Student ID: _____ Name: _____

MA205B - Differential Equations I - Quiz 4

TUESDAY, NOVEMBER 20, 2012

$$f(x) = \sum_{n=0}^{\infty} \frac{2^{-n}}{n+1}(x-5)^n$$

$x_0 = 5$

(5) **1.** Find the interval of convergence of f.

(3) **2.** Find a power series for $f'(x)$.

(2) **3.** Find the interval of convergence of f'.

1. Use the Ratio Test.

Let $L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$$= \lim_{n\to\infty} \frac{2^{-(n+1)}}{n+1} \cdot \frac{n}{2^{-n}}$$

$$= \lim_{n\to\infty} \frac{n}{2\cdot(n+1)}$$

$$= \frac{1}{2}$$

The radius of convergence is $\rho = \frac{1}{L} = 2$

The interval of convergence is $(5-2, 5+2)$

$= (3, 7)$.

$$f(x) = \sum_{n=0}^{\infty} \frac{2^{-n}}{n+1}(x-5)^n$$

2. $f'(x) = \sum_{n=1}^{\infty} \frac{2^{-n} \cdot n}{n+1}(x-5)^{n-1}$

3. $(3, 7)$ (Radius of convergence for f' is always the same as for f.)