seek to avoid this complexity by using an energy approach. Very conveniently, we are asked only for the deformation under load, so the work-energy method is OK.

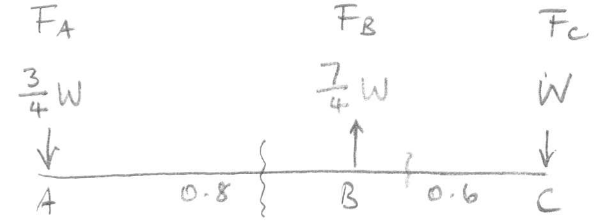
Compute $\int \frac{P^2}{2AE} dx$ for the wire and $\int \frac{M^2}{2EI}$ for the beam.

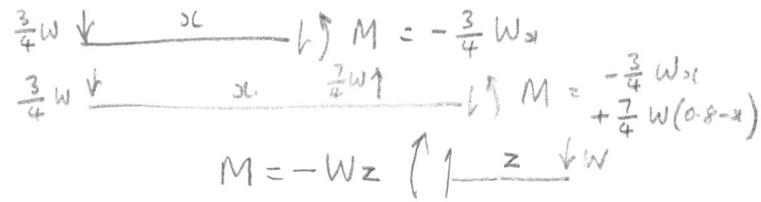
Sum them to find U and equate to $\frac{1}{2} W \delta$ to get δ.

As always, draw the FBD first!

(c) Draw FBD.







$$\Sigma M_A = P \times 0.8 \times \frac{3}{5} - W \times 1.4 = 0$$

$$\rightarrow P = \frac{35}{12} W = 2290 \text{ N}$$

Within the beam, the bending deformation are the largest

→ ignore axial deformations in beam

→ consider vertical force components only

$$\Sigma M_A = 0.8 F_B - 1.4 W = 0 \quad \rightarrow \quad F_B = \frac{7}{4} W$$

$$\Sigma M_B = 0.8 F_A - 0.6 W = 0 \quad \rightarrow \quad F_A = \frac{3}{4} W$$

Strain Energy: $U = \int \frac{P^2}{2AE} dx + \int \frac{M^2}{2EI} dx$.

$$U = \int_0^{1.0} \frac{P^2}{2EA} dx + \int_0^{0.8} \frac{(-\frac{3}{4}Wx)^2}{2EI} dx + \int_{0.8}^{1.4} \frac{(-\frac{3}{4}Wx + \frac{7}{4}W(0.8-x))^2}{2EI} dx$$

We could evaluate the third term as-is, but the algebra will be messy. Suggest measuring z from right → $z = 1.4 - x$.

→ Third integral becomes $\int_0^{0.6} \frac{(Wz)^2}{2EI} dz$

$$\rightarrow U = \frac{P^2 \, 1.0}{2EA} + \frac{3W^2 \, 0.8^3}{32 EI} + \frac{W^2 \, 0.6^3}{6EI}$$

$$= \frac{2290^2 \times 1.0}{2 \times 210 \times 10^9 \times \pi \times 0.002^2} + \frac{(80 \times 9.81)^2}{210 \times 10^9 \times 1.04 \times 10^{-6}} \left(\frac{3}{32} 0.8^3 + \frac{1}{6} 0.6^3 \right)$$

$$U = 0.99 + 0.24 = 1.23 \text{ J} = \frac{1}{2} W \cdot \delta$$

$$\rightarrow \delta = \frac{2U}{W} = \frac{2 \times 1.23}{80 \times 9.81} = \underline{1.56 \text{ mm}}$$

4.

(a) Explain the theoretical basis for the integration method for determining beam deflections.

The integration method for determining beam deflections is based on the basic beam result $\frac{\sigma}{-y} = \frac{M}{I} = \frac{E}{R}$ where the radius of the beam curvature R is mathematically related to the second derivative $\frac{d^2y}{dx^2} \approx \frac{1}{R}$ for small $\frac{dy}{dx}$.

Thus, $\frac{d^2y}{dx^2} = \frac{M}{EI}$. We can the determine y by integrating this equation twice to get y. The geometrical boundary conditions of the beam will supply the needed integration constants.

(b) Explain why the Macaulay method is used for beam deflection derivations and describe how it works.

