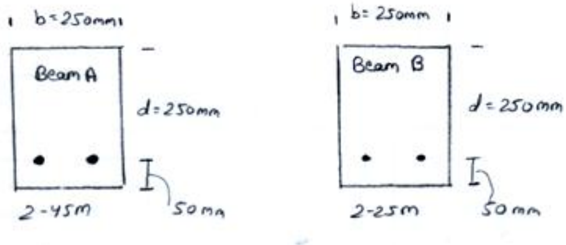
CIVE 3206 TA: Bishoy Alty

Assignment 4

Q.1 $f'_c = 30 MPa$ $\varepsilon'_c = -0.002$, $f_y = 400 MPa$



For Beam A

$f_r = 0.6\sqrt{f'_c} = 0.6\sqrt{30} = 3.29 MPa$

$A_s = 2(1500) = 3000 mm^2$

$E_c = 5500\sqrt{f'_c} = 30124.74 MPa$

Analysis Procedure

→ Select ε_{cc}
→ Guess c
→ Compute α_1, β_1
→ Check $T_s = C_c$ or $N = 0$
→ If $T_s \neq C_c$ or $N \neq 0$ guess another c
→ Compute $\phi = \varepsilon_{cc}/c$
→ Find $M = T_s\left(d - \frac{\beta_1 c}{2}\right)$

Cracking moment

$$I_g = \frac{bh^3}{12} = \frac{250(300)^3}{12} = 5.63\times10^8 mm^4$$

$$y_t = 300/2 = 150 mm$$

$$M_{cr} = \frac{f_r \times I_g}{y_t} = \frac{3.29 \times 5.63\times10^8}{150} = 12.3 kN.m$$

$$\varepsilon_{cr} = \frac{f_r}{E_c} = \frac{3.29}{30124.74} = 1.09\times10^{-4}$$

$$\phi_{cr} = \frac{\varepsilon_{cr}}{y_t} = \frac{1.09\times10^{-4}}{150} = 7.28\times10^{-7} = 0.728\times10^{-6}$$

Sample calculation

choose $\varepsilon_{cc} = -0.5 \times 10^{-3}$

Guess $c = 139.2$ mm mm

$$\beta_1 = \frac{4 - \varepsilon_{cc}/\varepsilon'_c}{6 - 2\varepsilon_{cc}/\varepsilon'_c} = \frac{4 - \left(\frac{-0.5\times10^{-3}}{-0.002}\right)}{6 - 2\left(\frac{-0.5\times10^{-3}}{-0.002}\right)} = 0.682$$

$$\alpha_1\beta_1 = \frac{\varepsilon_{cc}}{\varepsilon'_c} - \frac{1}{3}\left(\frac{\varepsilon_{cc}}{\varepsilon'_c}\right)^2 = 0.25 - \frac{1}{3}(0.25)^2 = 0.229$$

$$\alpha_1 = \frac{0.229}{0.682} = 0.336$$

$$C_c = \alpha_1 f'_c \beta_1 c b = 0.336\,(30)(0.682)(139.2)(250) = -239.2 \text{ kN}$$

$$T_s = \varepsilon_s E_s A_s = -\frac{\varepsilon_{cc}(d-c)}{c} \times E_s \times A_s$$

$$= \frac{0.0005\,(250 - 139.2)}{139.2} \times 200{,}000 \times 3000 = 239.2 \text{ kN}$$

$$N = T_s + C_c = 0$$

$$\phi = \varepsilon_{cc}/c = \frac{+0.0005}{139.2} = 3.59 \times 10^{-6}$$

$$M = T_s\left(d - \frac{\beta_1 c}{2}\right) = 239.2\left(250 - \frac{0.682\,(139.2)}{2}\right) = 48.38 \text{ kN.m}$$

$$\rho = \frac{A_s}{bd} = \frac{3000}{(250)(250)} = 0.048 > \rho_b = 0.0368$$

$\therefore$ Section is over-reinforced → Concrete crushes before yielding of steel

