Power Series for ODE

introduction

Interesting theorem on series

- Theorem 1: Wierestrass Criteria

Let us look at a series of function $\sum_{n=1}^{\infty} f_n(x)$. If there exist a convergent serie such as all the term of that convergent serie are greater or equal to all the function of the initial serie, for all n and all x in the domain, then $\sum_{n=1}^{\infty} f_n(x)$ is absolutely and uniformly convergent

- Theorem 2: derivative

on the board…

Power series

- Definition:

  A power series is a series that can be written in the form: $y \equiv \sum_{n=0}^{\infty} a_n (x - x_0)^n$
- Use to **represent** functions
- Interval of convergence…
- d'Alembert criteria (on the board)
- Ratio criteria

  Let us study the power series $y \equiv \sum_{n=0}^{\infty} a_n x^n$. The radius of convergence is given by:

$$R = \lim_{n \to \infty} \left| \frac{a_n}{a_{n+1}} \right|$$

Provided that the limit exist.

- Theorem on calculus using power series:

Let the power series $f(x) \equiv \sum_{n=0}^{\infty} a_n x^n$ have a radius of convergence R not zero. Then, f is differentiable on (-R,R) and, for x in the interval,

$$f'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

- Example on adding power series (on the board)

Solution of ODE

- Using the standard form of the second order ODE, a point is said to be ORDINARY if the P(x) and Q(x) coefficient are analytical at that point
- If not, a point is said to be singular.
- Theorem:

let x_0 be an ordinary point of a second order ODE. We can always find 2 linearly independent solution of that ODE in the form of a power series centered at that ordinary point. That solution converge on ± R

- Example Airy equation (simplest second order with turning point, …)