INDU471/4X- Stochastic Models in Industrial Engineering

Brief Solutions to Assignment Questions (Assignments #1-#5)

Assignment #1

Exercises 1

Problem 6,
An example partition will be A={(a),(b,c),(d,e)}

Problem 14
(I) (a) [5, 100]; (b) [10,50]; (c) [0,25); (d) [10,100]; (e) empty; (f) [5,10); (g) [10,100]
(h) [0,100], (i) empty, (j) {[5,10),(90,100]}

(II) One solution could be {(0, 0.5], (0.5, 0.75), [0.75, 1)}

Exercises 2

Problem 14

Let R and N represent regular and two-headed pennies, respectively. Let T and H represent tail and head, respectively. We have:

$$\Pr[N \mid H] = \frac{\Pr[N \cap H]}{\Pr[H]} = \frac{\Pr[N]}{\Pr[H]} = \frac{1/3}{2/3} = \frac{1}{2}$$

Problem 16

Use Bayes' formula to determine that

$$\Pr[A \mid B] = \frac{\Pr[B \mid A]\Pr[A]}{\Pr[B \mid A]\Pr[A] + \Pr[B \mid \overline{A}]\Pr[\overline{A}]} = 0.99$$

$$\Pr[\overline{A} \mid B] = 1 - \Pr[A \mid B] = 0.01$$

$$\Pr[A \mid \overline{B}] = \frac{\Pr[\overline{B} \mid A]\Pr[A]}{\Pr[\overline{B} \mid A]\Pr[A] + \Pr[\overline{B} \mid \overline{A}]\Pr[\overline{A}]} = 0.01$$

$$\Pr[\overline{A} \mid \overline{B}] = 1 - \Pr[A \mid \overline{B}] = 0.99$$

Problem 17

(a) $\Pr[T \mid O] = \dfrac{0.01p}{0.01p + 0.99(1-q)}$, (b) $\Pr[T \mid \overline{O}] = \dfrac{0.01(1-p)}{0.01(1-p) + 0.99q}$
(c) $\Pr[T \cap O] = 0.01p$, (d) $\Pr[T \cap \overline{O}] = 0.01(1-p)$

We want

$\Pr[T \mid O] = \dfrac{0.01p}{0.01p + 0.99(1-q)} \geq 0.99$, that is: $\dfrac{p}{p + 99(1-q)} \geq 0.99$ or

$p \geq (99)^2(1-q)$. Since $1 \geq p \geq (99)^2(1-q)$, we need: $1 \geq (99)^2(1-q)$ or $q \geq 1 - \dfrac{1}{(99)^2}$.

We also want

$\dfrac{1-p}{1-p+99q} \leq 0.02$ or $1-p \leq 0.02 - 0.02p + 1.98q$ or $p \geq -1.02q$ this is always the case.

In the end, we need: $p \geq (99)^2(1-q)$ or $1 - \dfrac{p}{(99)^2} \leq q \leq 1$ for $0 \leq p \leq 1$

Assignment #2

Exercises 3

Problem 2,

(a) $\Pr[X > 1] = 1 - \Pr[X \leq 1] = e^{-\lambda}$

(b) $\Pr[2 \leq X \leq 3] = e^{-2\lambda} - e^{-3\lambda}$

(c) same as in (b)

(d) $\Pr[X > y] = e^{-\lambda y}, y > 0$

Problem 3

Since $C \ln x = 1$ at $x = 2$, C=1.44.

$\underset{x\to-\infty}{Lim} F(x) = 0$, $\underset{x\to+\infty}{Lim} F(x) = 1$

$F(x)$ is non-decreasing.

Problem 4

(a) $\Pr[X \leq 30] = 1 - e^{-2} = 0.8647$

(b) $\Pr[X \leq 30] = 1 - e^{-0.2} = 0.1813$

(c) $\Pr[3 < X \leq 30] = 0.6834$

(d) same as (a).