Beam B

$A_s = 2(500) = 1000\ mm^2$

$f_r = 3.29\ MPa$

$I_g = 5.63 \times 10^9\ mm^4$

$M_{cr} = 12.3\ kN\ m$

$\phi_{cr} = 0.728 \times 10^{-6}$

Same procedure as Beam A

$$\rho = \frac{A_s}{bd} = \frac{1000}{(250)(250)} = 0.016 < \rho_b$$

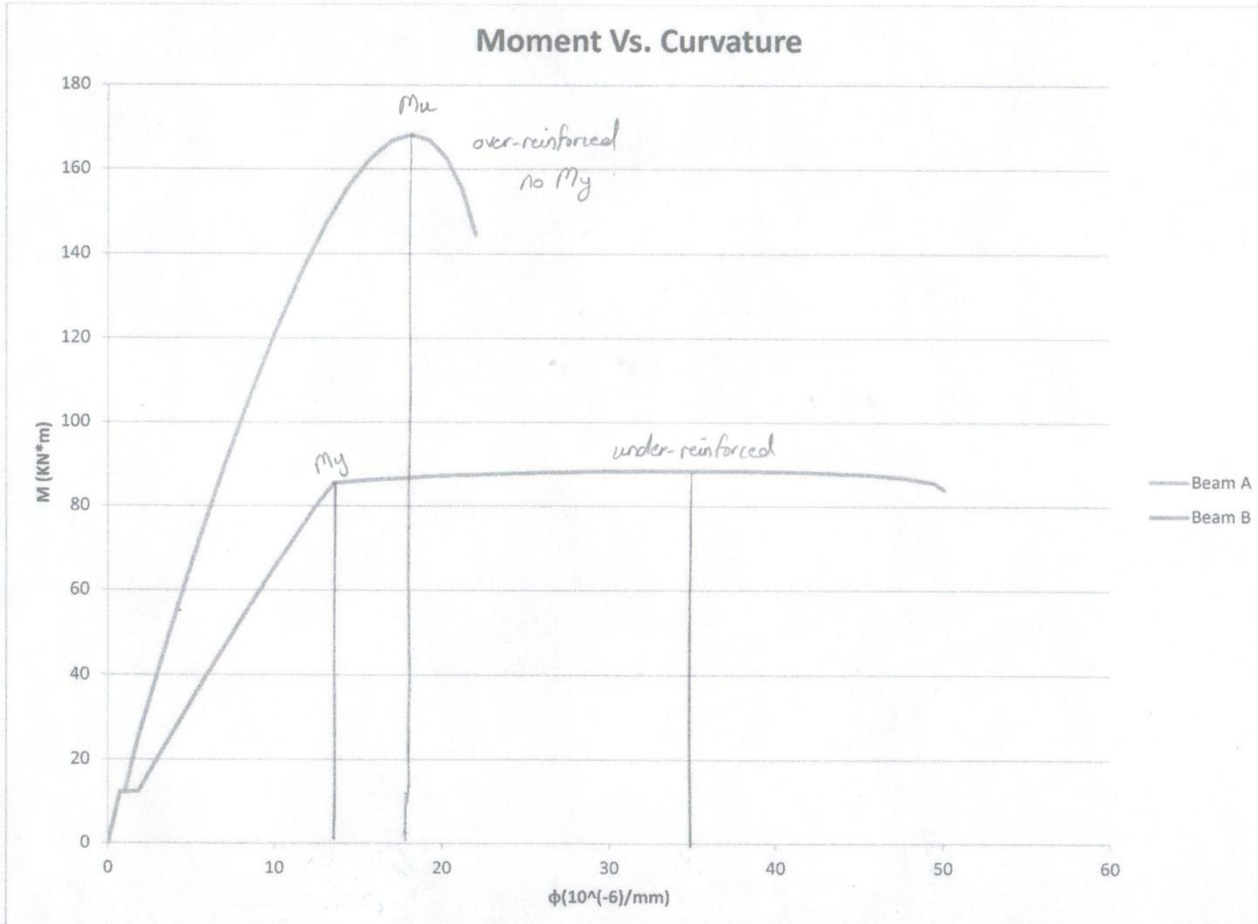
$\therefore$ Section is under-reinforced → steel yield before concrete crushes

There is a considerable amount of ductility in Beam B, the under-reinforced Beam. This is due to steel yielding.

