CIVE 3206 ASSIGNMENT 3 BY: TOMAS PLESNIK

WINTER 2013 SOLUTION 100741826

QUESTION 1a&b

$f'_c = 30 MPa$

$\varepsilon'_c = -2.0\times10^{-3}$

$E_s = 200\,000 MPa$

$f_y = 400 MPa$

$E_c = 5500\sqrt{f'_c} = 5500\sqrt{30} = 30125 MPa$

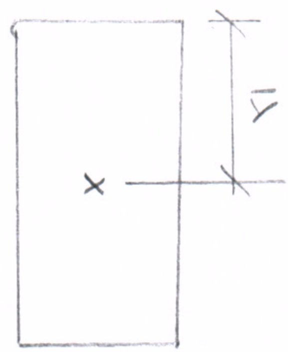
$f_r = 0.6\sqrt{f'_c} = 0.6\sqrt{30} = 3.3 MPa$

$A_s = 3(700mm^2) = 2100\ mm^2$

$$n = \frac{E_s}{E_c} = \frac{200\,000 MPa}{30125 MPa} = 6.64$$

$n-1 = 5.64$

Gross Properties



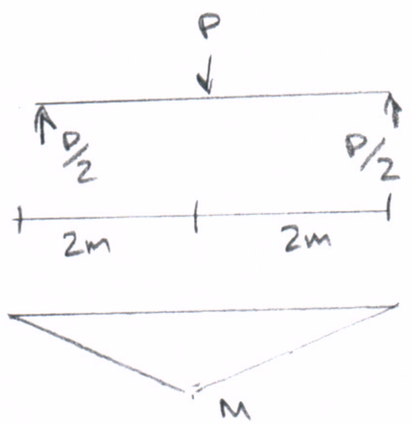
$$\bar{y} = \frac{h}{2} = \frac{500}{2} = 250mm$$

$$I_{gr} = \frac{bh^3}{12} = \frac{(300)(500)^3}{12} = 3.125\times10^9\ mm^4$$

$$M_{cr} = \frac{f_{cr} I_g}{\bar{y}} = \frac{(3.3MPa)(3.125\times10^9)}{250mm\ (10^6)} = 41.1kNm$$

$$\phi_{cr} = \frac{M_{cr}}{E_c I_g} = \frac{41.1kNm\ (10^6)}{(30125MPa)(3.125\times10^9)}$$

$$= 0.437\times10^{-6}\ rad/mm$$



M_{max} @ midspan.

$$M = \frac{P}{2}\cdot 2m = P(1m)$$

$$P = \frac{41.1kNm}{1m} = 41.1kN$$

Transformed Properties

$$\bar{y} = \frac{bh(h/2) + (n-1)A_s(d)}{bh + A_s(n-1)} = \frac{(300)(500)(\frac{500}{2}) + (5.64)(2100)(440)}{(300)(500) + (5.64)(2100)}$$

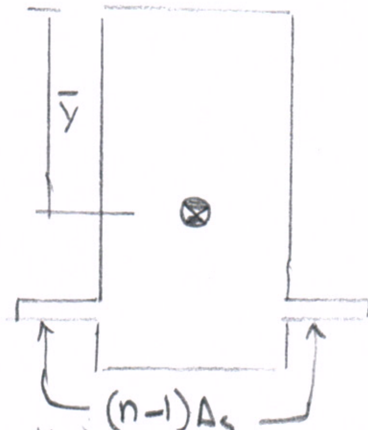
$$\bar{y} = 263.9 mm$$

$$I_{tr} = \frac{bh^3}{12} + bh\left(\frac{h}{2} - \bar{y}\right)^2 + (n-1)A_s\left(d - \bar{y}\right)^2$$

$$= \frac{(300)(500)^3}{12} + (300)(500)\left(\frac{500}{2} - 263.9\right)^2$$

$$+ (5.64)(2100)\left(440 - 263.9\right)^2$$

$$= \left[3.125 + 0.029 + 0.367\right] \times 10^9 = 3.52 \times 10^9 \, mm^4$$



$$M_{cr} = \frac{f_r I_{tr}}{\bar{y}} = \frac{(3.3\,MPa)(3.52 \times 10^9\,mm^4)}{(500 - 263.9\,mm)} = 49.2\,kNm$$

$$\phi_{cr} = \frac{M_{cr}}{E_c I_{tr}} = \frac{49.2\,kNm\,(10^6)}{(30125\,MPa)(3.52 \times 10^9\,mm^4)} = 0.464 \times 10^{-6}\,\frac{rad}{mm}$$