Problem 12

(a) $\Pr[X \geq 1] = 1 - \Pr[X = 0] = 1 - e^{-\lambda}$

(b) $\Pr[X < 2] = \Pr[X = 0] + \Pr[X = 1] = e^{-\lambda} + \lambda e^{-\lambda}$

(c) $\Pr[2 \leq X \leq 4] = \dfrac{\lambda^2 e^{-\lambda}}{2!} + \dfrac{\lambda^3 e^{-\lambda}}{3!} + \dfrac{\lambda^4 e^{-\lambda}}{4!}$

Assignment #3

Exercises 4, Problem 25

(a) $\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c} R_{ij} = 174.9$. Using the table with DOF=6 and 95% level of significance, the two variables are not independent.

(b) $\Pr[X = 1] = \frac{86.1}{282.4} = 0.305$, $\Pr[X = 2] = \frac{158.1}{282.4} = 0.560$, $\Pr[X = 3] = 0.135$

(c) $\Pr[Y = 1] = \frac{66.7}{282.4} = 0.236$, $\Pr[Y = 2] = 0.266$, $\Pr[Y = 3] = 0.069$, $\Pr[Y = 4] = 0.429$

(d) Let $p_{ij} = \Pr[Y = j \mid X = i]$,

$p_{11} = \frac{34.6}{86.1} = 0.402$, $p_{21} = 0.063$, $p_{31} = 0.002$, $p_{41} = 0.533$

$p_{12} = \frac{29.0}{158.1} = 0.183$, $p_{22} = 0.397$, $p_{32} = 0.007$, $p_{42} = 0.412$

$p_{13} = \frac{3.1}{38.2} = 0.081$, $p_{23} = 0.181$, $p_{33} = 0.476$, $p_{43} = 0.262$

Exercises 7, Problem 18

(a) Using Poisson approximation:

$$\Pr[X \le 3] = \sum_{i=0}^{3} \Pr[X = i] \approx \sum_{i=0}^{3} \frac{e^{-2}2^i}{i!} = 0.1353 + 0.2707 + 0.2707 + 0.1804 = 0.8571$$

The exact probability from binomial distribution is 0.8552.

(b) Using normal approximation:

$$\Pr[X \le 30] \approx \Pr\left(Z \le \frac{30-25}{4.33}\right) = 0.876$$

Exercises 7, Problem 21

We can find the mean for each of the random variable $X_i, i = 1,\ldots,12$ to be

$\bar{x}_i = \frac{1+2+3+4+5+6}{6} = 3.5, i = 1,\ldots,12$. Similarly, we can calculate the variance for each of these variables:

$$s_i^2 = \frac{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)} = \frac{6\sum_{i=1}^{6} i^2 - \left(\sum_{i=1}^{6} i\right)^2}{6(5)} = \frac{105}{30} = 3.5$$

For $Y = X_1 + X_2 + \ldots + X_{12}$, we have

$E[Y] = E[X_1] + E[X_2] + \ldots + E[X_{12}] = 12 \times 3.5 = 42$

$V[Y] = V[X_1] + V[X_2] + \ldots + V[X_{12}] = 12 \times 3.5 = 42$, $s_Y = \sqrt{V(Y)} = \sqrt{42} = 6.48$

$$\Pr[25 \le X \le 40] \approx \Pr\left(\frac{25-42}{6.48} \le Z \le \frac{40-42}{6.48}\right) = \Pr(-2.62 \le Z \le -0.31) = 0.9956 - 0.6217 = 0.3739$$

Assignment #4

Exercises 9

Problems 1, 2, 7 Plots

Problem 8

Since $\Pr[Z_3 = 1 \mid Z_2 = 0, Z_1 = 1] = \Pr[X_3 = 1 \mid X_2 = 1, X_1 = 1] = 0$ and
$\Pr[Z_3 = 1 \mid Z_2 = 0, Z_1 = -1] = \Pr[X_3 = 1 \mid X_2 = 1, X_1 = -1] = 0.5$
The process is not Markov.

Problem 11
(a)
$S_1 = \{1,0,-1,0,1,0,-1,\ldots\}$
$S_2 = \{-1,0,1,0,-1,0,1,\ldots\}$
(b) It's not Markov.

Assignment #5

Exercises 10

Problem 1.

It is Markov since each toss is independent. The P matrix is 6 by 6 with each entry being 1/6.