Though Beam A has higher moment of resistance, it will fail suddenly due to its brittle nature.

εcc(*10^-3)	c(mm)	εs(*10^-3)	fs(Mpa)	β	α	Cc(KN)	Ts(KN)	N(KN)	ϕ((10^-6)/m)	M(KN*m)	Comments
									0	0	
									0.73	12.32	Just before cracking
-0.13	125	0.13	26.00	0.670	0.095	-60	60	0	1.04	12.42	Just after cracking
-0.25	137.223	0.21	41.09	0.674	0.178	-123	123	0	1.82	25.12	
-0.5	139.2	0.40	79.60	0.682	0.336	-239	239	0	3.59	48.37	
-0.75	141.15	0.58	115.67	0.690	0.475	-347	347	0	5.31	69.85	
-1	143.17	0.75	149.24	0.700	0.595	-447	448	0	6.98	89.49	
-1.25	145.4	0.90	179.85	0.711	0.696	-540	540	0	8.60	107.02	
-1.5	147.75	1.04	207.61	0.722	0.779	-623	623	0	10.15	122.48	
-1.75	150.15	1.16	232.75	0.735	0.843	-698	698	0	11.66	136.02	
-2	152.75	1.27	254.66	0.750	0.889	-764	764	0	13.09	147.24	
-2.25	155.5	1.37	273.47	0.767	0.917	-820	820	0	14.47	156.20	
-2.5	158.45	1.44	288.89	0.786	0.928	-867	867	0	15.78	162.72	
-2.75	161.6	1.50	300.87	0.808	0.922	-903	903	0	17.02	166.74	
-3	164.95	1.55	309.37	0.833	0.900	-928	928	0	18.19	168.24	Ultimate Moment
-3.25	168.6	1.57	313.82	0.864	0.862	-942	941	0	19.28	166.82	
-3.5	172.5	1.57	314.49	0.900	0.810	-943	943	0	20.29	162.63	
-3.75	176.75	1.55	310.82	0.944	0.744	-932	932	0	21.22	155.29	
-4	181.43	1.51	302.35	1.000	0.667	-907	907	0	22.05	144.48	

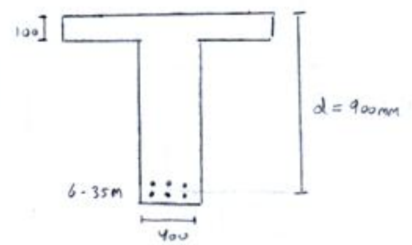
εcc(*10^-3)	c(mm)	εs(*10^-3)	fs(Mpa)	β	α	Cc(KN)	Ts(KN)	N(KN)	ϕ((10^-6)/m)	M(KN*m)	Comments
									0	0	
									0.73	12.32	Just before cracking
-0.169	92	0.29	58.05	0.671	0.122	-57	57	0	1.84	12.42	Just after cracking
-0.25	93.3	0.42	83.98	0.674	0.178	-84	84	0	2.68	18.35	
-0.5	95	0.82	163.16	0.682	0.336	-163	163	0	5.26	35.51	
-0.75	96.7	1.19	237.80	0.690	0.475	-238	238	0	7.76	51.51	
-1	98.5	1.54	307.61	0.700	0.595	-308	308	0	10.15	66.30	
-1.25	100.4	1.86	372.51	0.711	0.696	-373	373	0	12.45	79.84	
-1.37	101	2.02	400.22	0.716	0.738	-400	400	0	13.56	85.54	Steel Yields
-1.5	94.7	2.46	400.00	0.722	0.779	-400	400	0	15.84	86.32	
-1.75	86	3.34	400.00	0.735	0.843	-400	400	0	20.35	87.35	
-2	80	4.25	400.00	0.750	0.889	-400	400	0	25.00	88.00	
-2.25	75.8	5.17	400.00	0.767	0.917	-400	400	0	29.68	88.38	
-2.5	73.2	6.04	400.00	0.786	0.928	-400	400	0	34.15	88.50	Ultimate Moment
-2.75	71.6	6.85	400.00	0.808	0.922	-400	400	0	38.41	88.43	
-3	71.2	7.53	400.00	0.833	0.900	-401	400	0	42.13	88.13	
-3.25	71.6	8.10	400.00	0.864	0.862	-400	400	0	45.39	87.63	
-3.5	73.2	8.45	400.00	0.900	0.810	-400	400	0	47.81	86.82	
-3.75	75.8	8.62	400.00	0.944	0.744	-400	400	0	49.47	85.68	
-4	79.9	8.52	400.00	1.000	0.667	-400	400	0	50.06	84.02	