P at midspan:

$$P = \frac{M}{1m} = \frac{49.2\,kNm}{1m} = 49.2\,kN$$

(c) $$\frac{f_s}{n} = \frac{M(d-\bar{y})}{I_{tr}}$$

$$f_s = \frac{nM(d-\bar{y})}{I_{tr}} = \frac{(6.64)(49.2\,kNm)(440 - 263.9\,mm)(10^6)}{3.52 \times 10^9\,mm^4}$$

$$= 16.3\,MPa$$

(d) $$P = \frac{49.2\,kN}{2} = 24.6\,kN \rightarrow M = P(1m) = 24.6\,kNm$$

$$f_s = \frac{nM(d-\bar{y})}{I_{tr}} = \frac{(6.64)(24.6)(440-263.9)(10^6)}{3.52 \times 10^9}$$

$$= 8.17\,MPa$$

QUESTION 2

– Same properties as Q1 (f'_c, ε'_c, E_s, f_y, A_s)

$$E_c = 4500\sqrt{f'_c} = 4500\sqrt{30} = 24650 \text{ MPa}$$

$$n = \frac{E_s}{E_c} = \frac{200\,000}{24650} = 8.11$$

$$\rho = \frac{A_s}{bd} = \frac{2100\text{ mm}^2}{(300\text{mm})(440\text{mm})} = 0.0159 \quad (1.59\%) \qquad n\rho = 0.12895$$

$$k = -n\rho + \sqrt{(n\rho)^2 + 2(n\rho)} = -0.12895 + \sqrt{(0.12895)^2 + 2(0.12895)}$$

$$= 0.395$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.395}{3} = 0.868$$

when stress in concrete is 15 mpa:

$$f_{c,top} = \frac{2M}{jkbd^2}$$

$$15\text{MPa} = \frac{2M}{(0.868)(0.395)(300)(440)^2} \rightarrow \boxed{M = 149.3\text{ kNm}}$$

governs

when stress in steel is 200 MPa:

$$f_s = \frac{M}{A_s jd}$$

$$200\text{ MPa} = \frac{M}{(2100)(0.868)(440)} \rightarrow M = 160.4\text{ kNm}$$

corresponding P:

$$P = \frac{M}{1\text{m}} = \frac{149.3\text{kNm}}{1\text{m}} = 149.3\text{kN}$$

QUESTION 2 - ALTERNATE SOLUTION

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d-kd)^2$$

$$= \frac{(300)(0.395 \times 440)^3}{3} + (8.11)(2100)(440 - 0.395 \times 440)^2$$

$$= \left[0.525 + 1.207\right] \times 10^9 = 1.732 \times 10^9 \text{ mm}^4$$

$$f_{c,top} = \frac{M\,kd}{I_{cr}}$$

$$15\text{ MPa} = \frac{M(0.395 \times 440)}{1.732 \times 10^9} \rightarrow \boxed{M = 149.5\text{ kNm}}$$

governs

$$f_s = \frac{nM(d-kd)}{I_{cr}}$$

$$200\text{ MPa} = \frac{(8.11)M(440 - 0.395 \times 440)}{1.732 \times 10^9} \rightarrow M = 160.5\text{ kNm}$$

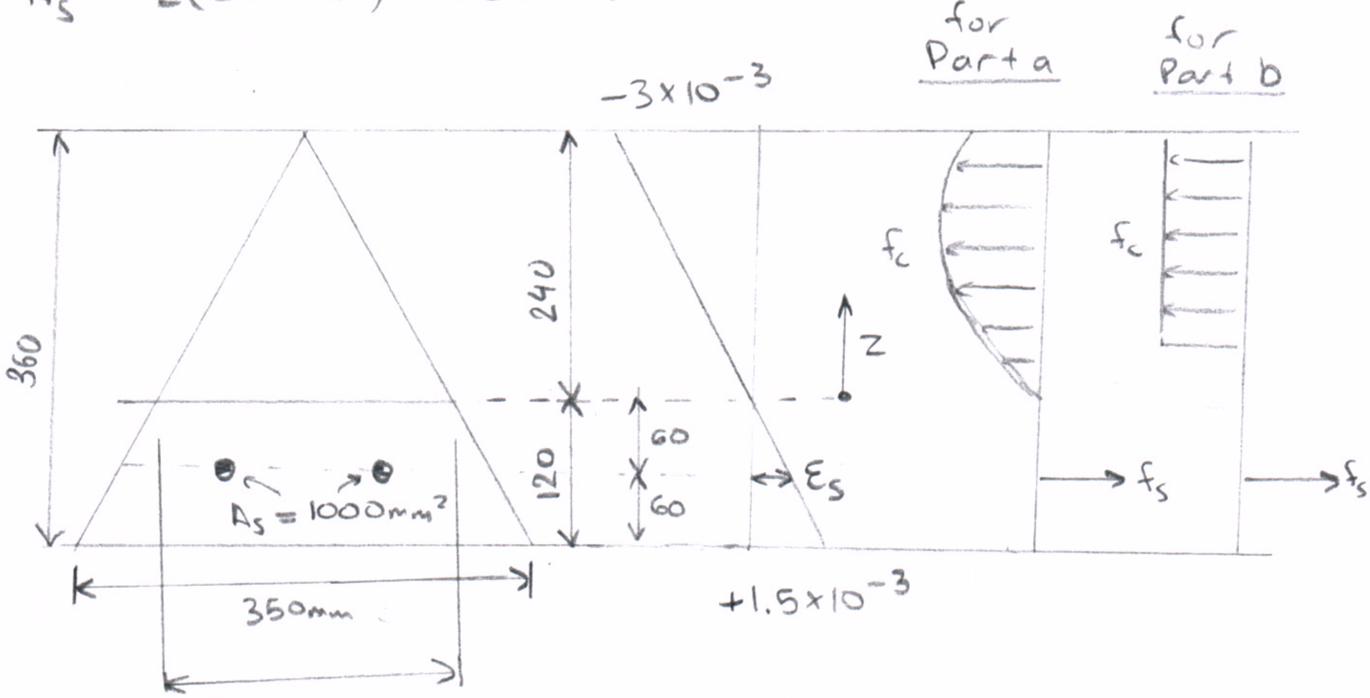
$$P = \frac{M}{1\text{m}} = \frac{149.5\text{ kNm}}{1\text{m}} = 149.5\text{ kN}$$

