Student ID: Quiz 5 Name: SOLUTIONS

MA205B - Differential Equations I - Quiz 5

TUESDAY, NOVEMBER 27, 2012

Consider the following differential equation:

$$y' - 3x^2 y = 0.$$

Assume that the solution y can be written as a power series about $x_0 = 0$.

(7) **a.** Find a recurrence relation for the coefficients in the power series.

(3) **b.** Find the first 5 non-zero terms in the power series.

Let $y = \sum_{n=0}^{\infty} a_n x^n$

Then $y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$

So, $\sum_{n=1}^{\infty} n a_n x^{n-1} - 3x^2 \sum_{n=0}^{\infty} a_n x^n = 0$

$\sum_{n=1}^{\infty} n a_n x^{n-1} - \sum_{n=0}^{\infty} 3 a_n x^{n+2} = 0$

Let $k = n-1$, $n = k+1$ ($n=1 \Rightarrow k=0$)

Let $k = n+2$, $n = k-2$ ($n=0 \Rightarrow k=2$)

$\sum_{k=0}^{\infty} (k+1) a_{k+1} x^k - \sum_{k=2}^{\infty} 3 a_{k-2} x^k = 0$

$a_1 + 2a_2 x + \sum_{k=2}^{\infty} \left[(k+1) a_{k+1} - 3 a_{k-2} \right] x^k = 0$

(a_1: $k=0$; $2a_2 x$: $k=1$)

Coef of x^0: $a_1 = 0$

Coef of x^1: $2a_2 = 0 \Rightarrow a_2 = 0$ } (*)

Coef of x^k for $k \geq 2$: $(k+1) a_{k+1} - 3 a_{k-2} = 0 \Rightarrow a_{k+1} = \frac{3}{k+1} a_{k-2}$

Let $n = k+1$, $k = n-1$ ($k \geq 2 \Rightarrow n \geq 3$)

$\Rightarrow$ **Recurrence Relation:** $a_n = \frac{3}{n} a_{n-3}$ for $n \geq 3$

$a_0 \rightarrow$ no information

$a_1 = 0$
$a_2 = 0$
} from (*)

Now use the recurrence relation

$$a_n = \frac{3}{n} a_{n-3} \quad \text{for} \quad n \geq 3 \qquad (**)$$

Since $a_1 = 0$, (**) implies $0 = a_1 = a_4 = a_7 = \cdots$

Since $a_2 = 0$, (**) implies $0 = a_2 = a_5 = a_8 = \cdots$

$$a_3 = \frac{3}{3} a_0 = a_0$$

$$a_6 = \frac{3}{6} a_3 = \frac{1}{2} a_0$$

$$a_9 = \frac{3}{9} a_6 = \frac{1}{6} a_0$$

$$a_{12} = \frac{3}{12} a_9 = \frac{1}{24} a_0$$

So, $y = a_0 \left[1 + x^3 + \frac{1}{2} x^6 + \frac{1}{6} x^9 + \frac{1}{24} x^{12} + \cdots \right]$