Fundamental Skills Module
MAT 1330 Fall 2010

September 8, 2010

1. You have 90 minutes to complete this test.
2. This is a multiple choice test. You have to record your answers on the attached optical response sheet.
3. For each question there is one and only one correct answer.
4. Only the TI 30 and equivalent calculators are allowed.
5. No books, phones, music players or other data storage devices are allowed.
6. There are a total of 20 questions, each is worth 1/2 point for a total of 10.
7. You HAVE TO mark your answers on the OPTICAL RESPONSE SHEET. Circling the answers on this booklet of questions will NOT give you any points.

Question 1:
Simplify the following expression

$$\frac{1-x^3}{1-x}$$

Answer:
A: x^2; B: $1+x^2$; C: $1+x+x^2$; D: $1-x+x^2$; E: $1-x-x^2$

Question 2:
Simplify the expression

$$\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}+\sqrt{5}}$$

Answer:
A: $\sqrt{15}+4$; B: $\sqrt{15}-4$; C: $4-\sqrt{15}$; D: $4\sqrt{15}$; E: $4+\sqrt{15}$;

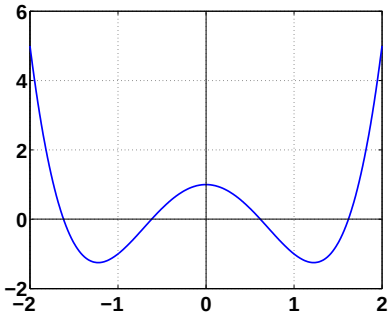
Question 3:
Simplify the following expression

$$\frac{1}{\frac{1}{x}+\frac{1}{x-1}}$$

Answer:

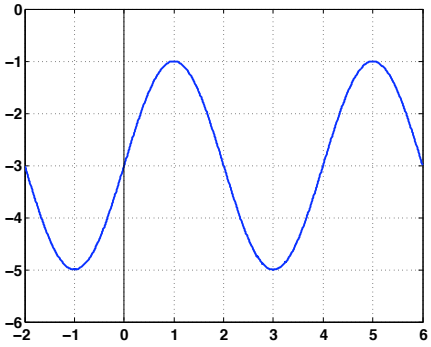
A: $\frac{2x-1}{x^2-x}$; B: $\frac{x^2-x}{2x-1}$; C: $\frac{x-1}{2x-1}$; D: $2x-1$; E: $\frac{1}{2x-1}$

Question 4:
Which of the following functions could have this graph?



Answer:
A: x^2+1; B: $-x^4+3x^2-1$; C: x^4-3x^2+1; D: x^4-3x^3+1; E: x^3-2x+1

Question 5:
Which of the following equations corresponds to the graph?



Answer:
A: $-2+4\cos\left(\frac{2\pi}{1}(x-3)\right)$; B: $-5+3\cos\left(\frac{2\pi}{2}(x-1)\right)$; C: $-1+2\cos\left(\frac{2\pi}{3}(x-4)\right)$;
D: $-3+4\cos\left(\frac{2\pi}{2}(x-1)\right)$; E: $-3+2\cos\left(\frac{2\pi}{4}(x-1)\right)$

Question 6:
Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = f(-x)$ for all x in $\mathbb{R}$. Which of the following is true for the function $g(x) = f(x + 1)$?

Answer:
A: g is an even function.
B: $g(x - 1) = g(1 - x)$ for all x in $\mathbb{R}$.
C: The graph of g is the graph of f, shifted by 1 to the right.
D: The graph of g is the graph of f, shifted by 1 to the left.
E: Such a function g does not exist.

Question 7: Find the equation of the line with slope 2 through the point $(1, 1)$.

Answer:
A: $y = x + 1$; B: $y = 2x + 1$; C: $y = 2x - 2$; D: $y = -x - 1$; E: $y = 2x - 1$

Question 8:
Let $g(x) = \frac{1}{\log(x)-1}$. Find the domain of g.

Answer:
A: all real numbers.
B: all positive real numbers.
C: all positive real numbers except for $x = 10$.
D: all real numbers except for $x = 10$.
E: all real numbers except for $x = 0$ and $x = 10$.

Question 9:
Which of the following is **not true** for positive real numbers a, b.
Answer:
A: $\log(a^x b^y) = x \log a + y \log b$; B: $\left(\frac{a}{b}\right)^x = a^x b^{-x}$; C: $a^b = 10^{b \log a}$;
D: $\log(a^x + b^x) = x \log(a + b)$; E: $\sqrt{a^x} = a^{x/2}$

Question 10:
The expression $3 \ln(x) + 5 \ln(y) - \ln(z)$ is equal to
Answer:
A: $\ln(-x^3 y^5 z)$; B: $\ln(3x + 5y - z)$; C: $\ln(x^3 y^5 z^{-1})$;
D: $\ln(3x + 5y) \cdot \frac{1}{z}$; E: $-\ln(3x \cdot 5y \cdot z)$