Moment Vs. Curvature
180
160
140
120
100
80
60
40
20
0
M (KN*m)
0
10
20
30
40
50
60
ϕ(10^(-6)/mm)
Beam A
Beam B
Mu
over-reinforced
no My
My
under-reinforced

Pg. 7

Q.2

20 points



$f'_c = 25\,MPa$
$f_y = 400\,MPa$
$b_f = 1000\,mm$
$h_f = 100\,mm$
$b_w = 400\,mm$

$\phi_s = 0.85$
$\phi_c = 0.65$

$A_s = 6000\,mm^2$

$d = 860 + 80/2 = 900\,mm$

$\alpha_1 = 0.85 - 0.0015f'_c = 0.85 - 0.0015(25) = 0.8125 > 0.67$

$\beta_1 = 0.97 - 0.0025f'_c = 0.97 - 0.0025(25) = 0.9075 > 0.67$

For M_n

$$a = \frac{A_s f_y}{\alpha_1 f'_c b_f} = \frac{6000(400)}{0.8125(25)(1000)} = 118.2 > h_f$$

$$\therefore a = \frac{A_s f_y - \alpha_1 f'_c (b_f - b_w) h_f}{\alpha_1 f'_c b_w} = \frac{6000(400) - 0.8125(25)(1000-400)100}{0.8125(25)(400)}$$

$a = 145.4\,mm$

$$A_{sf} = \frac{\alpha_1 f'_c (b_f - b_w) h_f}{f_y} = \frac{0.8125(25)(1000-400)(100)}{400} = 3046.9\,mm^2$$

$A_{sw} = 6000 - 3046.9 = 2953.1\,mm^2$

$$M_n = 3046.9(400)\left(900 - \frac{100}{2}\right) + 2953.1(400)\left(900 - \frac{145.4}{2}\right)$$

$= 2010\ kN{\cdot}m$

check yielding of steel.

$c = a/\beta_1 = 145.4/0.9075 = 160.2\ mm$ $\quad \frac{c}{d} = \frac{160.2}{900} = 0.178$

$\frac{700}{700+f_y} = \frac{700}{700+400} = 0.636$

$\therefore \frac{c}{d} < \frac{700}{700+f_y} \Rightarrow \therefore$ underreinforced

For $\underline{\underline{M_R}}$

$\alpha_1 = 0.8125$
$\beta_1 = 0.9075$

$a = \frac{\phi_s A_s f_y}{\alpha_1 \phi_c f'_c b_f} = \frac{0.85(6000)(400)}{0.8125(0.65)(25)(1000)} = 154.5 > h_f$

$\therefore a = \frac{\phi_s A_s f_y - \alpha_1 \phi_c f'_c (b_f - b_w) h_f}{\alpha_1 \phi_c f'_c b_w}$

