1

SOLUTIONS

APRIL 2013 — Final Exam — MATH 206

Simplify the expressions (without the calculator)

1 (a) $3\sqrt{5} + \sqrt{20} - 2\sqrt{45} = 3\sqrt{5} + \sqrt{4\cdot 5} - 2\sqrt{9\cdot 5}$

$= 3\sqrt{5} + \sqrt{4}\cdot\sqrt{5} - (2\sqrt{9}\cdot\sqrt{5})$

$= 3\sqrt{5} + 2\sqrt{5} - (2\cdot 3\sqrt{5})$

$= 3\sqrt{5} + 2\sqrt{5} - 6\sqrt{5}$

$= (3+2-6)\sqrt{5} = (-1)\sqrt{5} = \boxed{-\sqrt{5}}$

(b) $\log_5 10 + \log_5(3^3 - 12) - \log_5 30 = \log_5(10) + \log_5(27-12) - \log_5(30)$

$= \underbrace{\log_5(10) + \log_5(15)} - \log_5 30$

$= \log_5(10\cdot 15) - \log_5 30$

$= \underbrace{\log_5(150) - \log_5 30}$

$= \log_5\left(\frac{150}{30}\right) = \log_5(5) = \frac{\ln 5}{\ln 5} = \boxed{1}$

Rationalize the denominator

2 (a) $\frac{\sqrt{5}}{3-\sqrt{2}} = \frac{\sqrt{5}}{3-\sqrt{2}} \cdot \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{(\sqrt{5})(3+\sqrt{2})}{(3)^2 - (\sqrt{2})^2} = \frac{3\sqrt{5} + \sqrt{2}\cdot\sqrt{5}}{9-2}$

$= \boxed{\frac{3\sqrt{5} + \sqrt{10}}{7}}$

(b) $\frac{1+\sqrt{2}}{1-\sqrt{2}} = \frac{1+\sqrt{2}}{1-\sqrt{2}} \cdot \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{(1+\sqrt{2})(1+\sqrt{2})}{(1)^2 - (\sqrt{2})^2} = \frac{1+2\sqrt{2}+2}{1-2} = \frac{3+2\sqrt{2}}{-1}$

$= \boxed{-3-2\sqrt{2}}$

Simplify the expressions

3 (a) $6x(x^3 - x^2 - 3x) - [4x(3x^4 - 2x^3 + 3x^2 + x)] =$

$= 6x^4 - 6x^3 - 18x^2 - [12x^5 - 8x^4 + 12x^3 + 4x^2] =$

$= 6x^4 - 6x^3 - 18x^2 - 12x^5 + 8x^4 - 12x^3 - 4x^2$

$= \boxed{-12x^5 + 14x^4 - 18x^3 - 22x^2}$ or $\boxed{-2x^2(6x^3 - 7x^2 + 9x + 11)}$

factor form
(Thus, I can cancel terms)

(b) $\dfrac{4x^2-8x}{12x-24} = \dfrac{\cancel{4}x\,\cancel{(x-2)}}{\cancel{12}_3\,\cancel{(x-2)}} = \boxed{\dfrac{x}{3}}$

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(4) (a) $2x^2-x-6$

Factor Completely

↳ Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2\cdot a} = \frac{-(-1) \pm \sqrt{(-1)^2-4(2)(-6)}}{2\cdot 2} = \frac{1 \pm \sqrt{1+48}}{4}$$

$$= \frac{1 \pm \sqrt{49}}{4} = \frac{1 \pm 7}{4} \quad \boxed{\begin{matrix} 2 \\ -\frac{3}{2} \end{matrix}}$$

solutions for $2x^2-x-6=0$

But, you are asked to FACTOR:

$$2x^2-x-6 = 2(x-x_1)(x-x_2)$$

$$= \boxed{2(x-2)(x+3/2)}$$

$x_1 = 2$
$x_2 = -3/2$

CHECK your answer: (EXPAND)

$$2(x-2)(x+3/2) = 2\left(x^2+\frac{3}{2}x-2x-3\right)$$

$$= 2x^2+3x-4x-6$$

$$= 2x^2-x-6 \quad ✓$$

gives you back your initial polynomial!