Problem 2.

(a) $\Pr[X_3 = 2 \mid X_0 = 1] = \Pr[X_3 = 2] = 1/6$
(b) $\Pr[X_3 = 2, X_2 = 3, X_1 = 6 \mid X_0 = 1] = \Pr[X_3 = 2, X_2 = 3, X_1 = 6] = (1/6)^3$
(c) $\Pr[X_{75} = 5 \mid X_0 = 1] = \Pr[X_{75} = 5] = 1/6$
(d) 1/6, (e) 1/6

Problem 3.

It is a Markov process. The transition probabilities are:

$$p_{ij} = \begin{cases} 1/6, & \text{for} \quad i+1 \le j \le i+6 \\ 0, & \text{otherwise} \end{cases}$$

Problem 6.

(a) $$\begin{aligned} p(0,1,1) &= \Pr[X_0 = 1, X_1 = 1, X_2 = 1] = \Pr[X_2 = 1 \mid X_1 = 1, X_0 = 0] \times \Pr[X_1 = 1, X_0 = 0] \\ &= \Pr[X_2 = 1 \mid X_1 = 1] \times \Pr[X_1 = 1 \mid X_0 = 0] \times \Pr[X_0 = 0] = 1/16 \end{aligned}$$

(b) $$\begin{aligned} \Pr[X_1 = 1, X_2 = 1 \mid X_0 = 0] &= \frac{\Pr[X_1 = 1, X_2 = 1, X_0 = 0]}{\Pr[X_0 = 0]} = \frac{\Pr[X_1 = 1, X_2 = 1, X_0 = 0]}{\Pr[X_1 = 1, X_0 = 0]} \frac{\Pr[X_1 = 1, X_0 = 0]}{\Pr[X_0 = 0]} \\ &= \Pr[X_2 = 1 \mid X_0 = 0, X_1 = 1] \times \Pr[X_1 = 1 \mid X_0 = 0] = \Pr[X_2 = 1 \mid X_1 = 1] \times \Pr[X_1 = 1 \mid X_0 = 0] = p_{11} p_{01} \end{aligned}$$

(c) $$p_{01}^{(2)} = \sum_{k=0}^{2} p_{0k} p_{k1} = \frac{1}{4}\frac{3}{4} + \frac{3}{4}\frac{1}{3} + 0\frac{1}{4} = 7/16$$

Problem 10.

$$P=\begin{bmatrix}0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{10} & \frac{7}{10} & \frac{1}{10} & \frac{1}{10}\\ 0 & 0 & 0 & 1\\ \frac{1}{4} & \frac{3}{4} & 0 & 0\end{bmatrix}$$

Problem 11.

(a) Pr[“Rest of the week is dry”| “Monday is dry”]= $\left(\frac{3}{4}\right)^6$

(b) Pr[“Tuesday, Thursday, Saturday wet, Wednesday, Friday, Sunday dry”| “Sunday dry”]

$$=\frac{1}{4}\times\frac{1}{2}\times\frac{1}{4}\times\frac{1}{2}\times\frac{1}{4}\times\frac{1}{2}=\left(\frac{1}{8}\right)^3=$$

Problem 12.

(a) Pr[“Wet tomorrow”| “Wet yesterday”]= $\frac{1}{2}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{2}=0.375$

Assignment #6

Exercises 10

Problems 15, 16, 17, 19, 20 (a), 21 (a)

Problem 15

$\mathbf{p}^{(1)}=(p,0,q,r,0)$, $\mathbf{p}^{(2)}=(pq+p+rp,0,q^2,2qr,r^2)$,

$\mathbf{p}^{(3)}=(pq+p+rp+q^2p+2pqr+r^2p,r^3,q^3,rq^2+2rq^2,2qr^2+r^2q)$

Problem 16

$$p^{(1)}=\left(\frac{3}{16}+\frac{1}{4},\frac{1}{16}+\frac{2}{4}\right)=\left(\frac{7}{16},\frac{9}{16}\right)$$
$$p^{(2)}=\left(\frac{21}{64}+\frac{9}{48},\frac{7}{64}+\frac{18}{48}\right)=\left(\frac{99}{192},\frac{93}{192}\right)$$
$$p^{(3)}=\left(\frac{1263}{2304},\frac{1041}{2304}\right)$$