QUESTION 3a

$f'_c = 30\,MPa$ $\quad E_s = 200\,000\,MPa$

$f_y = 400\,MPa$

$A_s = 2(500\,mm^2) = 1000\,mm^2$



$$\frac{240}{360} \times 350 = 233\,mm$$

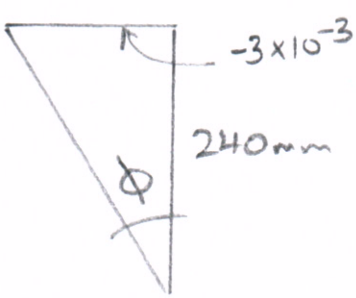
$$\varepsilon_s = +1.5\times10^{-3}\left(\frac{60\,mm}{120\,mm}\right) = +0.75\times10^{-3} < 0.002 = \frac{f_y}{E_s}$$

$\therefore$ steel does not yield.

$$f_s = \varepsilon_s E_s = (0.75\times10^{-3})(200\,000\,MPa) = 150\,MPa$$

$$T_s = f_s A_s = (150\,MPa)(1000\,mm^2) = 150\,kN$$ ← force in steel (tension)

$$\phi = \frac{-3\times10^{-3}}{240\,mm} = -1.25\times10^{-5}\ rad/mm$$



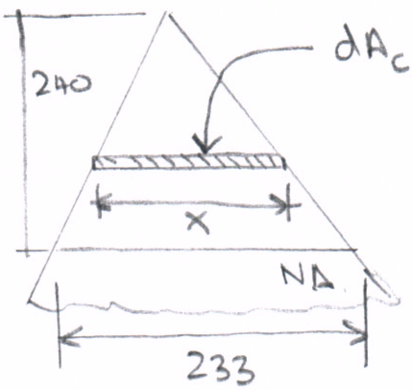
$$\varepsilon_c(z) = \phi z$$
$$= -1.25\times10^{-5} z$$

$$f_c(z) = -f'_c\left[2\left(\frac{\varepsilon'_c(z)}{\varepsilon'_c}\right) - \left(\frac{\varepsilon'_c(z)}{\varepsilon'_c}\right)^2\right]$$ ~ hognestad parabola

$$= -30\left[2\left(\frac{-1.25\times10^{-5}z}{-0.002}\right) - \left(\frac{-1.25\times10^{-5}z}{-0.002}\right)^2\right]$$

$$f_c(z) = -0.375z + 1.1719\times10^{-3}z^2$$

stress in concrete at a location z, above N.A.



$dA_c = x\,dz$

$$x = 233.3 - \frac{233.3}{240}z$$

$$x = 233.3 - 0.9721z$$

$$dA_c = (233.3 - 0.9721z)\,dz$$

$$C_c = \int_{A_c} f_c(z)\,dA_c$$

$$= \int_{z=0}^{z=240} (-0.375z + 1.1719\times10^{-3}z^2)(233.3 - 0.9721z)\,dz$$

$$= \int_0^{240} (-87.53z + 0.6381z^2 - 0.001139z^3)\,dz$$

$$= \left[-43.767z^2 + 0.2127z^3 - 0.000285z^4\right]_0^{240}$$

$$C_c = -525353 = -525.3\text{ kN}$$ ← force in concrete (compression)