(b) $x^7 - x^5 = (x^5 \cdot x^2) - (x^5)$

$$= \underbrace{x^5(x^2-1)}_{a^2-b^2=(a-b)(a+b)} = \boxed{x^5(x+1)(x-1)}$$

Check: (by expanding your answer)

$$x^5(x+1)(x-1) = x^5(x^2-x+x-1)$$

$$= x^5(x^2-1) = x^7-x^5 \quad ✓$$

gives you back your initial polynomial!

2

5 Perform the arithmetic operations & simplify

$$\boxed{\frac{x+4}{x^2-x-2}} - \boxed{\frac{2x+3}{x^2+2x-8}} =$$

① ②

① $\frac{x+4}{x^2-x-2} \longrightarrow$ factor it! $\longrightarrow = \frac{x+4}{(x-2)(x+1)}$

use Quadratic f.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2-4(1)(-2)}}{2 \cdot 1} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2} \begin{cases} 2 \\ -1 \end{cases}$$

② $\frac{2x+3}{x^2+2x-8} \longrightarrow$ factor it $\longrightarrow = \frac{2x+3}{(x-2)(x+4)}$

use Quadratic f:

$$x = \frac{-(2) \pm \sqrt{(2)^2-4(1)(-8)}}{2 \cdot 1} = \frac{-2 \pm \sqrt{4+32}}{2} = \frac{-2 \pm \sqrt{36}}{2} = \frac{-2 \pm 6}{2} \begin{cases} 2 \\ -4 \end{cases}$$

① − ② $= \frac{x+4}{(x-2)(x+1)} - \frac{2x+3}{(x-2)(x+4)}$

LCM: $(x-2)(x+1)(x+4)$

$$= \frac{x+4}{(x-2)(x+1)} \cdot \frac{(x+4)}{(x+4)} - \frac{2x+3}{(x-2)(x+4)} \cdot \frac{(x+1)}{(x+1)}$$

$$= \frac{(x+4)(x+4) - (2x+3)(x+1)}{(x-2)(x+1)(x+4)}$$

$$= \frac{x^2+8x+16-(2x^2+2x+3x+3)}{(x-2)(x+1)(x+4)}$$

$$= \frac{x^2+8x+16-2x^2-2x-3x-3}{(x-2)(x+1)(x+4)} =$$

$$= \boxed{\frac{-x^2+3x+13}{(x-2)(x+1)(x+4)}}$$

⑥ Solve the equations :

(a) $\frac{x+1}{x^2+2x} - \frac{x+4}{x^2+x} = \frac{-3}{x^2+3x+2}$ $\longrightarrow$ use quadratic f.

$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(2)}}{2 \cdot 1}$

$= \frac{-3 \pm 1}{2} < \begin{matrix} -1 \\ -2 \end{matrix}$

$\Rightarrow x^2+3x+2 = (x+1)(x+2)$

$\frac{x+1}{x(x+2)} - \frac{x+4}{x(x+1)} = \frac{-3}{(x+1)(x+2)}$

$\frac{x+1}{x(x+2)} - \frac{x+4}{x(x+1)} + \frac{3}{(x+1)(x+2)} = 0$

LCM: $(x)(x+1)(x+2)$

$\frac{x+1}{x(x+2)} \cdot \frac{(x+1)}{(x+1)} - \frac{x+4}{x(x+1)} \cdot \frac{(x+2)}{(x+2)} + \frac{3}{(x+1)(x+2)} \cdot \frac{(x)}{(x)} = 0$

$x(x+1)(x+2) \cdot \left(\frac{(x+1)^2 - (x+4)(x+2) + 3x}{x(x+1)(x+2)} \right) = (0) \cdot (x)(x+1)(x+2)$

$(x+1)^2 - (x+4)(x+2) + 3x = 0$

$x^2 + 2x + 1 - (x^2 + 2x + 4x + 8) + 3x = 0$

$\cancel{x^2} + \cancel{2x} + 1 - \cancel{x^2} - \cancel{2x} - 4x - 8 + 3x = 0$

$-x - 7 = 0$

$\boxed{x = -7}$

(b) $\frac{3(169^x)}{3} = \frac{39}{3}$

$\underbrace{169^x = 13}_{\text{Exponential}}$ $\longrightarrow$ $\log_{169} 13 = x$