Problem 17

$$f_{ij}^{(1)}=\begin{bmatrix}3/4 & 1/4\\ 1/3 & 2/3\end{bmatrix},\ f_{ij}^{(2)}=\begin{bmatrix}1/12 & 3/16\\ 2/9 & 1/12\end{bmatrix},$$

Problem 19

$f_{20}^{(1)} = p,\ f_{20}^{(2)} = pq + pr,\ f_{20}^{(3)} = pq^2 + pqr + pr^2,\ f_{20}^{(4)} = pq^3 + 2pq^2r + pqr^2,$
$f_{21}^{(1)} = 0,\ \ f_{21}^{(2)} = 0,\ \ f_{21}^{(3)} = r^3 \quad f_{21}^{(4)} = 3qr^3$

Problem 20 (a)
States 0 and 3 are clearly absorbing states, since once they are entered they are never left. States 1 and 2 are transient, since they may be left and never revisited.

Problem 21 (a)
The closed communicating classes are {0, 1} and {4}; the transient class is {2, 3}.

Assignment #7

Exercises 10

Problem 25.
(a) State 3 is an absorbing state and is a closed communicating class. All others are transient.
(b) (2,4) I sa closed communicating class.

Problem 26. $v_3 = 1$ and all other probabilities are 0.

Problem 28

$$\mathbf{P} = \begin{bmatrix} 0.3 & 0.4 & 0.3 & 0 & 0 & 0 & 0 \\ 0.3 & 0.4 & 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0.4 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0.4 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0.4 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0.3 & 0.4 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 0.3 & 0.7 \end{bmatrix}$$

Solving

$$[v_0, v_1, v_2, v_3, v_4, v_5, v_6] = [v_0, v_1, v_2, v_3, v_4, v_5, v_6] \begin{bmatrix} 0.3 & 0.4 & 0.3 & 0 & 0 & 0 & 0 \\ 0.3 & 0.4 & 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0.4 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0.4 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0.4 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0.3 & 0.4 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 0.3 & 0.7 \end{bmatrix}$$

gives: $[v_0, v_1, v_2, v_3, v_4, v_5, v_6] = [\frac{1}{20}, \frac{7}{60}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}]$

Problem 35. The particular state may be visited at any time period. As it communicates with all other states, then none of the other states can be periodic. So the chain is aperiodic.

Problem 38. The particular state does not communicate with other states then by definition it is a closed communicating class with one state.
Optional Problems

Exercises 1

Problem 11
Similar to that shown in Figure 2.3, Page 31.

Problem 12
$\overline{E}_1$: at least one is not working
$\overline{E}_2$: both are working
$E_2 = \overline{E}_1$

Exercises 2

Problem 18

$$\Pr[H_1 \mid A] = \frac{\Pr[A \mid H_1]\Pr[H_1]}{\Pr[A \mid H_1]\Pr[H_1] + \Pr[A \mid H_2]\Pr[H_2] + \Pr[A \mid H_3]\Pr[H_3]} = \frac{.75 \times .4}{.75 \times .4 + .80 \times .5 + .85 \times .1} = 0.382$$

$\Pr[H_2 \mid A] = .509 \quad \Pr[H_3 \mid A] = .108$

Exercises 3

Problem 6

(a) 0.2894, (b) 0, (c) 0.3589, (d) 0.2894, (e) 0.6483, (f) 0.0087

Problem 15

$c = \left(\frac{2}{b-a}\right)^2$ (a) 0.5, (b) 0.5556, (c) 0.2222 , (d) $F(x) = \begin{cases} 0 & x < 1 \\ x^2/2 - x & 1 \le x \le 2 \\ 3x - x^2/2 & 2 \le x \le 3 \\ 1 & x > 3 \end{cases}$