Question 11:
Find all values of k for which the following equation has only one solution:

$$x^2 + 2kx + 9k - 8 = 0$$

Answer:
A: $k = 1$ and 2; B: $k = 9$ and 8; C: $k = 3$; D: $k = 8$ and 1; E: $k = 8$

Question 12:
Find the solution of the equation

$$x - a = \frac{1}{x}$$

Answer:
A: $\pm\sqrt{1-a}$; B: $\pm\sqrt{a}$; C: $\frac{1}{2}(a \pm \sqrt{a^2+4})$; D: $\frac{1}{2}(a \pm \sqrt{a^2-4})$; E $\pm\sqrt{1+a}$:

Question 13:
Consider the equation $3x(1-x) = hx$, where h is any real number. Which of the following is true:

Answer:
A: There are two distinct solutions for all values of h.
B: There is a choice of h such that the equation has no solution.
C: This equation has no solution for any value of h.
D: This equation has always at least one positive solution.
E: There is a choice of h such that the equation has exactly one solution.

Question 14:
Which of the following has precisely two real solutions?

Answer:
A: $|x^2+5| = 5$; B: $|x^2-5| = 5$; C: $|x^2+5| = 1$; D: $|x^2+5| = 10$; E: $|x^2-5| = 1$

Question 15:
The inequality

$$|16 - x| < 9$$

is true for all x that satisfy

Answer:
A: $7 < x < 25$; B: $x < 7$ or $x > 25$; C: $x > 25$;
D: $x < 7$; E: none of the above

Question 16:
Simplify the following expression:

$$\left(\frac{x^{3/4}y^3}{x^{-1/4}y}\right)^2 \left(\frac{xy}{\sqrt[3]{y}}\right)$$

Answer:
A: $x^3y^{7/5}$; B: $\frac{x}{y}$; C: $x^2y^{14/3}$; D: x^3y^3; E: $x^3y^{14/3}$

Question 17:
Find all x for which the following inequality is true

$$\frac{1}{x-1} > \frac{1}{5}$$

Answer:
A: all real numbers except 1; B: $1 < x < 6$; C: $x < 1$ or $x > 6$;
D: $x < 1$; E: $x > 6$

Question 18:
Determine the amplitude and the period for the function $f(x) = 2\pi \sin(4x)$.

Answer:
A: Amplitude 4, Period π;
B: Amplitude 2π, Period π;
C: Amplitude 2π, Period 1;
D: Amplitude 2, Period 2π;
E: Amplitude 2π, Period $1/4$.

Question 19:
For which values of x in the interval $(\frac{3\pi}{2}, 2\pi)$ do we have $\sin(x) > \cos(x)$?

Answer:
A: $(\frac{7\pi}{4}, 2\pi)$; B: $(\frac{3\pi}{2}, 2\pi)$; C: $(\frac{3\pi}{2}, \frac{7\pi}{4})$; D: $(\frac{7\pi}{4}, \frac{15\pi}{8})$; E: none of the above;

Question 20:
What is the range of the function $f(x) = (2x-1)^4 - 2$?

Answer:
A: $[0, \infty)$; B: $[2, \infty)$; C: $[-2, \infty)$; D: $[-\infty, 2)$; E: $[-\infty, -2)$;

Additional practice problems – harder

Question 1:
Find $f(a+3)$ when the function f is given by

$$f(x) = \frac{x^2-2}{x+1}$$

Answer:
A: $\frac{a^2+6a+11}{a+4}$; B: $\frac{a^2-2}{a+1}+3$; C: $\frac{a^2+6a+9}{a+1}$; D: $\frac{a^2+6a+9}{a+4}$; E: $\frac{a^2+6a+7}{a+4}$

Question 2:
Find all solutions of

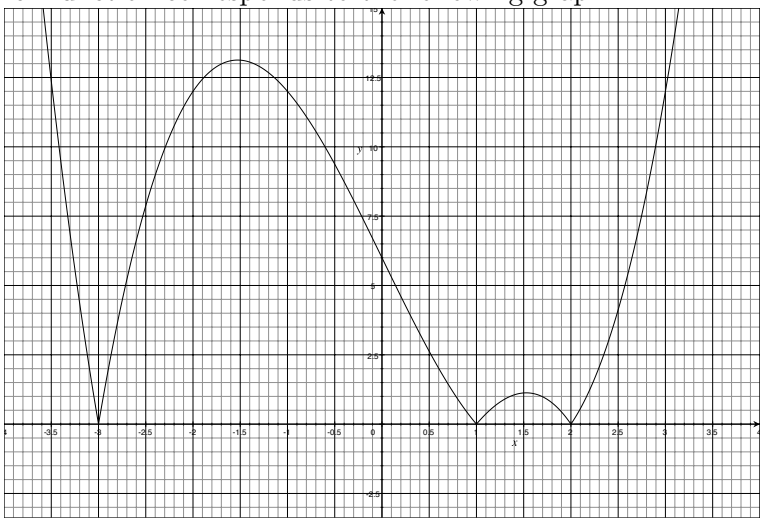
$$\log(x+2)+\log(x+3)=\log(2)$$

Answer:
A: $-3/2$; B: $3/2$; C: 1; D: $-1,-4$; E: -1

Question 3:
In the following "proof" that 1=2, which is the first line that is not true?