Exponential

base : 169

Logarithmic base 169

$\frac{\ln 13}{\ln 169} = \boxed{x = \frac{1}{2}}$

use calculator

③

(c) $\log_4(5-2x) = -2$

$\downarrow$

in exponential form base 4 :

$$4^{-2} = 5-2x$$

$$\frac{1}{4^2} = 5-2x$$

$$2x = 5 - \frac{1}{16} = \frac{79}{16}$$

$$x = \frac{\frac{79}{16}}{2} \longrightarrow \boxed{x = \frac{79}{32}}$$

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⑦ Solve the inequalities, express your answer using set notation or interval notation:

(a) $1 \le \frac{-2-3x}{7} < 4$

$$7(1) \le \cancel{7}\left(\frac{-2-3x}{\cancel{7}}\right) < 7(4)$$

$$7 \le -2-3x < 28$$

$$7+2 \le \cancel{-2}-3x\cancel{+2} < 28+2$$

$$9 \le -3x < 30$$

negative → $\frac{9}{-3} \quad \frac{-3x}{-3} \quad \frac{30}{-3}$

$$\boxed{-3 \ge x > -10} \Rightarrow$$

$$\Updownarrow$$

$$\boxed{-10 < x \le -3}$$

Answer :

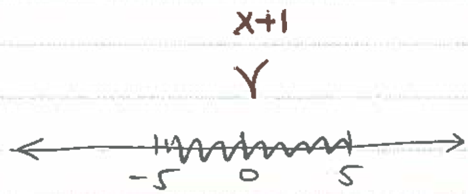
① set notation : $\{x \mid -10 < x \le -3\}$

or

② Interval notation : $(-10, -3]$

(b) $2|x+1|-3 \le 7$

- $2|x+1|-3+3 \le 7+3$
- $2|x+1| \le 10$
- $\frac{2|x+1|}{2} \le \frac{10}{2}$
- $|x+1| \le 5$



$\Downarrow$

$-5 \le x+1 \le 5$

$-5-1 \le x+1-1 \le 5-1$

$-6 \le x \le 4$

Answer:

① set notation: $\{x \mid -6 \le x \le 4\}$

OR ② Interval notation: $[-6, 4]$

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⑧ Solve the system of equations: (ie. Find all the values of x & y that work for All the equations)

$$\begin{cases} 2x^2 + y^2 = 1 & (1) \\ 2x - y = -1 & (2) \end{cases}$$

By SUBSTITUTION:

(2) $2x - y = -1 \longrightarrow \boxed{y = 2x+1}$

(1) $2x^2 + y^2 = 1$

$2x^2 + (2x+1)^2 = 1$

$2x^2 + 4x^2 + 4x + 1 = 1$

$6x^2 + 4x = 0$

$2x(3x+2) = 0$

$2x = 0$ for $x = 0$

$3x+2=0$ for $x = -\frac{2}{3}$

Backsubstitution into $y = 2x+1$ to find y:

if $x = 0$, $y = 2(0)+1 = 1$

if $x = -\frac{2}{3}$, $y = 2\left(-\frac{2}{3}\right)+1 = -\frac{1}{3}$

Answer: For (x,y): $(0,1)$ & $\left(-\frac{2}{3}, -\frac{1}{3}\right)$

OR

$\boxed{\begin{matrix} x=0 \\ y=1 \end{matrix}}$ $\boxed{\begin{matrix} x=-2/3 \\ y=-1/3 \end{matrix}}$

4

9

(a) which of ~~the~~ points A (9,3), B (4,5) is closer to the point C (13,4)?

(i) First, find the distance between A & C and B & C.