$= \frac{0.85(6000)(400) - 0.8125(0.65)(25)(1000-400)(100)}{0.8125(0.65)(25)(400)}$

$a = 236.3\ mm$

$C_{cf} = \alpha_1 \phi_c f'_c (b_f - b_w) h_f$

$= 0.8125(0.65)(25)(1000-400)100$

$= 792.2\ kN.$

$C_{cw} = \phi_c \alpha_1 f'_c a (b_w)$

$= 0.65 \times 0.8125 \times 25 \times 236.3 \times 400$

$C_{cw} = 1247.9\ kN$

$M_R = C_{cf}(d - \frac{h_f}{2}) + C_{cw}(d - a/2)$

$= 792.2(900 - \frac{100}{2}) + 1247.9(900 - \frac{236.2}{2})$

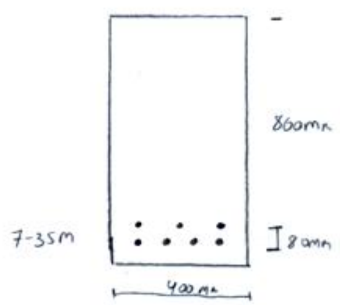
$M_R = 1650.0\ kN.m$

check yielding

$C = \frac{a}{\beta_1} = \frac{236.3}{0.9075} = 260.4$ mm $\qquad \frac{c}{d} = \frac{260.4}{900} = 0.2893$

$0.2893 < \frac{700}{700+900} = 0.636 \rightarrow$ ok

$\boxed{10 \text{ points}}$



$f'_c = 40 MPa$

$f_y = 400 MPa$

$A_s = 7(1000) = 7000 mm^2$

$$d = \frac{3000 \times 860 + 4000(940)}{7000}$$

$d = 906 mm$

$\alpha_1 = 0.85 - 0.0015 f'_c = 0.85 - 0.0015(40) = 0.79$

$\beta_1 = 0.97 - 0.0025 f'_c = 0.97 - 0.0025(40) = 0.87$

$$a = \frac{\phi_s A_s f_y}{\alpha_1 \phi_c f'_c b} = \frac{0.85(7000)(400)}{0.79(0.65)(40)(400)} = 289.68 mm$$

$$M_r = \phi_s A_s f_y (d - a/2) = 0.85(7000)(400)(906 - \frac{289.68}{2})$$

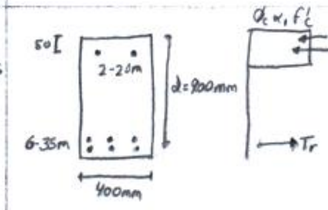
$M_r = 1810$ kN m.

<u>check yielding</u>

$$\frac{a/\beta_1}{d} = \frac{289.68/0.87}{906} = 0.368 < \frac{700}{700+f_y} = 0.636 \quad \therefore ok$$

Pg11

10 points



$f'_c = 40\,mPa$
$f_y = 400\,mPa$
$\phi_c = 0.65$
$\phi_s = 0.85$
$b = 400\,mm$
$h = 940\,mm$

$d = 860 + 40 = 900\,mm$
$A_s = 6000\,mm^2$
$A_s' = 600\,mm^2$
$\alpha_1 = 0.79$
$\beta_1 = 0.87$

$T_r = \phi_s f_y A_s = 0.85(400)(6000) = 2040\,kN$

Force in compression Steel (C'_c)

Assume Compression Steel has yielded $f'_s = f_y$

$T_r' = \phi_s f'_s A_s' = 0.85(400)(600) = 204\,kN$

$$a = \frac{T_r - T_r'}{\alpha_1 \phi_c f'_c b} = \frac{(2040 - 204) \times 10^3}{0.79(0.65)(40)(400)} = 223.466\,mm$$

$$M_r = \phi_s f_s A_s'(d - d') + \phi_s f_y (A_s - A_s')\left(d - \frac{a}{2}\right)$$