$$\begin{aligned}
\text{Suppose} \quad & a = b \qquad (a, b \neq 0) \\
\text{Multiply both sides by } b\text{:} \quad & ab = b^2 && (A) \\
\text{Subtract } a^2 \text{ from both sides:} \quad & ab - a^2 = b^2 - a^2 && (B) \\
\text{Factor:} \quad & a(b-a) = (b+a)(b-a) && (C) \\
\text{Cancel } b-a\text{:} \quad & a = a+b && (D) \\
\text{But, since } a = b, \quad & a = 2a && (E) \\
\text{Therefore,} \quad & 1 = 2
\end{aligned}$$

Question 4:
Which function corresponds to the following graph?



Answer:
A: $y = |(x-1)(x-2)(x+3)|$; B: $y = |(x-1)(x-2)(x-3)|$;
C: $y = |(x-1)(x+2)(x+3)|$; D: $y = |(x+1)(x+2)(x-3)|$;
E: $y = |(x+1)(x-2)(x+3)|$

Question 5:
Suppose $\tan\theta = -\dfrac{5}{12}$ and that the terminal point for θ is in quadrant II. What are $\cos\theta$ and $\sin\theta$?

Answer:
A: $\cos\theta = \dfrac{12}{13}$ and $\sin\theta = \dfrac{5}{13}$

B: $\cos\theta = \dfrac{12}{13}$ and $\sin\theta = -\dfrac{5}{13}$

C: $\cos\theta = -\dfrac{5}{13}$ and $\sin\theta = \dfrac{12}{13}$

D: $\cos\theta = -\dfrac{12}{13}$ and $\sin\theta = \dfrac{5}{13}$

E: $\cos\theta = -\dfrac{13}{12}$ and $\sin\theta = \dfrac{13}{5}$

Question 6:
Let $t \in \mathbb{R}$. Which of the following statements are true?

1. t and $t + 61\pi$ have the same terminal point.
2. The point $P\left(\frac{\sqrt{7}}{4}, -\frac{3}{4}\right)$ is on the unit circle.
3. The terminal point of $\frac{3\pi}{2}$ is $P(0, -1)$.
4. If $a > 0$, then $\log_a(e) + \log_a\left(\frac{1}{e}\right) = 0$
5. There is a t for which $\sin(t) = 2010$.
6. $\sin(t) = 0$ if $t = k\pi$ with $k \in \mathbb{Z}$.

Answer:

A: (a), (b), (c), (d) and (f).

B: (b), (c), (d) and (f).

C: (b) and (c).

D: (b), (c) and (e).

E: (c), (d) and (f).

Question 7:
Which of the following statements are true?

1. $\{1, 2, 3\} \cap \{2, 3, 4, 5\} = \{2, 3\}$
2. $\dfrac{88}{101} \in \mathbb{Q}$
3. $\sqrt{3} \notin \mathbb{R}$
4. $(-\infty, 3] \cap (-1, 7) = (-1, 3]$

Answer:
A: (1); B: (2) and (4); C: (1) and (3); D: (1), (2) and (3); E: (1), (2) and (4).

Question 8:
Which of the following is the expression $\tan(\sin^{-1}(x))$ equal to?

Answer:

A: $\sqrt{1 - x^2}$

B: x

C: $\dfrac{x}{\sqrt{1 - x^2}}$

D: $\dfrac{\sqrt{1 - x^2}}{x}$

E: The expression is undefined.

Question 9: Find the solutions of the equation $\tan(3x) = 1$ in the interval $[0, \pi)$.

Answer:

A: $\dfrac{3\pi}{4}$

B: This equation has no solutions.

C: $\dfrac{\pi}{12}$

D: $\dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{9\pi}{12}, \dfrac{13\pi}{12}$

E: $\dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{9\pi}{12}$

Question 10
The inequality

$$|16 - x^2| < 9$$

is true for all x that satisfy

Answer:
A: $5 < |x| < \sqrt{7}$; B: $\sqrt{7} < |x| < 5$; C: $x > \sqrt{7}$;
D: $x > 5$ or $x < -5$; E: $x > \sqrt{7}$ or $x < -\sqrt{7}$

Question 11:
The inequality

$$|12 - x^2| > 3$$

is true for all x that satisfy
Answer:
A: $|x| < 3$ or $|x| > \sqrt{15}$; B: $|x| < 3$ or $|x| > 15$; C: $|x| > 3$ or $|x| < \sqrt{15}$;
D: $|x| > 3$ or $|x| < 15$; E: no x will satisfy the inequality;

Question 12:
Find all x for which the following inequality is true

$$\frac{1}{x-3} > \frac{1}{x+2}$$

Answer:
A: all real numbers; B: $-2 < x < 3$; C: all real numbers except 3, -2;
D: $x < -2, x > 3$; E: no solution