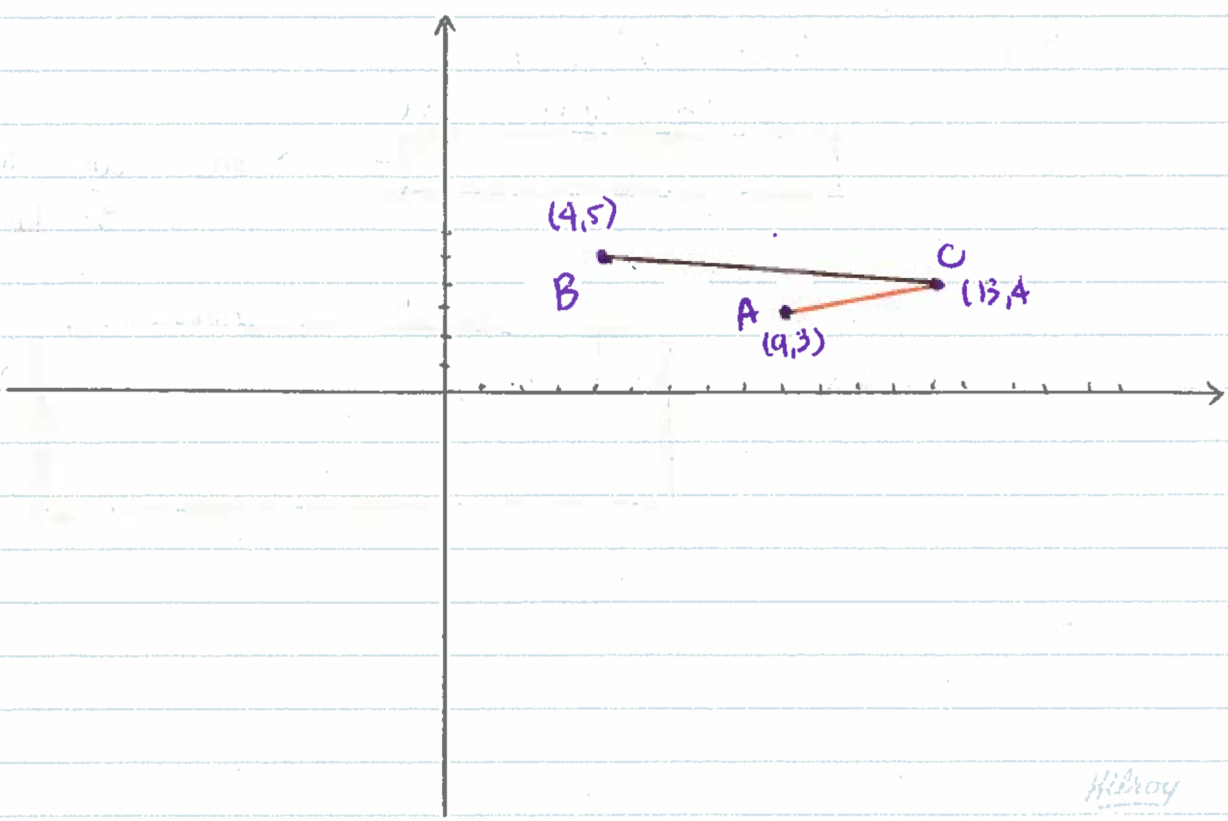
let $(x_1, y_1) = A(9,3)$
$(x_2, y_2) = B(4,5)$
$(x_3, y_3) = C(13,4)$

$\boxed{d(A,C)} = \sqrt{(x_3-x_1)^2 + (y_3-y_1)^2} = \sqrt{(13-9)^2 + (4-3)^2} = \sqrt{4^2 + 1^2} = \sqrt{9} = 3$

$\boxed{d(B,C)} = \sqrt{(x_3-x_2)^2 + (y_3-y_2)^2} = \sqrt{(13-4)^2 + (4-5)^2} = \sqrt{9^2 + (-1)^2} = \sqrt{82} \approx 9.06$

(ii) Compare both distances & pick the smallest one (the point closer to c!)

$d(A,C) < d(B,C) \Rightarrow$ **Point A is closer to c**
$3 < \sim 9.06$



Kilroy

Show the equation represents a circle &

(b) Find coordinates of the center & radius of the circle

$$x^2 + y^2 - 8x + 20y + 107 = 0$$

$$\boxed{x^2 - 8x} + \boxed{y^2 + 20y} + 107 = 0$$

① ②

① $(x+a)^2 = x^2 + \boxed{2ax} + a^2$

$= x^2 \boxed{-8x} + \boxed{}$

$2ax = -8x$

$2a = -8 \Rightarrow \boxed{a = -4} \therefore a^2 = 16$

$$\Rightarrow x^2 - 8x = x^2 - 8x + (-4)^2 - (-4)^2$$

$$= \boxed{x^2 - 8x + 16} - 16$$

$$= (x-4)^2 - 16$$

② $(y+b)^2 = y^2 + \boxed{2by} + b^2$

$= y^2 + \boxed{20y} + \boxed{}$

$2by = 20y$

$2b = 20 \Rightarrow \boxed{b = 10} \therefore b^2 = 100$

$$\Rightarrow y^2 + 20y = y^2 + 20y + (10)^2 - (10)^2$$

$$= \boxed{y^2 + 20y + 100} - 100$$

$$= (y+10)^2 - 100$$

Thus, $x^2 - 8x + y^2 + 20y + 107 = 0$

$(x-4)^2 - 16 + (y+10)^2 - 100 + 107 = 0$

$\boxed{(x-4)^2 + (y+10)^2 = 9}$ → this shows the given equation represents a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