Same as

$$M_r = T_r'(d - d') + C_r\,(d - a/2)$$

$$\therefore M_r = 204 \times 10^3 (900 - 50) + 0.85(400)(6000 - 600)\left(900 - \frac{223.466}{2}\right)$$

$= 1620\ kN{\cdot}m$

<u>Yield Check</u>

Strain in compression steel ε_s'

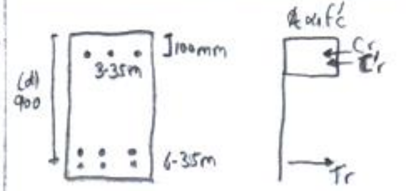
$$\varepsilon_s' = \varepsilon_{c\,max}\left(1 - \frac{d'}{c}\right) = 0.0035\left(1 - \frac{50}{256.86}\right) = 0.00282$$

Yield Strain $\varepsilon_y = \frac{f_y}{E_y} = \frac{400}{200{,}000} = 0.002$

$\varepsilon_s' > \varepsilon_y$ (Compression steel Yields)

Pg 12

10 points



$f_c' = 40\,mPa$
$f_y = 400\,mPa$
$\phi_c = 0.65$
$\phi_s = 0.85$
$b = 400\,mm$
$h = 940\,mm$

$d = 900\,mm$ $\quad A_s = 6000\,mm^2$ $\quad \alpha_1 = 0.69$
$\quad A_s' = 3000\,mm^2$ $\quad \beta_1 = 0.87$

Determine depth of equivalent rectangular stress block "a"

For T_r, assume $f_s = f_y$ (Assuming Tension Steel has yielded)

* $T_r = \phi_s A_s f_y = 0.85(400)(6000) = 2040\,kN$

Force in Compression Steel T_r' (Assume $f_s' = f_y$)

* $T_r' = \phi_s f_s A_s' = 0.85(400)(3000) = 1020\,kN$

Depth of Compression Stress block

$$a = \frac{T_r - T_r'}{\alpha_1 \phi_c f_c' b} = \frac{2040 - 1020}{0.69(0.65)(40)(400)} = 124.148\,mm$$

$$M_r = T_r'(d - d') + C_r\left(d - \frac{a}{2}\right) = T_r'(d - d') + \phi_s f_y (A_s - A_s')\left(d - \frac{a}{2}\right)$$

$$M_r = 1020 \times 10^3 (900 - 100) + 0.85(400)(6000 - 3000)\left(900 - \frac{124.148}{2}\right)$$

$$M_r = 1670 \quad kN\cdot m$$

Strain in Compression Steel

$$\varepsilon_s' = \varepsilon_{cmax}\left(1 - \frac{d'}{c}\right) = 0.0035\left(1 - \frac{100}{142.7}\right) = 0.0010473$$

$c = \frac{a}{\beta_1} = \frac{124.148}{0.87}$ $\qquad \varepsilon_y = \frac{400}{200{,}000} = 0.002$

$c = 142.7$

$\varepsilon_s' < \varepsilon_y$ Compression steel does not yield
(NOT OK)

Strain in Tension Steel

$$\frac{\varepsilon_s}{d-c} = \frac{\varepsilon_{cmax}}{c} \qquad \varepsilon_s = \left(\frac{d-c}{c}\right)\varepsilon_{cmax}$$

$$= \left(\frac{250-142.7}{142.7}\right)(0.0035)$$

$$\varepsilon_s = 0.002632 > 0.002$$

Tension Steel yields (o.k.)