The Macaulay method, using the angle brackets, is useful because it enables integration to be done on a beam with intermediate loads using a single equation for the whole beam. This avoids the need to solve separate equations for the different parts of the beam and having to handle all the interior continuity conditions. The discontinuities at the interior loads are handled by using angle bracket notation

$$\langle x - a \rangle^n = (x-a)^n \quad \text{if} \quad x - a > 0$$
$$= 0 \quad \text{if} \quad x - a \leq 0$$

These terms can be integrated or differentiated as needed, but care should be taken to retain their format.

Some Engineering Material Properties:

Property	Steel	Aluminum	Brass
Young's Modulus, **E**	210 GPa (30 psi x 10^6)	70 GPa (10 psi x 10^6)	105 GPa (15 psi x 10^6)
Shear Modulus, **G**	81 GPa (11.6 psi x 10^6)	26 GPa (3.7 psi x 10^6)	39 GPa (5.6 psi x 10^6)
Poisson's Ratio, $\boldsymbol{\nu}$	0.30	0.33	0.35
Thermal Expansion, $\boldsymbol{\alpha}$	$11\text{x}10^{-6}$ /°C ($6\text{x}10^{-6}$ °/F)	$22\text{x}10^{-6}$ /°C ($12\text{x}10^{-6}$ °/F)	$20\text{x}10^{-6}$ /°C ($11\text{x}10^{-6}$ °/F)
Density, $\boldsymbol{\rho}$	7850 kg/m^3 (0.28 lb/in^3)	2720 kg/m^3 (0.10 lb/in^3)	8410 kg/m^3 (0.30 lb/in^3)

Moments of Area for Common Shapes:

Shape	Cross-section A	I_x	I_y	$J = I_x + I_y$
solid circle	πc^2	$\pi c^4 / 4$	$\pi c^4 / 4$	$\pi c^4 / 2$
thin-wall circle	$2 \pi t c$	$\pi t c^3$	$\pi t c^3$	$2 \pi t c^3$
solid rectangle	$b h$	$b h^3 / 12$	$h b^3 / 12$	$b h (b^2 + h^2) / 12$

Elastic relationships:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E} + \alpha \Delta T \qquad \gamma = \frac{\tau}{G} \qquad G = \frac{E}{2(1+\nu)} \qquad K = \frac{E}{3(1-2\nu)}$$

Failure Criteria: (for principal stresses $\sigma_1 \geq \sigma_2 \geq \sigma_3$)

Tresca: $\sigma_1 - \sigma_3 = \sigma_Y$ **Mises**: $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_Y^2$ **Mohr**: $\sigma_1/\sigma_{UT} - \sigma_3/\sigma_{UC} = 1$

Thin-Walled Pressure Vessels:

$$\text{Cylinder:} \quad \sigma_\theta = \frac{p R}{t} \qquad \sigma_a = \frac{p R}{2 t} \qquad \sigma_r \approx 0 \qquad \text{Sphere:} \quad \sigma_\theta = \sigma_a = \frac{p R}{2 t} \qquad \sigma_r \approx 0$$

Shaft Torsion:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{G \phi}{L}$$

Beam Bending:

$$\frac{\sigma}{-Y} = \frac{M}{I} = \frac{E}{R}$$

Parallel Axis Theorem:

$$I(d) = I_0 + A d^2$$

Power Transmission:

$$P = \frac{2\pi f J \tau_{max}}{c} \text{ watts} \quad \textit{(metric units)} \qquad P = \frac{15.87 \times 10^{-6} \text{ rpm } J \tau_{max}}{c} \text{ h.p.} \quad \textit{(lb.in units)}$$

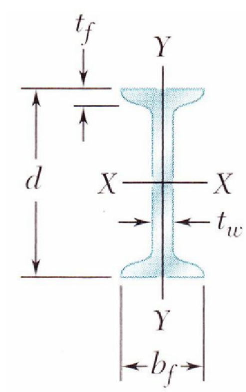
Beam Formulas:

$$V = \frac{dM}{dx} \qquad w = \frac{dV}{dx} \qquad \frac{d^2 y}{dx^2} = \frac{M}{EI} \qquad q = \frac{VA\bar{y}}{I} = \frac{VQ}{I} \quad \text{where } Q = \int y \, dA$$

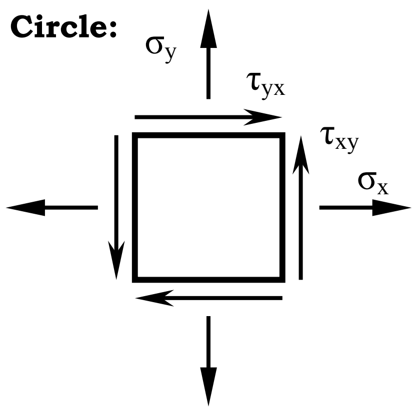
$$\langle x - a \rangle = (x - a) \text{ if } x - a > 0, \quad = 0 \text{ if } x - a \leq 0$$

