Quiz 3 Solutions

Student ID: _____ Name: _____

MA205B - Differential Equations I - Quiz 3

TUESDAY, NOVEMBER 13, 2012

1. Use Laplace transforms to solve the following initial value problem.

$$y'' + 6y' + 5y = 12e^t, \qquad y(0) = -1, \quad y'(0) = 7$$

Let $Y(s) = \mathcal{L}\{y(t)\}$.

Then $\mathcal{L}\{y'(t)\} = sY(s) - y(0) = sY(s) + 1$

and $\mathcal{L}\{y''(t)\} = s^2Y(s) - sy(0) - y'(0) = s^2Y + s - 7$

$$(s^2Y + s - 7) + 6(sY + 1) + 5Y = \frac{12}{s-1}$$

$$(s^2 + 6s + 5)Y + s - 1 = \frac{12}{s-1}$$

$$(s^2+6s+5)Y = -s + 1 + \frac{12}{s-1} = \frac{-s^2 + 2s + 11}{s-1}$$

$$Y = \frac{-s^2 + 2s + 11}{(s^2+6s+5)(s-1)} = \frac{-s^2 + 2s + 11}{(s+5)(s+1)(s-1)}$$

$$= \frac{A}{s+5} + \frac{B}{s+1} + \frac{C}{s-1}$$

$$= \frac{A(s+1)(s-1) + B(s+5)(s-1) + C(s+5)(s+1)}{(s+5)(s+1)(s-1)}$$

So $-s^2+2s+11 = A(s+1)(s-1) + B(s+5)(s-1) + C(s+5)(s+1)$

$s=1 \Rightarrow \quad 12 = 12C \quad \Rightarrow \quad C = 1$

$s=-1 \Rightarrow \quad 8 = -8B \quad \Rightarrow \quad B = -1$

$s=-5 \Rightarrow \quad -24 = 24A \quad \Rightarrow \quad A = -1$

So, $Y(s) = -\frac{1}{s+5} - \frac{1}{s+1} + \frac{1}{s-1}$

So, $y(t) = \mathcal{L}^{-1}\{Y(s)\}$

$= -e^{-5t} - e^{-t} + e^{t}$