$$N = T_s + C_c$$

$$= 150\text{ kN} + (-525.3\text{ kN}) = -375\text{ kN}$$ ← Axial force

moment – steel component

$$\int_{A_s} f_s z \, dA_s = (150\,kN)(120 - 60\,mm) = 9.00\,kNm$$

moment – concrete component

$$\int_{A_c} (f_c(z))\, dA_c = \int_0^{240} (-87.534z + 0.6381z^2 - 0.001139z^3)\, z\, dz$$

$$= \left[-29.1781z^3 + 0.15953z^4 - 0.000228z^5 \right]_0^{240}$$

$$= 55450460 = +55.45\,kNm$$

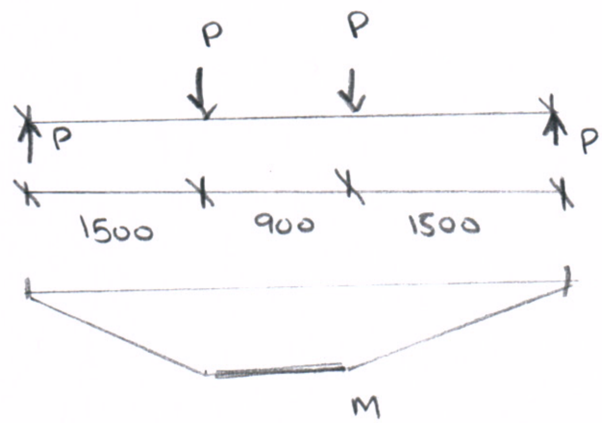
$$M = 9.00 + 55.45 = 64.45\,kNm$$

location

$$\bar{z} = \frac{\int_{A_c} F_c(z)\, dA_c}{\int_{A_c} F_c\, dA_c} = \frac{55.45\,kNm}{525.3\,kN} = 0.1055\,m$$

$$= 105\,mm$$

Force P



$$M_{max} = P(1500\,mm)$$

$$P = \frac{M}{1.5m}$$

$$= \frac{64.45\,kNm}{1.5m}$$

$$= 43.0\,kN$$

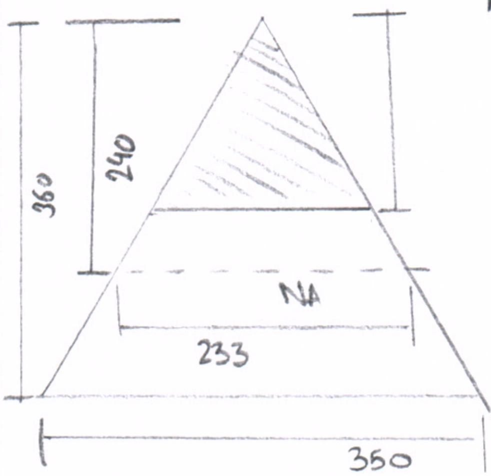
(b)

$$\frac{\varepsilon_{cc}}{\varepsilon_c'} = \frac{-3.0\times10^{-3}}{-2.0\times10^{-3}} = 1.5$$

$$\alpha_1\beta_1 = \left(\frac{\varepsilon_{cc}}{\varepsilon_c'}\right) - \frac{1}{3}\left(\frac{\varepsilon_{cc}}{\varepsilon_c'}\right)^2 = 1.5 - \frac{1}{3}(1.5)^2 = 0.75$$

$$\beta_1 = \frac{4-\left(\frac{\varepsilon_{cc}}{\varepsilon_c'}\right)}{6-\left(\frac{\varepsilon_{cc}}{\varepsilon_c'}\right)2} = \frac{4-1.5}{6-(1.5)2} = 0.833 \qquad \alpha_1 = \frac{0.75}{0.833} = 0.90$$

Stress block



Area of stress block (triangle)

$$= \frac{bh}{2} = \frac{[\beta(233)][\beta(240)]}{2}$$

$$= \frac{(0.833)(233)(0.833)(240)}{2}$$

$$= 19401\ \text{mm}^2$$

$$C_c = \alpha_1 f_c' (19401\ \text{mm}^2) = (0.90)(30\text{MPa})(19401\ \text{mm}^2)$$

$$= 523831 = 524\ \text{kN} \quad \text{(compression)}$$

$T_s = 150$ kN (from part a)

$N = 150 - 524 = -374$ kN ≈ -375 kN (direct integration)

Moment

$$M = C_c\left(c - \frac{2}{3}\beta_1 c\right) + T_s(d-c)$$

$$= 524\,\text{kN}\left(240 - \frac{2}{3}(0.833)240\right) + 150\,\text{kN}(300-240)$$

$$= 56.03 + 9.00 = 65.0\ \text{kNm}$$

$$P = \frac{M}{1.5\text{m}} = \frac{65.0}{1.5} = 43.3\ \text{kN}$$

MARKING

Q1 – part a & b	–	20	30
part c	–	5	
part d	–	5	
Q 2	–	15	15
Q3 – part a	–	30	45
part b	–	15	
"Professionalism"	–	10	
		100	