- center : $(h,k) = (4,-10)$
- radius : $r = 3$

→ Coordinates of the center & radius

5

10 Find the domain & Range of the functions

(a) $f(x) = \frac{3}{x^2+9}$ — denominator $\neq 0$

$x^2+9 = 0$

$x^2 = -9$] impossible!

⇒ no matter the value of x, the denominator will never be zero. Thus, no restrictions.

$\boxed{\text{Domain: all real numbers.}}$

→ what is the inverse of f ?

$y = \frac{3}{x^2+9} \Longrightarrow x = \frac{3}{y^2+9}$

$x(y^2+9) = 3$

$xy^2 + 9x = 3$

$xy^2 = 3 - 9x$

$y^2 = \frac{3-9x}{x}$

$y = \sqrt{\frac{3-9x}{x}}$

$y = -\sqrt{\frac{3-9x}{x}}$

For $\sqrt{\frac{3-9x}{x}}$

① $\boxed{x \neq 0}$ (Denominator $\neq 0$)

② $x^2\left(\frac{3-9x}{x}\right) \geq (0)x^2$

$3x - 9x^2 \geq 0 \Longrightarrow 3x \geq 9x^2 \Rightarrow \boxed{x \leq 1/3}$

$1 \geq 3x$

$3x(1-3x) \geq 0$

notice if x is negative, $3x$ = negative # , $(1-3x)$ = positive # } ⇒ the answer of $3x(1-3x)$ = negative #

⇒ x has to be positive $\boxed{(x>0)}$

Range of f = Domain f^{-1} : $\{x \mid 0 < x \leq 1/3\}$

thus: $\boxed{\text{Range } f = \{ y \mid 0 < y \leq 1/3 \}}$

(b) $g(x) = \sqrt{4-3x}$

has to be ≥ 0

$4-3x \geq 0$

$4 \geq 3x$

$\boxed{\frac{4}{3} \geq x}$

$\boxed{\text{Domain } g : \{x \mid x \leq \frac{4}{3}\}}$

what is the inverse of g ?

$y = \sqrt{4-3x} \Rightarrow x = \sqrt{4-3y}$

$x^2 = (\sqrt{4-3y})^2$

$x^2 = 4-3y$

$3y = 4-x^2$

$\boxed{y = \frac{4-x^2}{3}} = g^{-1}(x)$

$\boxed{\text{Range } g = \text{Domain of } g^{-1} = \text{all real numbers}}$

(c) $h(x) = |x-4|$

from $y = |x| \longrightarrow h(x) = |x-4|$

there was a horizontal shift to the right of 4 units.

$\boxed{\begin{array}{l}\text{Domain } h: \text{ all real numbers} \\ \text{Range } h : \{y \mid y \geq 0\}\end{array}}$ based on the properties of the absolute value function.

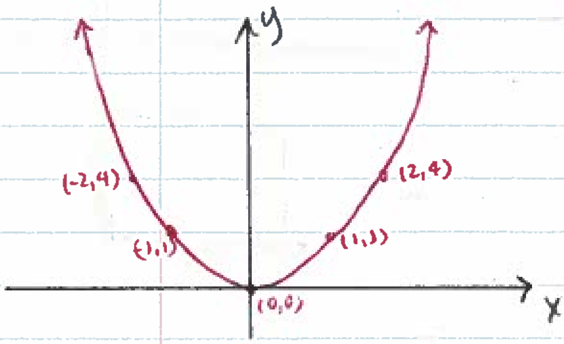
Note : the absolute value function is not a one-to-one function. Thus, it doesn't have an inverse function.

6

(11) Sketch the graph of the function $f(x) = -2(x-3)^2$, starting from the graph of the function $g(x) = x^2$ & using appropriate transformations.