Exercises 4

Problem 1

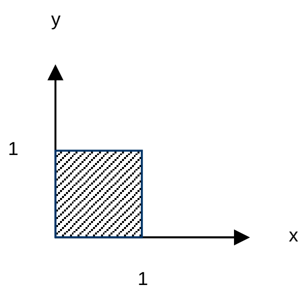
(a) 1/6
(b)

x	$F_X(x)$
0	2/3
1	1

y	$F_Y(y)$
0	1/6
1	7/12
2	1

(c) 1/6
(d) 1/3

Problem 2
(a)



(c) $F_X(x)=\begin{cases}0, & x\le 0\\ x, & 0<x<1\\ 1, & x\ge 1\end{cases}$ $F_Y(y)=\begin{cases}0, & y\le 0\\ y, & 0<y<1\\ 1, & y\ge 1\end{cases}$

(d)
(i) $\Pr[X>0.5 \;\; and \;\; Y>0.5]=F(1,1)-F(0.5,1)-F(1,0.5)+F(0.5,0.5)=1-0.5-0.5+0.25=0.25$
(ii) $\Pr[X>0.5 \;\; and \;\; Y<2]=\Pr[X>0.5]=1-F_X(0.5)=0.5$
(ii) $\Pr[X=0.5 \;\; and \;\; Y=0.5]=0$

Problem 22

P(1,1)=0.388, P(2,2)=0.051, P(3,3)=0.156
P(2,1)=0.388, P(2,2)=0.018, P(3,3)=0.024
P(3,1)=0.388, P(2,2)=0.023, P(3,3)=0.135

Problem 23

$\chi^2=43.098$. It does not support that the variables are independent.

Exercises 7

Problem 19
With p=0.081, the required probability is 0.7788

Problem 20
With mean = 81, s.d.=8.83, the required probability is 0.1011.

Exercises 9

Problem 9

(a) Markovian

$$\Pr[X_{n+1} = j \mid X_n = i] = \begin{cases} \frac{i}{100-n}, & j = i-1, i > 0 \\ 1 - \frac{i}{100-n}, & j = i \\ 0, & otherwise \end{cases}$$

$\Pr[X_{n+1} = 0 \mid X_n = 0] = 1$

(b) $\Pr[X_{10} = 47 \mid X_9 = 18] = 48/91$

(c) $\Pr[X_{20} = 4 \mid X_{19} = 48] = 0$

(d) shown in (a)

(e) No.

Problem 10

(a) Yes.

(b) $\Pr[X_{10} = 47 \mid X_9 = 18] = 48/200$

(c) $\Pr[X_{20} = 4 \mid X_{19} = 48] =$

(d)

$$\Pr[X_{n+1} = j \mid X_n = i] = \begin{cases} \frac{i}{100}, & j = i-1, i > 0, \quad \Pr[X_{n+1} = 0 \mid X_n = 0] = \frac{1}{2} \\ \frac{1}{2}, & j = i \quad i > 0, \quad \Pr[X_{n+1} = 0 \mid X_n = 0] = \frac{1}{2} \\ \frac{100-i}{200}, & j = i+1, i > 0 \end{cases}$$

for other $i, j, \Pr[X_{n+1} = j \mid X_n = i] = 0$

(e) Yes, it is stationary.

Exercises 10

7. (a) $\Pr[X_0=0, X_1=1, X_2=1] = \Pr[X_1=1 \mid X_0=0]\cdot \Pr[X_2=1 \mid X_0=0, X_1=1]\cdot \Pr[X_0=0]$

$$= \Pr[X_2=1 \mid X_1=1]\cdot \Pr[X_1=1 \mid X_0=0]\cdot \Pr[X_0=0] = \tfrac{1}{3}\cdot\tfrac{1}{2}\cdot a$$

$$= \left(\tfrac{1}{6}\right) a$$

(b) $\Pr[X_0=0] = a$, $\Pr[X_0=1] = b$

$\Pr[X_1=0] = \frac{2}{3}a + \frac{1}{2}b$ $\quad \Pr[X_1=1] = \frac{1}{3}a + \frac{1}{2}b$

$\Pr[X_2=0] = \frac{11}{18}a + \frac{7}{12}b$ $\quad \Pr[X_2=1] = \frac{7}{18}a + \frac{5}{12}b$