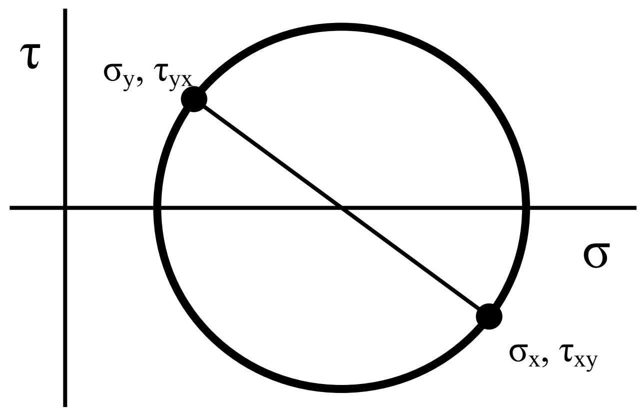
Some Typical I-Beams:

Designation†	Area A, mm²	Depth d, mm	Flange Width b_f, mm	Flange Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X I_x 10^6 mm⁴	Axis X-X S_x 10^3 mm³	Axis X-X r_x mm	Axis Y-Y I_y 10^6 mm⁴	Axis Y-Y S_y 10^3 mm³	Axis Y-Y r_y mm
S150 × 25.7	3260	152	90.7	9.12	11.8	10.9	143	57.9	0.953	21.0	17.1
18.6	2360	152	84.6	9.12	5.89	9.16	120	62.2	0.749	17.7	17.8
S130 × 15	1890	127	76.2	8.28	5.44	5.12	80.3	52.1	0.495	13.0	16.2
S100 × 14.1	1800	102	71.1	7.44	8.28	2.81	55.4	39.6	0.369	10.4	14.3
11.5	1460	102	67.6	7.44	4.90	2.52	49.7	41.7	0.311	9.21	14.6
S75 × 11.2	1420	76.2	63.8	6.60	8.86	1.21	31.8	29.2	0.241	7.55	13.0
8.5	1070	76.2	59.2	6.60	4.32	1.04	27.4	31.2	0.186	6.28	13.2



Mohr's Circle:





Use sign convention "***in the kitchen, the <u>clock is above</u> and the <u>counter is below</u>***" for the shear stresses. Then, rotations in the Mohr's circle have the same direction and double the rotation angle of the physical stresses. For Mohr's circle of strain, use $\boldsymbol{\varepsilon}$ and $\boldsymbol{\gamma/2}$ in place of $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$.

Column Buckling:

$$P_{CR} = \frac{\pi^2 EI}{L_e^{\,2}} = \frac{\pi^2 EA}{(L_e/r)^2}$$

where: $L_e = L$ (pinned-pinned) $\quad L_e = 2\,L$ (free-fixed)
$L_e = 0.7\,L$ (pinned-fixed) $\quad L_e = 0.5\,L$ (sliding-fixed)

$$\text{Eccentric load: } y_{max} = e\left(\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{CR}}}\right) - 1\right) \qquad \text{Initial curvature: } y_{max} = \frac{a\,P}{P_{CR} - P}$$

$$I = A\,r^2 \qquad \text{For "short" steel columns (Johnson): } \frac{L_e}{r} < \sqrt{\frac{2\pi^2 E}{\sigma_Y}} \qquad \sigma_{CR} = \sigma_Y - \left(\frac{\sigma_Y^{\,2}}{4\pi^2 E}\right)\left(\frac{L_e}{r}\right)^2$$

Strain Energy:

$$\text{rod: } U = \int \frac{P^2}{2EA}\,dx \qquad \text{beam: } U = \int \frac{M^2}{2EI}\,dx \qquad \text{shaft: } U = \int \frac{T^2}{2GJ}\,dx$$

Castigliano's Theorem:

$$\delta_j = \frac{\partial U}{\partial P_j} \qquad \theta_j = \frac{\partial U}{\partial M_j} \qquad \text{rod: } \delta_j = \int \frac{P}{EA}\frac{\partial P}{\partial P_j}\,dx \qquad \text{beam: } \delta_j = \int \frac{M}{EI}\frac{\partial M}{\partial P_j}\,dx$$