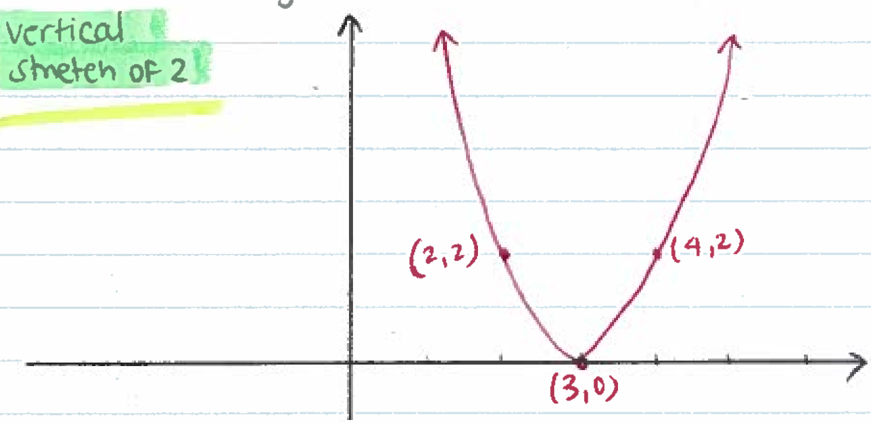
(1) $y = g(x) = x^2 \longrightarrow y = (x-3)^2$

horizontal shift to the right of 3 units.



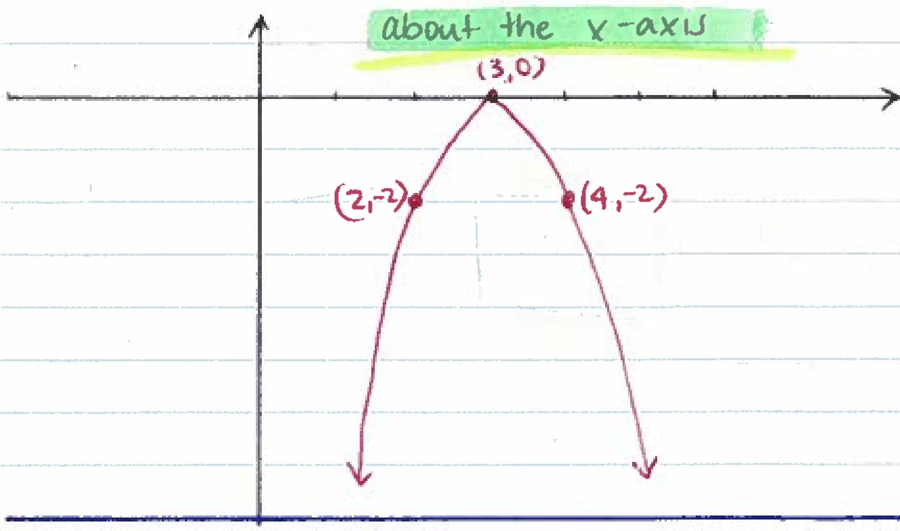
(2) $y = (x-3)^2 \longrightarrow y = 2(x-3)^2$

vertical stretch of 2



(3) $y = 2(x-3)^2 \longrightarrow f(x) = -2(x-3)^2$

reflection about the x-axis



⑫ $f(x) = \dfrac{x+3}{2x-4}$ $\qquad g(x) = \dfrac{3x-1}{x-2}$

(a) $fg = \dfrac{x+3}{2x-4} \cdot \dfrac{3x-1}{x-2} = \dfrac{(x+3)(3x-1)}{(2x-4)(x-2)} = \dfrac{3x^2-x+9x-3}{2x^2-4x-4x+8}$

$= \boxed{\dfrac{3x^2+8x-3}{2x^2-8x+8}} = \boxed{\dfrac{3x^2+8x-3}{2(x^2-4x+4)}}$

(b) $\dfrac{f}{g} = \dfrac{\frac{x+3}{2x-4}}{\frac{3x-1}{x-2}} = \dfrac{x+3}{2x-4} \cdot \dfrac{x-2}{3x-1} = \dfrac{(x+3)\cancel{(x-2)}}{2\cancel{(x-2)}(3x-1)} = \boxed{\dfrac{x+3}{2(3x-1)}}$

(c) $f\circ g = f(g(x)) = f\left(\dfrac{3x-1}{x-2}\right) = \dfrac{\left(\frac{3x-1}{x-2}\right)+3}{2\left(\frac{3x-1}{x-2}\right)-4}$