$\Pr[X_3=0] = \frac{65}{108}a + \frac{43}{72}b$ $\quad \Pr[X_3=1] = \frac{43}{108}a + \frac{29}{72}b$

From (a) above, $\Pr[X_n=0, X_{n+1}=1, X_{n+2}=1] = \left(\frac{1}{6}\right)\Pr[X_n=0]$

$n=1$: $\Pr[X_1=0, X_2=1, X_3=1] = \left(\frac{1}{6}\right)\left(\frac{2}{3}a+\frac{1}{2}b\right) = \frac{1}{9}a + \frac{1}{12}b$

$n=2$: $\Pr[X_2=0, X_3=1, X_4=1] = \left(\frac{1}{6}\right)\left(\frac{11}{18}a+\frac{7}{12}b\right) = \frac{11}{108}a + \frac{7}{72}b$

$n=3$: $\Pr[X_3=0, X_4=1, X_5=1] = \left(\frac{1}{6}\right)\left(\frac{65}{108}a+\frac{43}{72}b\right) = \frac{65}{648}a + \frac{43}{432}b$

(c) $\Pr[X_n=0, X_{n+2}=1] = \Pr[X_n=0, X_{n+1}=0, X_{n+2}=1] + \Pr[X_n=0, X_{n+1}=1, X_{n+2}=1]$

$$= \tfrac{2}{9}\Pr[X_n=0] + \tfrac{1}{6}\Pr[X_n=0]$$

$$= \tfrac{7}{18}\Pr[X_n=0]$$

From (b), we have $\Pr[X_n=0]$.

$n=1$: $\Pr[X_1=0, X_3=1] = \frac{7}{18}\left(\frac{2}{3}a+\frac{1}{2}b\right)$

$n=2$: $\Pr[X_2=0, X_4=1] = \frac{7}{18}\left(\frac{11}{18}a+\frac{7}{12}b\right)$

$n=3$: $\Pr[X_3=0, X_5=1] = \frac{7}{18}\left(\frac{65}{108}a+\frac{43}{72}b\right)$

(d) $\Pr[X_{n+2}=1] = \Pr[X_n=0, X_{n+2}=1] + \Pr[X_n=1, X_{n+2}=1]$

$$= \tfrac{7}{18}\Pr[X_n=0] + \tfrac{5}{12}\Pr[X_n=1]$$

$n=1$: $\Pr[X_3=1] = \frac{43}{108}a + \frac{29}{72}b$

$n=2$: $\Pr[X_4=1] = \frac{259}{648}a + \frac{173}{432}b$

$n=3$: $\Pr[X_5=1] = \frac{1555a}{3888} + \frac{1337}{2592}b$

(e) $\Pr[X_{n+2}=1 \mid X_n=0] = \dfrac{\Pr[X_{n+2}=1, X_n=0]}{\Pr[X_n=0]}$ (since $\Pr[X_n=0]\neq 0$)

From (c), $\Pr[X_{n+2}=1, X_n=0] = \frac{7}{18}\Pr[X_n=0]$

$\Rightarrow \Pr[X_{n+2}=1 \mid X_n=0] = \frac{7}{18}$

(f) Only (a) and (e) do not depend on n; the others do.

8. From Tables 9.2 and 9.3,

(a) $Pr[X=1] = Pr[X_1=1 | X_0=1] = 0.65$ (since $Pr[X_0=1]=1$)

This is the probability that the first failure was in system 1.

(b) $Pr[X_2=1 | X_0=1] = 0.602$

This is the probability that the second failure was in system 1 given the starting failure was also in system 1.

(c) $Pr[X_2=1] = Pr[X_2=1 | X_0=1] = 0.602$

This is the probability that the second failure was in system 1.

(d) $Pr[X_2=1, X_1=1 | X_0=1] = Pr[X_2=1 | X_1=1] \cdot Pr[X_1=1 | X_0=1]$

$= (0.65)^2 = 0.4275$

This is the probability that the first two failures were in system 1 given the starting failure was also in system 1.

(e) $Pr[X_3=1 | X_2=1, X_1=1, X_0=1] = Pr[X_3=1 | X_2=1] = 0.65$

This is the probability that the third failure was in system 1 given all the previous failures were also in system 1.