Recompute c and fs

$$C_r + T_r' = T_r$$

$$\alpha_1 \phi_c f_c' b \beta_1 c + E_s \varepsilon_s' \phi_s A_s' = \phi_s A_s f_y \quad \left(\begin{array}{c}\text{Ignoring strain}\\ \text{hardening}\end{array}\right)$$

$$0.79(0.65)(40)(400)0.87\,c + 2\times10^5(0.0035)(3000)\left(1-\frac{100}{c}\right)(0.85)$$

$$= 0.85(400)(6000)$$

$$\Rightarrow 7147.92\,c + 1.785\,10^6\left(1-\frac{100}{c}\right) = 2.04\times10^6$$

$$7147.92\,c - \frac{1.785\times10^6}{c} = 255 \times10^3$$

$$7147.92\,c^2 - 255 \times10^3 c - 1.785\times10^6 = 0$$

$$c = 176.87 \qquad a = \beta_1 c = 0.87(176.87) = 153.9\text{mm}$$

$$\varepsilon_s' = 0.0035\left(1-\frac{100}{176.87}\right) = 0.00152$$

$$f_s' = 2\times10^5(0.00152) = 304.2\text{ mPa}$$

$$C_r' = 0.85(304.2)(3000) = 775.8 \times10^3\text{kN}$$

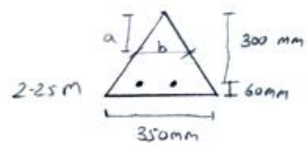
$$M_r = C_r'(d-d') + \phi_c \alpha_1 f_c' b a \left(d-\frac{a}{2}\right)$$

$$M_r = 775.8\times10^3(900-100) + 0.65(0.79)(40)(400)(153.9)\left(900-\frac{153.9}{2}\right)$$

$$= 1661 \text{ kN.m}$$

Pg. 14

$\boxed{\text{10 points}}$



$\alpha_1 = 0.79$ $\quad f'_c = 40MPa$ $\quad A_s = 2 \cdot 500 = 1000\ mm^2$

$\beta_1 = 0.87$ $\quad f_y = 400\ MPa$

Using similar triangles

$C_c = T_s$ $\qquad \frac{350}{360} = \frac{b}{a} \Rightarrow b = \frac{350}{360}(a)$

$\phi_c \alpha_1 f'_c\ a\ b \frac{1}{2} = \phi_s A_s f_y$

$\phi_c \alpha_1 f'_c\ a^2 \frac{350}{360} \cdot \frac{1}{2} = \phi_s A_s f_y$

$$a^2 = \frac{720\ \phi_s A_s f_y}{\phi_c \alpha_1 f'_c (350)} = \frac{720\ (0.85)(1000)(400)}{0.65\ (0.79)(40)(350)}$$

$a = 184.5\ mm$

$$M_r = \phi_s A_s f_y \left(d - \tfrac{2}{3}a\right) = 0.85\ (2)(500)(400)\left(300 - \tfrac{2}{3}(184.5)\right)$$

$M_r = 60.18\ kN\,m$

check yielding

$\frac{a/\beta_1}{d} = \frac{184.5/0.87}{300} = 0.7 > 0.636$ $\quad$ NOT ok

Recompute c

$$\alpha_1 \phi_c f'_c \frac{350}{360} \frac{(\beta_1 c)^2}{2} = E_s \varepsilon_s \phi_s A_s$$

$$0.79\,(0.65)(40) \frac{350}{360} \cdot \frac{(\beta_1 c)^2}{2} = 2\times10^5\ (0.0035)\left(\frac{300 - c}{c}\right)(0.85)(1000)$$

$$7.557\,c^3 + 595000\,c - 178{,}500{,}000 = 0$$

$$c = 199.4 \text{ mm}$$

$$a = \beta_1 c = 0.87\,(199.4) = 173.5 \text{ mm}$$

$$\varepsilon_s = 0.0035\left(\frac{300 - 199.4}{199.4}\right) = 0.00177$$

$$f_s = \varepsilon_s E_s = 0.00177\,(2\times10^5) = 353 \text{ MPa}$$

$$M_r = A_s \phi_s f_s \left(d - \tfrac{2}{3}a\right)$$

$$= 2(500)(0.85)(353)\left(300 - \tfrac{2}{3}(173.5)\right)$$

$$M_r = 55.3 \text{ kN.m}$$