$= \dfrac{\frac{3x-1}{x-2} + \frac{3(x-2)}{(x-2)}}{\frac{2(3x-1)}{x-2} - \frac{4(x-2)}{(x-2)}} = \dfrac{\frac{3x-1+3(x-2)}{x-2}}{\frac{2(3x-1)-4(x-2)}{x-2}} = \dfrac{3x-1+3(x-2)}{\cancel{(x-2)}} \cdot \dfrac{\cancel{(x-2)}}{2(3x-1)-4(x-2)}$

$= \dfrac{3x-1+3x-6}{6x-2-4x+8} = \boxed{\dfrac{6x-7}{2x+6}} = \boxed{\dfrac{6x-7}{2(x+3)}}$

(d) $g\circ f = g(f(x)) = g\left(\dfrac{x+3}{2x-4}\right) = \dfrac{3\left(\frac{x+3}{2x-4}\right)-1}{\left(\frac{x+3}{2x-4}\right)-2}$

$= \dfrac{\frac{3(x+3)}{2x-4} - \frac{(2x-4)}{(2x-4)}}{\frac{x+3}{2x-4} - \frac{2(2x-4)}{(2x-4)}} = \dfrac{\frac{3(x+3)-(2x-4)}{2x-4}}{\frac{(x+3)-2(2x-4)}{2x-4}} = \dfrac{3(x+3)-(2x-4)}{\cancel{(2x-4)}} \cdot \dfrac{\cancel{(2x-4)}}{(x+3)-2(2x-4)}$

$= \dfrac{3x+9-2x+4}{x+3-4x+8} = \boxed{\dfrac{x+13}{-3x+11}}$

7

13 1

(a) Find the inverse $f(x) = \dfrac{2x+5}{x+7}$

$$y = \frac{2x+5}{x+7} \longrightarrow x = \frac{2y+5}{y+7}$$

$$x(y+7) = 2y+5$$
$$xy + 7x = 2y+5$$
$$xy - 2y = 5 - 7x$$
$$y(x-2) = 5 - 7x$$
$$\boxed{y = \frac{5-7x}{x-2} = f^{-1}(x)}$$

(b) find the vertical & horizontal asymptotes of both f & f^{-1} above

For $f(x) = \dfrac{2x+5}{x+7}$

1- $f(x)$ is in proper terms. then,

2- Find the zeros of the denominator:

$$x+7 = 0$$

$\boxed{x = -7}$: vertical asymptote

3- Is the degree of the denominator > the degree of the numerator?

NO, they have the same degree.

⇒ use the long division:

$$\begin{array}{r} 2 \\ x+7 \overline{)\ 2x+5} \\ -(2x+14) \\ \hline -9 \end{array} \quad \frac{2x+5}{x+7} = 2 + \frac{-9}{x+7}$$

as $x \to \infty$ (very big)

$$\frac{-9}{x+7} \longrightarrow 0$$

then, $\boxed{y = 2}$: horizontal asymptote

For $f^{-1}(x) = \dfrac{5-7x}{x-2}$

1- $f^{-1}(x)$ is in proper terms, then,

2- Find the zeros of the denominator:

$$x-2=0$$

$\boxed{x=2}$: vertical asymptote

3- Is the degree of the denominator > the degree of the numerator?

NO, they have the same degree. Use long division:

$$\begin{array}{r} -7 \\ x-2 \,\overline{)\, -7x+5} \\ -(-7x+14) \\ \hline -9 \end{array} \qquad \frac{5-7x}{x-2} = -7 + \frac{-9}{x-2}$$

as $x \to 0$ (BIG)

$$\frac{-9}{x-2} \to 0.$$

Thus, $\boxed{y=-7}$: horizontal asymptote

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(14) Connie : 2 hours $\longrightarrow$ $\boxed{\frac{1}{2}}$ of work done in 1 hour

Alvaro : t hours $\longrightarrow$ $\boxed{\frac{1}{t}}$ of work done in 1 hour

15mins = $\frac{1}{4}$ of an hour

Connie + Alvaro : 1h 15mins = $1\frac{1}{4} = \frac{5}{4}$ hours $\longrightarrow$ $\frac{1}{5/4} = \boxed{\frac{4}{5}}$ of work done in 1 hour

Connie time + Alvaro time = time for both them (if they work together)

$$\frac{1}{2} + \frac{1}{t} = \frac{4}{5}$$

$$\frac{1}{t} = \frac{4}{5} - \frac{1}{2} = \frac{4}{5}\cdot\frac{2}{2} - \frac{1}{2}\cdot\frac{5}{5} = \frac{8-5}{10} = \frac{3}{10}$$

$$\frac{1}{t} = \frac{3}{10}$$

$$3t = 10$$

$$\boxed{t = \frac{10}{3} \text{ hours} \approx 3.33 \text{ hours}}$$

8

15 Let x = amount invested in bonds
y = amount invested in stocks

- "A total of \$18,000 is invested, some in stocks & some in bonds" $\Downarrow$

$$18000 = x+y$$

- "If the amount invested in bonds is half that invested in stocks..." $\Downarrow$

$$x = \frac{1}{2}y \quad \text{or} \quad 2x = y$$

$\Downarrow$

system of equations!

$$\begin{cases} x + y = 18000 & (1) \\ 2x = y & (2) \end{cases}$$

→ Substitution: (2) in (1)

$$(1)\quad x + y = 18000$$
$$x + (2x) = 18000$$
$$3x = 18000$$
$$\boxed{x = 6000}$$

→ Back substitution in (2): $2(6000) = \boxed{y = 12000}$

Answer: \$6000 was invested in bonds (x)
& \$12000 was invested in stocks (y)

population (in grams) of bacterium

carrying capacity

(16) $P(t) = \dfrac{1000}{1+32.33\,e^{-0.439t}}$; t = hours (time)

(a) what is the population after 9 hours?

$t=9 \quad \therefore \quad P(9) = \dfrac{1000}{1+32.33\,e^{-0.439\cdot 9}}$ $\boxed{\approx 616.57 \text{ grams}}$

(b) When will the population be 700grams?

$t = ?$

$$P(t) = 700 = \frac{1000}{1+32.33e^{-0.439t}}$$

$$700(1+32.33e^{-0.439t}) = 1000$$

$$700 + 22631e^{-0.439t} = 1000$$

$$22631e^{-0.439t} = 300$$

$$e^{-0.439t} = \frac{300}{22631}$$

$\Downarrow$ — in Log base e : Ln

$$\ln\left(\frac{300}{22631}\right) = -0.439t$$

$$t = \frac{\ln\left(\frac{300}{22631}\right)}{-0.439} \quad \boxed{\approx 9.85 \text{ hours}}$$

(c) How long does it take for the population to reach one-half the carrying capacity?

$t = ?$

$$P(t) = \frac{1}{2}(1000) = 500 = \frac{1000}{1+32.33e^{-0.439t}}$$

$$500 + 16165e^{-0.439t} = 1000$$

$$e^{-0.439t} = \frac{500}{16165} \longrightarrow \ln\left(\frac{500}{16165}\right) = -0.439t$$

$$t = \frac{\ln\left(\frac{500}{16165}\right)}{-0.439} \quad \boxed{\approx 7.92 \text{ hours}}$$

(16) $P(t) = \dfrac{1000}{1 + 32.33\, e^{-0.439t}}$; population (in grams) of bacterium; 1000: carrying capacity; t = hours (time)

(a) what is the population after 9 hours?

$t = 9 \quad \therefore \quad P(9) = \dfrac{1000}{1 + 32.33\, e^{-0.439 \cdot 9}}$ $\boxed{\approx 616.57 \text{ grams}}$

(b) When will the population be 700 grams?

$t = ?$

$P(t) = 700 = \dfrac{1000}{1 + 32.33 e^{-0.439t}}$

$700(1 + 32.33 e^{-0.439t}) = 1000$

$700 + 22631 e^{-0.439t} = 1000$

$22631 e^{-0.439t} = 300$

$e^{-0.439t} = \dfrac{300}{22631}$

$\Downarrow$ → in Log base e : Ln

$\ln\left(\dfrac{300}{22631}\right) = -0.439t$

$t = \dfrac{\ln\left(\frac{300}{22631}\right)}{-0.439}$ $\boxed{\approx 9.85 \text{ hours}}$

(c) How long does it take for the population to reach one-half the carrying capacity? $t = ?$

$P(t) = \frac{1}{2}(1000) = 500 = \dfrac{1000}{1 + 32.33 e^{-0.439t}}$

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$t = \dfrac{\ln\left(\frac{500}{16165}\right)}{-0.439}$ $\boxed{\approx 7.92 \text{ hours}}$